Colonies – the simplest grammar systems

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Abstract. Brief introduction to the colonies, the grammars systems interacting on common passive environment with components individually producing finite languages, is presented. An overview of the topic is completed by large number of references.

Keywords: grammar system, colony, generative power, hierarchy

1 Introduction

Colonies were introduced in [19], where basic fact on the motivation can be found. Overviews on the topic are in [20], [29]. The present paper was prepared to introduce the topic, namely to call attention to sequential and parallel models of colonies. It ends with the list of research topics inside the theory of colonies and with actual open problems. For the further information on the topic we recommend www.sztaki.hu/mms/bib.html, where also abstracts of the listed papers can be found.

2 Colony – general model

A colony is a collection of (very simple) grammars operating in a common string. By "very simple" we mean a grammar producing *a finite language*.

First, we present general model of a colony. Different acting possibilities realized by different derivation steps, due to different motivations lead to various variants of colonies.

Definition 1. *A colony* $\mathcal{C}$ *is a 3-tuple* $\mathcal{C} = (V, T, \mathcal{F})$, *where*

(i) V *is an alphabet of the colony, and*
(ii) $T \subseteq V$ *is a terminal alphabet of the colony,*
(iii) $\mathcal{F} = \{(S_i, F_i) : S_i \in V, F_i \subseteq (V - S_i)^*,\ F_i \text{ is finite },\ 1 \leq i \leq n\ \}$.
A pair (S_i, F_i) *is called i-th component of* $\mathcal{C}$, S_i *is its start symbol and* F_i *is the language of i-th component.*

Note 1. a) Any symbol of the strings from F_i can occur as a start symbol S_j of another component of the colony, $j \neq i$.
b) Components can be determined in more detail, by complete specifications of (regular) grammars $G_i = (N_i, T_i, S_i, P_i)$ producing from S_i words of F_i, $L(G_i) = F_i$. See the original definition in [19]. This case is useful for study processes in the level of components.
c) By the definition, $V = T$ is also possible. This special case is discussed in [28], [10].

An activity of components in a colony is realized by string transformation on a common tape.

Generally, denote by $\overset{x}{\Longrightarrow}$ the relation on strings representing an elementary string transformation realized by components and called a derivation step. Specific types of x introduced later define various variants of colonies.

As usual, $\overset{x}{\Longrightarrow}^*$ stays for a reflexive and transitive closure of $\overset{x}{\Longrightarrow}$. It represents string transformations, called derivation, realized by finite sequences of elementary transformations.

Language $L_x(\mathcal{C}, w_0)$ determined by a colony $\mathcal{C} = (V, T, \mathcal{F})$ and an initial string (axiom) $w_0 \in V^*$ by $\overset{x}{\Longrightarrow}$ derivation consists of all terminal strings derived from the axiom, i.e.

$$L_x(\mathcal{C}, w_0) = \{v | \ w_0 \overset{x}{\Longrightarrow}^* v, \ v \in T^*\}.$$

By COL_x we denote the class of all languages generated by colonies with $\overset{x}{\Longrightarrow}$ derivation.

The subscript x, above, can be omitted in the case when it is clear which derivation step is considered.

Basic differences among various possibilities of a behaviour of a colony, due to the number of components used in one step. The sequential model and parallel models discussed in the next sections are two examples of behaviour of colonies. The intermediate cases, colonies working in teams are discussed in [10], [42].

3 Sequentional Colonies

In the sequential models of a colony an elementary change of strings is realized by single component. Sequential colonies can differ in the amount of symbols rewritten by a component in one derivation step.

Basic derivation step $\overset{b}{\Longrightarrow}$ corresponds to rewriting a single symbol. Total derivation step $\overset{t}{\Longrightarrow}$ corresponds to a parallel behaviour of the chosen component.

3.1 Sequential colonies with sequentialy acting components b mode of derivation

The simplest case of rewriting in sequential colonies is the case when chosen component rewrites exactly one letter in a derivation step. Formally:

Definition 2. *For $x, y \in V^*$ we define basic derivation step:*

$$x \overset{b}{\Longrightarrow} y \text{ iff } x = x_1 S_i x_2,\ y = x_1 z x_2 \text{ and } z \in F_i \text{ for some } i, 1 \leq i \leq n.$$

In accordance with general case, the language of the colony with derivation step $\overset{b}{\Longrightarrow}$ is denoted $L_b(\mathcal{C}, w_0)$ and COL_b is the corresponding class of all languages.

Theorem 1. $COL_b = CF$

Proof. Let $L \in CF$. There is a context free grammar $G = (N, T, P, S)$ such that $L = L(G)$. To construct a colony $\mathcal{C} = (V, T, \mathcal{F})$ such that $L(G) = L_b(\mathcal{C}, w)$ for some w, we have to determine finite languages of components. To do it we eliminate of all rules with direct recursion from G. Let $\bar{N} = \{Z_A : A \in N\}$, where Z_A are new nonterminals.

We replace rules of the type $A \to \alpha$, where A occurs in α by two rules
$A \to \alpha_A, Z_A \to A$,
where α_A denotes the word obtained from α by replacing all occurrences of A by Z_A.
Denote by $\bar{G}$ resulting grammar $\bar{G} = (N \cup \bar{N}, T, \bar{P}, S)$. Evidently $L(G) = L(\bar{G})$.
Consider the colony $\mathcal{C} = (V, T, \mathcal{F})$, where $V = N \cup \bar{N} \cup T$ and
$\mathcal{F} = \{(X, F_X) :\ X \in N \cup \bar{N}, F_X = \{\alpha : X \to \alpha \in \bar{P}\}$.

Evidently, $\mathcal{C}$ is the colony and $L(\bar{G}) = L_b(C, S)$.

Let $L \in COL_b$. There is a colony $\mathcal{C} = (V, T, \mathcal{F})$ and an axiom w_0 such that $L = L_b(C, w_0)$. To construct an equivalent context-free grammar we have to determine the set of nonterminals N to be strictly disjoin with the terminal set T. Consider $\bar{V} = \{\bar{X} : X \in V \cap T\} \cup \{S\}$, where $\bar{X}$ are new pairwise different nonterminals and $S \notin V$ is a new start symbol.

Associate with a word $w \in V^*$ following set of words
$W = \{u_1 \bar{x}_1 \ldots u_n \bar{x}_n u_{n+1} : w = u_1 x_1 \ldots u_n x_n u_{n+1},\ u_i \in V^*,\ x_i \in V \cap T\}$.

Words $u \in W$ differs from w only in some letters from $V \cap T$, not necessary all, which are replaced by corresponding barred letters.

Construct the grammar $G = ((V - T) \cup \bar{V}, T, \bar{P}, S)$, where
$\bar{P} = \{S \to w_0\} \cup \{S \to u : u \in W_0\}$
$\cup\ \{X \to u : X \in V - T,\ (X, F_X) \in \mathcal{F},\ w \in F_X,\ u \in W\ \}$
$\cup\ \{\bar{X} \to u : X \in V \cap T,\ (X, F_X) \in \mathcal{F},\ w \in F_X,\ u \in W\}$.

It means that rules of the grammar are constructed from components of the colony in such a way, that they rewrite the start symbol of the component by a word from associated finite language. In case when original start symbol of some component was also a terminal symbol, corresponding barred letter is used for the derivation. Evidently, $L(G) = L_b(\mathcal{C}, w)$. □

Note 2. In the previous proof we presented possibility to transform a colony to an equivalent context-free grammar and vice versa. This makes possible to transform many known results on context-free grammars and languages to colonies. Normal form theorems, results on ε-rules and many others are examples of above mentioned properties. Descriptional complexity of colonies measured by the number of components necessary to produce the language is discussed in [19].

The case $T = V$ and axiom is a letter, corresponds to languages of the sentential forms.

3.2 Sequential colonies with parallely acting components t mode of derivation

In this section we discuss a sequential model of colony with parallelism inside the components. Parallelism at the level of a single component, denoted here by $\stackrel{t}{\Longrightarrow}$, means that in one derivation step one component rewrites all appearances of its start symbol S_i in an actual string to a (not necessary same) word of F_i.

Definition 3. *For a colony $\mathcal{C} = (V, T, \mathcal{F})$ and $x, y \in V^*$ we define a terminal derivation step*

$$\begin{aligned} x \stackrel{t}{\Longrightarrow} y \quad \text{iff}\, x &= x_1 S_i x_2 S_i x_3 \dots x_m S_i x_{m+1}, \\ & x_1 x_2 \dots x_{m+1} \in (V - \{S_i\})^*, \\ & y = x_1 w_1 x_2 w_2 x_3 \dots x_m w_m x_{m+1}, \\ & \text{where } w_j \in F_i, \text{ for each } j, 1 \le j \le m \\ & \text{and for some } i, 1 \le i \le n. \end{aligned}$$

In accordance with general case, a language of a colony with derivation step $\stackrel{t}{\Longrightarrow}$ is denoted $L_t(\mathcal{C}, w_0)$ and COL_t is the corresponding class of all languages.

Example 1. Consider a colony with

- $V = \{A, B, a\}$,
- $T = \{a\}$ and
- $\mathcal{F} = \{(A, \{BB\}), (B, \{A\}), (B, \{a\})\}$.

$A \stackrel{t}{\Longrightarrow} BB \stackrel{t}{\Longrightarrow} AA \stackrel{t}{\Longrightarrow} BBBB \stackrel{t}{\Longrightarrow} aaaa$
is an example of derivation in $\mathcal{C}$.

$$L_t(\mathcal{C}, A) = \{a^{2^i} : i \ge 1\}.$$

We compare class COL_t with class COL_b as well as with classes of languages generated by other known models of parallel grammars, namely L systems and Indian parallel grammars.

Theorem 2. [10] $COL_b \subset COL_t$

Proof. Let $\mathcal{C}$ be a colony with b mode of derivation. Let $\bar{\mathcal{C}}$ be a colony equivalent to $\mathcal{C}$ with all components having different start symbols. We can construct such a colony by cumulating all components of $\mathcal{C}$ with the same start symbol S to one component (S, F) of $\bar{\mathcal{C}}$. Set F of new component is the union of the original sets F_j of cumulated components from $\mathcal{C}$.

It holds $L_b(\bar{\mathcal{C}}, w) = L_t(\bar{\mathcal{C}}, w)$, i.e. the words produced by a colony $\bar{\mathcal{C}}$ with b mode of derivation are identical with that of t mode of derivation. The derivations of the same word in these colonies can differ only in their length.

The inclusion in the theorem is proper. The language in presented in previous example is in $COL_t - COL_b$. □

Furthermore we compare colonies with t mode of derivation with languages determined by Indian parallel grammars.

An Indian parallel grammar is a quadruple $G = (N, T, P, S)$ specified as in CF grammars with derivation step $\stackrel{ip}{\Longrightarrow}$:

$$\begin{aligned} x \stackrel{ip}{\Longrightarrow} y \ \text{ iff } & x = x_1 A x_2 A x_3 \ldots x_m A x_{m+1}, \\ & y = x_1 w x_2 w x_3 \ldots x_m w x_{m+1}, \\ & x_1 x_2 \ldots x_{m+1} \in (V - \{A\})^* \\ & \text{and } A \to w \in P, \text{ for some } A \in N. \end{aligned}$$

By $L(IP)$ we denote the class of all languages generated by Indian parallel grammar.

Note 3. [13] $L(IP)$ and CF are incomparable.

Theorem 3. $L(IP) \subset COL_t$

Proof. Let L be a language given by an Indian parallel grammar $G = (N, T, P, S)$. We can assume that none of the rules contain direct recursion, i.e. for $A \to \alpha \in P$, α does not contain A.
To construct $\mathcal{C} = (V, T, \mathcal{F})$ we put $V = T \cup N$ and for every $A \to \alpha \in P$ the pair $(A, \{\alpha\}) \in \mathcal{F}$. We have $L_t(\mathcal{C}, S) = L$.
Inclusion in the theorem is proper. Example of the language in $COL_t - L(IP)$ is the Dyck language. □

Another type of parallel grammars are L-systems.

An $ET0L$ system is a quadruple $G = (V, T, \mathcal{P}, S)$, where V is a total alphabet, $T \subset V$ is a terminal alphabet, $S \in V$, $\mathcal{P} = \{P_i : 1 \le i \le n\}$ and $P_i = \{A \to w : A \in V, w \in V^*\}$.
Moreover, P_i, $1 \le i \le n$ contains at least one rule for each $A \in V$.
Derivation step $\stackrel{L}{\Longrightarrow}$ of L system specifies totally parallel rewriting:

$x \stackrel{L}{\Longrightarrow} y$ iff $x = x_1 x_2 \ldots x_m$, $y = y_1 y_2 \ldots y_m$, where $x_1, x_2, \ldots, x_m \in V$ and $x_j \to y_j \in P_i$, for some i and for $j, 1 \le j \le m$.

Theorem 4. $COL_t \subset ET0L$

Proof. Stronger result, the equality $COL_t = ET0L_{[1]}$ was proved in [10], where $ET0L_{[1]}$ are languages generated by tables P_i which rewrites at most one letter of V to non identical string. □

4 Parallel Colonies

For a derivation in parallel model of colony is typical that several components are active in a derivation step of the colony. Parallel colonies were introduced in the principle that components of a colony, which *can* work on the tape *must* work simultaneously and each component rewrites at most one occurrence of its start symbol. To formalize mentioned requirement the case when more components have the same associated nonterminal requires a special discussion:

* If (S, F_i), and (S, F_j) are two components of a colony $\mathcal{C}$ and if (at least) two symbols S appear in a current string, then both these components must be used, each rewriting one occurrence of S.

* If only one S appears in a current string, then each component *can* be used, but not both in parallel, hence in such a case we discuss two possibilities

(i) derivation is blocked – *strongly competitive parallel* way of derivation, and

(ii) derivation continues and maximal number of components is used, non-deterministically chosen from all the components which can be used – *weakly competitive parallel* way of derivation.

This corresponds to the following derivation steps:

For $x, y \in V^*$ define a *strongly competitive parallel* derivation step by

$x \stackrel{sp}{\Longrightarrow} y$ iff $x = x_1 S_{i_1} x_2 S_{i_2} ... x_k S_{i_k} x_{k+1}$,
$y = x_1 z_{i_1} x_2 z_{i_2} ... x_k z_{i_k} x_{k+1}, z_{i_j} \in F_{i_j}, 1 \leq j \leq k$,
$i_u \neq i_v$ for all $u \neq v, 1 \leq u, v \leq k$,
(one component is allowed to rewrite at most one occurrence of its start symbol)
$|x|_{S_t} > 0$ implies $t = i_j$ for some j, $1 \leq j \leq k$
(if component F_t *can* be used, then it *must* be used).

In accordance with general case, the language of a colony with derivation step $\stackrel{sp}{\Longrightarrow}$ is denoted $L_{sp}(\mathcal{C}, w_0)$ and COL_{sp} is the corresponding class of all languages.

Example 2. Let
$\mathcal{C} = (\{A, B, C, D, 0, 1\}, \{0, 1\}, \mathcal{F})$, where
$\mathcal{F} = \{(A, \{0B, 1C\}), (A, \{0C, 1B\}), (B, \{A, E\}), (C, \{A, E\}),$
$(E, \{\varepsilon\}), (E, \{\varepsilon\})\}$.

In the colony we can derive for example
$AA \stackrel{sp}{\Longrightarrow} 0B0C \stackrel{sp}{\Longrightarrow} 0A0A \stackrel{sp}{\Longrightarrow} 01C01B \stackrel{sp}{\Longrightarrow} 01E01E \stackrel{sp}{\Longrightarrow} 0101$

Let the derivation start with
$AA \stackrel{sp}{\Longrightarrow} 0B1B$.

Next sentential form contains either single occurrence of A or E. The colony has two components to rewrite A and two components to rewrite E. In both cases derivation stops.

$$L_{sp}(\mathcal{C}, AA) = \{ww : w \in \{0,1\}^+\}.$$

For $x, y \in V^*$ define a *weakly competitive parallel* derivation step by

$$x \stackrel{wp}{\Longrightarrow} y \text{ iff } x = x_1 S_{i_1} x_2 S_{i_2} ... x_k S_{i_k} x_{k+1},$$
$$y = x_1 z_{i_1} x_2 z_{i_2} ... x_k z_{i_k} x_{k+1}, z_{i_j} \in F_{i_j}, 1 \le j \le k,$$
$$i_u \ne i_v \text{ for all } u \ne v, 1 \le u, v \le k,$$

(one component is allowed to rewrite at most one occurrence of its startsymbol)

$|x|_{S_t} > 0$ implies $S_t = S_{i_j}$ for some j, $1 \le j \le k$,

k is the maximal integer with the previous properties.

In accordance with general case, the language of a colony with derivation step $\stackrel{wp}{\Longrightarrow}$ is denoted $L_{wp}(\mathcal{C}, w_0)$ and COL_{wp} is the corresponding class of all languages.

Example 3. Let
$\mathcal{C} = (\{S, A, B, C, D, E, F, a, b, c\}, \{a, b, c\}, \mathcal{F})$, where
$\mathcal{F} = \{(S, \{ABC\}), (Y, \{Z\}), (Z, \{Y\}),$
$(A, \{aD, X\}), (B, \{bE, X\}), (C, \{cF, X\}),$
$(D, \{A\}), (E, \{B\}), (F, \{C\}),$
$(X, \{\varepsilon\}), (X, \{\varepsilon\}), (X, \{Y\})\}.$

A succesfull derivation in the colony ends by rewriting all occurrences of X by ε. There are at most three occurrences of X in sentential forms produced by the colony but only words containing at most two X-es can be rewritten to terminal words.

Successful derivation:
$S \stackrel{wp}{\Longrightarrow} ABC \stackrel{wp}{\Longrightarrow} XbEcF \stackrel{wp}{\Longrightarrow} bBcC \stackrel{wp}{\Longrightarrow} bXccF \stackrel{wp}{\Longrightarrow} bccC$
$\stackrel{wp}{\Longrightarrow} bccX \stackrel{wp}{\Longrightarrow} bcc$

Blocked derivation
$S \stackrel{wp}{\Longrightarrow} ABC \stackrel{wp}{\Longrightarrow} aDbEcF \stackrel{wp}{\Longrightarrow} aAbBcC \stackrel{wp}{\Longrightarrow} aXbXcX \stackrel{wp}{\Longrightarrow} abY$
$L_{wp}(\mathcal{C}, S) = \{a^i b^j c^k \mid i, j, k \ge 0, i \ne j \text{ or } j \ne k \text{ or } i \ne k\}.$

If the start symbols of all components are different in a parallel colony, then both $\stackrel{sp}{\Longrightarrow}$ and $\stackrel{wp}{\Longrightarrow}$ define the same relation denoted by $\stackrel{p}{\Longrightarrow}$. For the generative power of COL_p we have

Theorem 5. *a)* $COL_p = CF$
b) $COL_p \subset COL_{sp}$
c) $COL_p \subset COL_{wp}$

Proof. a) Let $L \in CF$, $L = L(G)$ for a context-free grammar $G = (N, T, P, S)$. Assume, that A does not appear in z for each rule $A \to z \in P$ and $N = \{A_1, A_2, ..., A_n\}$. Consider the colony

$$\mathcal{C} = (N \cup T, T, \mathcal{F}),\ \mathcal{F} = \{(A_i, \{z; A_i \to z \in P\}), 1 \le i \le n\}.$$

Clearly, $\mathcal{C}$ contains exactly the rules of G and $L_p(\mathcal{C}, S) \subseteq L(G)$. Also the converse inclusion is true. Take a derivation $D : S \Longrightarrow^* x$ in G. The order of applying the rules in D can be modified in such a way to obtain another derivation, D', described by the same derivation tree as D (hence producing the same string x), but consisting of subderivations $u \Longrightarrow v$ with respect to G such that for every $A_i \in N$ which appears in u, exactly one occurrence of it is rewritten. The derivation D' is a parallel derivation with respect to $\mathcal{C}$, hence $x \in L_p(\mathcal{C}, S)$. We have obtained the inclusion $CF \subseteq COL_p$.

Conversely, let $L \in COL_p$. There is a colony $\mathcal{C} = (V, T, \mathcal{F})$ and an axiom w such that $L = L_p(\mathcal{C}, w)$. To construct an equivalent context-free grammar we have to determine set of nonterminals N to be strictly disjoin with the terminal set T. Exactly as for COL_b we construct $G = ((V - T) \cup \bar{V}, T, \bar{P}, S)$. Rules of the grammar are constructed from components of the colony in such a way, that they rewrite start symbol of the component by a word from associated finite language. In the case when original start symbol of some component was also a terminal symbol, corresponding barred letter is used for the derivation.

The inclusion $L_p(\mathcal{C}, w) \subseteq L(G)$ is obvious, the converse inclusion can be obtained as previously, hence $L_p(\mathcal{C}, w) = L(G)$ and $COL_p \subseteq CF$.

b) and c) Clearly, $COL_p \subseteq COL_{sp} \cap COL_{wp}$.
For $L_{sp} = L_{sp}(\mathcal{C}, AA)$ from the Example 4.1 we have $L_{sp} \in COL_{sp} - COL_p$.
For $L_{wp} = L_{wp}(\mathcal{C}, S)$ from the Example 4.2 we have $L_{wp} \in COL_{wp} - COL_p$. □

To specify the generative power of parallel colonies more detailly we use matrix grammars with appearance checking An $ET0L$ system is a construct $G = (\Sigma, T, H, w)$, where Σ is a vocabulary, $T \subseteq \Sigma$, $w \in \Sigma^+$, and H is a finite set of tables, that is of finite substitutions $h : \Sigma^* \longrightarrow 2^{\Sigma^*}$. For $h \in H$ and $x \in \Sigma^*$ we define the 1-limited image $h(x)$ as the set of all strings in Σ^* which can be obtained from x by replacing for each $a \in \Sigma$ which appears in x exactly one occurrence by an element of $h(a)$. Thus, the 1-limited generated language is

$$L_{1l}(G) = \{z \in T^*; z \in h_n(h_{n-1}(...(h_1(w))...)), n \ge 0, h_j \in H, \text{ for all } j\}$$

Note 4. A *matrix grammar* (with appearance checking) is a construct $G = (N, T, M, S, F)$, where N is a nonterminal vocabulary, T is a terminal vocabulary, $S \in N$ is the a symbol, M is a finite set of finite sequences of context-free rules and F is a set of occurrences of rules in M. The derivations start from S and consist of steps of using matrices in M; when using a matrix $(A_1 \to x_1, ..., A_n \to x_n)$, all the rules are used, in this order, and this means the symbols A_i are effectively rewritten by x_i if they appear in the current string, with the possibility to skip the rules $A_i \to x_i$ which appear in F but A_i does not appear in the current string. and 1-limited $ET0L$ systems.

We denote by MAT_{ac} the family of languages generated by matrix grammars with appearance checking (without using ε-rules) and by $1lET0L$ the family of languages generated by 1-limited $ET0L$ systems. It is known that $CF \subset MAT_{ac} \subset CS$, strict inclusions for ε free grammars. $1lET0L$ is strictly included in MAT_{ac}.

Theorem 6. [12] $\quad COL_{sp} \subseteq MAT_{ac}$ *and* $COL_{wp} \subset 1lET0L$

Open problems: What is the relation between COL_{wp} and COL_{sp}.

Is the inclusion $COL_{sp} \subseteq MAT_{ac}$ proper?

What is the influence of ε in components of the colony on the generative power.

5 Further Research Topics and Open Problems

Large number of problems were investigated for colonies. We briefly summarize them with references to the original papers.
First we mention various variants of the basic model of a colony. The choice of terminal alphabets influences the generative capacity. Cases $T = V$ and $V \cap T \neq \emptyset$ for sequential colonies were discussed in [10], [28].
Results on parallel colonies can be found in [12], [43].
The role of ε rules in different models has to be discussed. No result in this direction was done and at least for some types of colonies the absence of ε rules will reduce their generative power.
Descriptional complexity of colonies characterized for example by the number of components were discussed in [19], [42].
Topics like stability and inference were open in [19] and [43], respectively.
There are several extensions or modifications of the basic model. Between sequential colonies with t and b mode of derivation the colonies with k active components in one step can be treated [42].
Consequences of restriction of using components in the sense of frequency or fixed period of forbidden activity of components were treated in [19], [23], [22].
Different motivations leads to more significant modifications of the basic model. Colonies with point mutations, PM colonies [41], [40], [30], [31] have special type of rules allowing to add at most one symbols and moreover also position of components in the environment plays role in this model.
Symbiosis and parasitism were inspiration for models introduced in [11].
Inspirations from automata theory influenced papers [1], [2].
Unreliable colonies are introduced and discussed in [14], [15], [16].
One can found also treatment to use ideas of colonies to model low level economy [35].
Application of the idea in linguistics due to [24],[3].
Colonies have constant environment. Models, where environment can be changed by inner rules, namely e-colonies and eco-colonies, were also introduced. These models came up combining ideas concerning the internal behaviour of the environment in the eco-grammar systems [8], [29], [38], [38] and single letter rewriting

behaviour of the components of the colonies. [46], [47], [48].
In last few years one very interesting type of colonies was introduced on the base of membrane systems. So called P colonies are studied in [25], [21], [4], [5], [7], [9], [27].

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