Structures of the Environment in Colonies

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Abstract. We study sequential colonies introduced in [5], [9] from the point of view of their environmental structures. We give expressions for the languages Life, Garden-of-Eden, Doomsday and Non-life and we present conditions for the emptiness of these languages for the sequential colonies with basic and terminal mode of the derivation.

Keywords: *b* mode colony, *t* mode colony, garden of Eden, life, doomsday and nonlife states and languages, characteristic vector of the environment

1 Introduction

Grammars and grammar systems can be treated not only as language generative devices, but also as rewriting systems for the states of the environment, which are represented by strings over the fixed alphabets. From this point of view, the rules of the system and the way how are they applied are important for the development of the environment, while the starting string and the terminal alphabet play no role. Typical states of the environment, the *garden-of-Eden*, *life*, *doomsday* and *non-life*, we will deal with, are known from the investigation of theory of cellular automata. This classification of the states of the environment is determined by (im)possibility of each state to produce next state as well as by (im)possibility of each state to be produced by another state.

A state is called a *garden of Eden*, if it cannot be derived from another state and it can produce next state. A state is called a *doomsday* if it is derived from another state and it cannot produce any other state. A state *life* can be derived from another state and can produce a new state and a state *nonlife* neither can be derived nor can produce any new state.

In the present paper we study the above mentioned states and their sets (languages) for grammar systems [2],[4], namely for their special case called the *colonies*. By a colony we mean a grammar system with simple components, introduced in [5]. The components of the colony, each of which is able to produce only the finite language, rewrite the common string by given protocol of the cooperation. The study of the states *garden-of-Eden*, *life*, *doomsday* and *non-life* and their sets was for colonies initiatized and motivated in [9], [10], where colonies with point mutation, PM colonies for short, were introduced and studied. Results presented in [9] include among the others also the regularity of the above languages for PM colonies. Environmental structures are studied more detaily in [7], namely conditions for emptiness and finiteness of languages are discussed.

In the present paper the *sequential colonies* will be investigated from the same view point. So we will continue the study of the structure and properties of the garden-of-Eden, life, doomsday and non-life for sequential colonies. We will discuss both b and t mode of the derivation in colonies. We will characterize languages of these structures of the environment, discuss their emptiness, (in)finiteness. We present new results for b mode of derivation. Results for t mode are revised and extended version of our results from [8].

In Section 2 we introduce the languages Garden-of-Eden, Life, Doomsday and Non-life generally, for any string rewriting systems, and the characteristic vector of the structure of the environment of the rewriting system. Some basic properties of these notions are included.

In Section 3 we turn to colonies and discuss above mentioned topics in detail first for b-mode colonies and then for t-mode colonies.

2 States of the environment and their dynamical properties

Assume the states of the environment to be given by V^*, the set of all strings over fixed alphabet V. Let a binary relation on V^*, the derivation step $\Longrightarrow$, defines global transformation of the states of the environment determined by local rewriting rules P. Let $\mathcal{S} = (V^*, P, \Longrightarrow)$ determine the rewriting system.

To characterize some low level dynamic of $\mathcal{S}$ we will study following sets of states:

A state $w \in V^*$ is said to be *alive* in $\mathcal{S}$ if there is a state $z \in V^*, z \neq w$ such that $w \Longrightarrow z$. A state which is not alive is said to be *dead*.

A state $w \in V^*$ is said to be *reachable* in $\mathcal{S}$ if there is a state $z \in V^*, z \neq w$ such that $z \Longrightarrow w$. A state which is not reachable is said to be *unreachable* in $\mathcal{S}$.

We denote by $Alive(\mathcal{S})$, $Dead(\mathcal{S})$, $Reachable(\mathcal{S})$ and $Unreachable(\mathcal{S})$ the languages of all alive, dead, reachable and unreachable states, respectively. By intersecting the classes in the two classifications above, we get four languages: the Garden-of-Eden of $\mathcal{S}$ – $GE(\mathcal{S})$, the Life of $\mathcal{S}$ – $LF(\mathcal{S})$, the Doomsday of $\mathcal{S}$ – $DD(\mathcal{S})$ and the Non-life of $\mathcal{S}$ – $NL(\mathcal{S})$.

Definition 1.
$$\begin{aligned} GE(\mathcal{S}) &= Unreachable(\mathcal{S}) \cap Alive(\mathcal{S}), \\ LF(\mathcal{S}) &= Reachable(\mathcal{S}) \cap Alive(\mathcal{S}), \\ DD(\mathcal{S}) &= Reachable(\mathcal{S}) \cap Dead(\mathcal{S}), \\ NL(\mathcal{S}) &= Unreachable(\mathcal{S}) \cap Dead(\mathcal{S}). \end{aligned}$$

To study the emptiness of the above languages for a given rewriting system $\mathcal{S}$ we will use a characteristic function χ of the language L defined as

$$\chi(L) = 0 \quad \text{for } L = \emptyset$$
$$\chi(L) = 1 \quad \text{otherwise.}$$

The characteristic vector $\chi(\mathcal{S})$ of the environment of $\mathcal{S}$ is defined as

$$\chi(\mathcal{S}) = (\ \chi(GE(\mathcal{S})),\ \chi(LF(\mathcal{S})),\ \chi(DD(\mathcal{S})),\ \chi(NL(\mathcal{S}))\ )$$

Directly from the definitions we have

Lemma 1. *At least one of the sets $GE(\mathcal{S}), LF(\mathcal{S}), DD(\mathcal{S})$ and $NL(\mathcal{S})$ is infinite for any $\mathcal{S}$.*
If $\chi(GE(\mathcal{S})) = 1$ then $\chi(LF(\mathcal{S})) = 1$ or $\chi(DD(\mathcal{S})) = 1$.
If $\chi(DD(\mathcal{S})) = 1$ then $\chi(LF(\mathcal{S})) = 1$ or $\chi(GE(\mathcal{S})) = 1$.

Corollary 1. *For arbitrary $\mathcal{S}$*
$$\chi(\mathcal{S}) \in \{\ (0,1,0,1),(0,1,1,1),(1,0,1,0),(1,0,1,1),(1,1,0,0),(0,1,1,0)$$
$$(1,1,0,1),(0,1,0,0),(0,0,0,1),(1,1,1,0),(1,1,1,1)\ \}.$$

Proof. $DD(\mathcal{S}), GE(\mathcal{S}), LF(\mathcal{S}), NL(\mathcal{S})$ form the partition on V^*, so at least one component of the $\chi(\mathcal{S})$ is equal 1 and $\chi(\mathcal{S}) = (0,0,0,0)$ for no $\mathcal{S}$.
The condition $\chi(GE(\mathcal{S})) = 1$ from Lemma 1 gives $\chi(\mathcal{S}) = (1,0,0,1)$ for no $\mathcal{S}$ and $\chi(\mathcal{S}) = (1,0,0,0)$ for no $\mathcal{S}$.
The condition $\chi(DD(\mathcal{S})) = 1$ from Lemma 1 gives $\chi(\mathcal{S}) = (0,0,1,0)$ for no $\mathcal{S}$ and $\chi(\mathcal{S}) = (0,0,1,1)$ for no $\mathcal{S}$. □

In this paper we will discuss vectors $\chi(\mathcal{S})$ for rewriting systems called colonies.

3 Colonies

Colonies were introduced in [5] as grammar systems [2],[4] consisting of a finite collection of very simple grammars rewriting symbols on common string environment. Each agent is allowed to trasform its start symbol into finite set of words.

Definition 2. *A colony $\mathcal{C}$ is 3-tuple $\mathcal{C} = (V, T, \mathcal{R})$, where*
V is a finite non-empty alphabet of the colony,
$T \subset V$ is a non-empty terminal alphabet,
$\mathcal{R} = \{(S,F) \mid S \in V,\ F \subseteq (V - \{S\})^*, F$ finite, $F \neq \emptyset\}$
is a finite multiset of components (S,F), where S is a start symbol of the component (S,F) and F is a finite language generated by the component (S,F).

Note 1. Terminal alphabet T, which plays basic role in the definition of the language determined by a colony will play no role in our considerations. Nevertheless, we decided to present here the original, i.e. grammar system, definition of the colony rather then grammar scheme version $\mathcal{C} = (V, \mathcal{R})$.

For $\mathcal{C} = (V, T, \mathcal{R})$ and $\mathcal{R} = \{c_1, \ldots, c_n\}$, where $c_i = (S_i, F_i)$ for $1 \le i \le n$ we fix the notations:

$Dom\ c_i = S_i$ $\qquad$ $Val\ c_i = F_i$

$Dom\ \mathcal{C} = \{Dom\ c_i : 1 \le i \le n\}$ $\qquad$ $Val\ \mathcal{C} = \bigcup_{i=1}^{n} Val\ c_i$

Depending on the motivation, several ways were considered to introduce the derivation step $\Longrightarrow$ in colonies $\mathcal{C} = (V, T, \mathcal{R})$. This led to the different variants of colonies introduced e.g. in [3], [9], etc.

In next section we will consider sequential colonies $\mathcal{C} = (V, T, \mathcal{R})$ with basic and terminal modes of the derivation, where the derivation steps will be denoted by $\overset{x}{\Longrightarrow}$ for $x \in \{b, t\}$. Corresponding rewriting systems specified by $(V^*, \mathcal{R}, \overset{x}{\Longrightarrow})$ will be denoted as $\mathcal{C}_x$ for $x \in \{b, t\}$.

In a sequential colony, there is active exactly one component, in each derivation step. In a basic mode the active component is allowed to rewrite one occurrence of its start symbol in an actual string – we speak on b -mode derivation and b -mode colony. In a terminal mode the active component has to rewrite all the occurrences of its start symbol in an actual string – we speak on t -mode derivation and t -mode colony. In next subsections, formal definitions of the rewriting steps and the characterization of structures of these colonies will be presented.

3.1 Colonies with b-mode derivation

For the sequential colony we first recall the definition of the basic mode of derivation.

Definition 3. *Let $\mathcal{C} = (V, T, \mathcal{R})$ be a colony and $\mathcal{R} = \{(S, F) \mid S \in V, F \subseteq (V - \{S\})^*, F$ finite, $F \neq \emptyset\}$. Then*
$x \overset{b}{\Longrightarrow} y$ iff $x = x_1 S x_2$, $y = x_1 w x_2$ for some component $(S, F) \in \mathcal{R}$ and $w \in F$.

We denote by $\mathcal{C}_b$ the rewriting system determined by the colony $\mathcal{C} = (V, T, \mathcal{R})$ and by the derivation step $\overset{b}{\Longrightarrow}$, i.e. $\mathcal{C}_b = (V^*, \mathcal{R}, \overset{b}{\Longrightarrow})$ and we denote by COL_b the collection of all rewriting systems $\mathcal{C}_b$.

To express languages of environment for $\mathcal{C}_b$ we have directly from the definition

Lemma 2.
$$Alive(\mathcal{C}_b) = V^* Dom\ \mathcal{C}_b V^*$$
$$Dead(\mathcal{C}_b) = (V - Dom\ \mathcal{C}_b)^*$$
$$Reachable(\mathcal{C}_b) = V^* Val\ \mathcal{C}_b V^*$$
$$Unreachable(\mathcal{C}_b) = V^* - V^* Val\ \mathcal{C}_b V^*$$

This leads to the following expressions for our languages.

Theorem 1.
$$LF(\mathcal{C}_b) = V^* Dom\ \mathcal{C}_b V^* \cap V^* Val\ \mathcal{C}_b V^*$$
$$GE(\mathcal{C}_b) = V^* Dom\ \mathcal{C}_b V^* - V^* Val\ \mathcal{C}_b V^*$$
$$DD(\mathcal{C}_b) = (V - Dom\ \mathcal{C}_b)^* (Val\ \mathcal{C}_b - V^* Dom\ \mathcal{C}_b V^*)(V - Dom\ \mathcal{C}_b)^*$$
$$NL(\mathcal{C}_b) = (V - Dom\ \mathcal{C}_b)^* - V^* Val\ \mathcal{C}_b V^*$$

Proof. Follows from the definitions and Lemma 2.
Each word in $LF(\mathcal{C}_b)$ has to contain an element of $Dom\ \mathcal{C}_b$ and a subword from $Val\ \mathcal{C}_b$.
Each word in $GE(\mathcal{C}_b)$ has to contain an element of $Dom\ \mathcal{C}_b$ and no subword from $Val\ \mathcal{C}_b$.
Each word in $DD(\mathcal{C}_b)$ has to contain subword from $Val\ \mathcal{C}_b$ and no element of $Dom\ \mathcal{C}_b$.
Each word in $NL(\mathcal{C}_b)$ has to contain no element of $Dom\ \mathcal{C}_b$ neither a subword from $Val\ \mathcal{C}_b$. □

Depending on the mutual position of symbols from $Dom\ \mathcal{C}_b$ and words from $Val\ \mathcal{C}_b$ we can express the life states as follows

Corollary 2. $LF(\mathcal{C}_b) = V^*Val\ \mathcal{C}_bV^*Dom\ \mathcal{C}_bV^* \cup V^*Dom\ \mathcal{C}_b\ V^*Val\ \mathcal{C}_bV^*$
$\cup V^*(V^*Dom\ \mathcal{C}_bV^* \cap Val\ \mathcal{C}_b)V^*$

For (non)emptiness of the languages above we obtain following conditions:

Theorem 2. *a)* $LF(\mathcal{C}_b) \neq \emptyset$ *for any* $\mathcal{C}_b$
b) $GE(\mathcal{C}_b) \neq \emptyset$ *iff* $V^*Dom\ \mathcal{C}_bV^* - V^*Val\ \mathcal{C}_bV^* \neq \emptyset$
c) $DD(\mathcal{C}_b) \neq \emptyset$ *iff* $Val\ \mathcal{C}_b - V^*Dom\ \mathcal{C}_bV^* \neq \emptyset$
d) $NL(\mathcal{C}_b) \neq \emptyset$ *iff* $(V - Dom\ \mathcal{C}_b)^* - V^*Val\ \mathcal{C}_bV^* \neq \emptyset$

Proof. a) By the definition we have $S \in Dom\ \mathcal{C}_b$ and $w \in F$ for some $(S, F) \in \mathcal{R}$ which gives $Sw \in LF(\mathcal{C}_b)$.
c) Evidently $Val\ \mathcal{C}_b - V^*Dom\ \mathcal{C}_bV^* \subset DD(\mathcal{C}_b)$. On the other side if $w \in DD(\mathcal{C}_b)$, then $w \in (V - Dom\ \mathcal{C}_b)^*$, and $w = w_1uw_2$ for some $u \in Val\ \mathcal{C}_b$. This gives $u \in Val\ \mathcal{C}_b - V^*Dom\ \mathcal{C}_bV^*$.
Points b) and d) follow directly from the Theorem 2. □

Corollary 3. *a)* $LF(\mathcal{C}_b)$ *is infinite for any* $\mathcal{C}_b$.
b) If $DD(\mathcal{C}_b) \neq \emptyset$ *then it is infinite.*

Proof. a) For the string Sw from the proof of Theorem 2 we have $S^+w \subset LF(\mathcal{C}_b)$.
b) For $u \in DD(\mathcal{C}_b)$ we have also $u^+ \in DD(\mathcal{C}_b)$.

Note 2. Nonempty $GE(\mathcal{C}_b)$ and nonempty $NL(\mathcal{C}_b)$ can be either finite or infinite.

Denote by $\chi(COL_b)$ the set of all characteristic vectors of $\mathcal{C}_b$.

Theorem 3. $\chi(COL_b) = \{\ (1,1,0,1), (1,1,1,0), (1,1,1,1), (1,1,0,0),$
$(0,1,1,1), (0,1,1,0), (0,1,0,1), (0,1,0,0)\ \}.$

Proof. By Corollary 1 and Theorem 2 we have
$\chi(COL_b) \subseteq \{\ (1,1,0,1), (1,1,1,0), (1,1,1,1), (1,1,0,0), (0,1,1,1),$
$(0,1,1,0), (0,1,0,1), (0,1,0,0)\ \}.$
All these vectors can be reached.
$\chi(\mathcal{C}_b) = (1,1,0,1)$ for $\mathcal{C}_b$ with $\mathcal{R} = \{(a, \{b, cb\}), (b, \{a\}), (d, \{a\}), \}$ and
$d^+ \subset GE(\mathcal{C}_b)$, $\quad a^+ \subset LF(\mathcal{C}_b)$, $\quad DD(\mathcal{C}_b) = \emptyset$, $\quad c^+ \subset NL(\mathcal{C}_b)$.

$\chi(\mathcal{C}_b) = (1,1,1,0)$ for $\mathcal{C}_b$ with $\mathcal{R} = \{(a, \{b,d\}), (b, \{a\}), (c, \{a\})\}$ and
$c^+ \subset GE(\mathcal{C}_b)$, $b^+ \subset LF(\mathcal{C}_b)$, $d^+ \subset DD(\mathcal{C}_b)$, $NL(\mathcal{C}_b) = \emptyset$.

$\chi(\mathcal{C}_b) = (1,1,1,1)$ for $\mathcal{C}_b$ with $\mathcal{R} = \{(a, \{bb\}), (b, \{ce\})\}$ and
$a^+ \subset GE(\mathcal{C}_b)$, $(bb)^+ \subset LF(\mathcal{C}_b)$, $(ce)^+ \subset DD(\mathcal{C}_b)$, $c^+ \subset NL(\mathcal{C}_b)$.

$\chi(\mathcal{C}_b) = (1,1,0,0)$ for $\mathcal{C}_b$ with $\mathcal{R} = \{(a, \{bb\}), (b, \{c\}, (c, \{b\})\}$ and
$a^+ \subset GE(\mathcal{C}_b)$, $(bb)^+ \subset LF(\mathcal{C}_b)$, $DD(\mathcal{C}_b) = \emptyset$, $NL(\mathcal{C}_b) = \emptyset$.

$\chi(\mathcal{C}_b) = (0,1,1,1)$ for $\mathcal{C}_b$ with $\mathcal{R} = \{(a, \{b, cc\}), (b, \{a, cc\})\}$ and
$GE(\mathcal{C}_b) = \emptyset$, $a^+ \subset LF(\mathcal{C}_b)$, $cc^+ \subset DD(\mathcal{C}_b)$, $c \in NL(\mathcal{C}_b)$.

$\chi(\mathcal{C}_b) = (0,1,1,0)$ for $\mathcal{C}_b$ with $\mathcal{R} = \{(a, \{b\}), (b, \{a, c\})\}$ and
$GE(\mathcal{C}_b) = \emptyset$, $a^+ \subset LF(\mathcal{C}_b)$, $c^+ \subset DD(\mathcal{C}_b)$, $NL(\mathcal{C}_b) = \emptyset$.

$\chi(\mathcal{C}_b) = (0,1,0,1)$ for $\mathcal{C}_b$ with $\mathcal{R} = \{(a, \{b, cb\}), (b, \{a\})\}$ and
$GE(\mathcal{C}_b) = \emptyset$, $a^+ \subset LF(\mathcal{C}_b)$, $c^+ \subset NL(\mathcal{C}_b)$, $DD(\mathcal{C}_b) = \emptyset$.

$\chi(\mathcal{C}_b) = (0,1,0,0)$ for $\mathcal{C}_b$ with $\mathcal{R} = \{(a, \{b\}), (b, \{a\})\}$ and
$\{a,b\}^+ = LF(\mathcal{C}_b)$ and all the other sets are empty. □

3.2 Colonies with t-mode derivation

Another possibility to define a derivation step in a sequential colony is that an active component is allowed to rewrite all occurrences of its start symbol in an actual string. We speak on a terminal mode of derivation (t-mode for short).

Definition 4. *Let* $\mathcal{C} = (V, T, \mathcal{R})$ *be a colony and* $\mathcal{R} = \{(S, F) \mid S \in V, F \subseteq (V - \{S\})^*, F \textit{finite}, F \neq \emptyset\}$. *Then*

$$x \stackrel{t}{\Longrightarrow} y \quad \textit{iff} x = x_1 S x_2 S x_3 \dots x_m S x_{m+1},\ x_1 x_2 \dots x_{m+1} \in (V - \{S\})^*,$$
$$y = x_1 w_1 x_2 w_2 x_3 \dots x_m w_m x_{m+1},$$
$$\textit{for some } (S, F) \in \mathcal{R} \textit{ and } w_j \in F, 1 \le j \le m.$$

We denote by $\mathcal{C}_t$ the rewriting system determined by the colony $\mathcal{C} = (V, T, \mathcal{R})$ and by the derivation step $\stackrel{t}{\Longrightarrow}$, i.e. $\mathcal{C}_t = (V^*, \mathcal{R}, \stackrel{t}{\Longrightarrow})$ and we denote by COL_t the collection of all rewriting systems $\mathcal{C}_t$.
To express languages of environment for $\mathcal{C}_t$ we have directly from the definition:

Lemma 3. $Alive(\mathcal{C}_t) = V^* Dom\ \mathcal{C}_t V^*$
$Dead(\mathcal{C}_t) = (V - Dom\ \mathcal{C}_t)^*$
$Reachable(\mathcal{C}_t) = \bigcup_{c \in \mathcal{R}} (V - Dom\ c)^* Val\ c\ (V - Dom\ c)^*$.
$Unreachable(\mathcal{C}_t) = V^* - \bigcup_{c \in \mathcal{R}} (V - Dom\ c)^* Val\ c\ (V - Dom\ c)^*$.

Note that $\mathcal{C}_b$ and $\mathcal{C}_t$ differs in the sets *Reachable* and *Unreachable* but not in *Alive* and *Dead*.

Theorem 4. $LF(\mathcal{C}_t) = \bigcup_{c\in\mathcal{R}}(V - Dom\ c)^* Val\ c\ (V - Dom\ c)^* \cap V^* Dom\ \mathcal{C}_t V^*$
$GE(\mathcal{C}_t) = V^* Dom\ \mathcal{C}_t V^* - \bigcup_{c\in\mathcal{R}}(V - Dom\ c)^* Val\ c\ (V - Dom\ c)^*$
$DD(\mathcal{C}_t) = (V - Dom\ \mathcal{C}_t)^*(Val\ \mathcal{C}_t - V^* Dom\ \mathcal{C}_t V^*)(V - Dom\ \mathcal{C}_t)^*$
$NL(\mathcal{C}_t) = (V - Dom\ \mathcal{C}_t)^* - \bigcup_{c\in\mathcal{R}}(V - Dom\ c)^*\ Val\ c\ (V - Dom\ c)^*$

Proof. It follows from Lemma 3 and definitions. □

Theorem 5. *a)* $GE(\mathcal{C}_t) \neq \emptyset$ *for any* $\mathcal{C}_t$.
b) $LF(\mathcal{C}_t) \neq \emptyset$ *iff* $|Dom\ \mathcal{C}_t| \geq 2$
c) $DD(\mathcal{C}_t) \neq \emptyset$ *iff* $(Val\ \mathcal{C}_t - V^* Dom\ \mathcal{C}_t V^*) \neq \emptyset$
d) $NL(\mathcal{C}_t) \neq \emptyset$ *iff* $(V - Dom\ \mathcal{C}_t)^* - V^* Val\ \mathcal{C}_t V^* \neq \emptyset$

Proof. a) Let $Dom\ \mathcal{C}_t = \{S_1, \ldots, S_n\}$. Then $(S_1 S_2 \ldots S_n)^+ \subset GE(\mathcal{C}_t)$. It follows from the condition that $F \subseteq (V - \{S\})^*$ for $(S, F) \in \mathcal{R}$.
b) Let $|Dom\ \mathcal{C}_t| \geq 2$ and $S_1, S_2 \in Dom\ \mathcal{C}_t$. Let $w \in F$ for $(S_1, F) \in \mathcal{R}$. Then $(wS_2)^+ \subset LF(\mathcal{C}_t)$.
Let $Dom\ \mathcal{C}_t = \{S\}$. Then words containing S cannot be derived and words not containing S do not produce any word. Therefore $LF(\mathcal{C}_t) = \emptyset$.
Points c) and d) follow from definitions. □

Corollary 4. $GE(\mathcal{C}_t)$ *is infinite.*
If $LF(\mathcal{C}_t) \neq \emptyset$ *then it is infinite.*
If $DD(\mathcal{C}_t) \neq \emptyset$ *then it is infinite.*

Proof. Results for GE and LF follows from the previous proof. Let $DD(\mathcal{C}_t) \neq \emptyset$ and $u \in Val\ \mathcal{C}_t - V^* Dom\ \mathcal{C}_t V^*$. Then $u^+ \subset DD(\mathcal{C}_t)$. □

Note 3. Nonempty $NL(\mathcal{C}_t)$ can be either finite or infinite.

Denote by $\chi(COL_t)$ the set of all characteristic vectors of $\mathcal{C}_t$.

Theorem 6. $\chi(COL_t) = \{\ (1,0,1,0), (1,0,1,1), (1,1,1,1), (1,1,0,0), (1,1,0,1), (1,1,1,0)\ \}$.

Proof. By Corollary 1 and Theorem 5 we have
$\chi(COL_t) \subseteq \{\ (1,0,1,0), (1,0,1,1), (1,1,1,1), (1,1,0,0), (1,1,0,1), (1,1,1,0)\ \}$.
All these vectors can be reached.
$\chi(\mathcal{C}_t) = (1,0,1,0)$ for $\mathcal{C}_t$ with $\mathcal{R} = \{(a, \{b\})\}$ and
$GE(\mathcal{C}_t) = \{a,b\}^+ - b^+, \quad LF(\mathcal{C}_t) = \emptyset, \quad DD(\mathcal{C}_t) = b^+, \quad NL(\mathcal{C}_t) = \emptyset.$

$\chi(\mathcal{C}_t) = (1,0,1,1)$ for $\mathcal{C}_t$ with $\mathcal{R} = \{(a, \{b, ccc\})\}$ and
$\{a,b\}^+ - b^+ \subset GE(\mathcal{C}_t), \quad LF(\mathcal{C}_t) = \emptyset, \quad b^+ \subset DD(\mathcal{C}_t), \quad \{c, cc\} \subset NL(\mathcal{C}_t).$

$\chi(\mathcal{C}_t) = (1,1,1,1)$ for $\mathcal{C}_t$ with $\mathcal{R} = \{(a, \{b, bdd\}), (b, \{c\})\}$ and
$a^+ \subset GE(\mathcal{C}_t), \quad b^+ \subset LF(\mathcal{C}_t), \quad c^+ \subset DD(\mathcal{C}_t), \quad d^+ \subset NL(\mathcal{C}_t).$

$\chi(\mathcal{C}_t) = (1,1,0,0)$ for $\mathcal{C}_t$ with $\mathcal{R} = \{(a, \{b\}), (b, \{a\})\}$ and
$GE(\mathcal{C}_t) = \{a,b\}^+ - a^+ - b^+, \quad LF(\mathcal{C}_t) = a^+ \cup b^+, \quad DD(\mathcal{C}_t) = \emptyset, \ NL(\mathcal{C}_t) = \emptyset.$

$\chi(\mathcal{C}_t) = (1,1,0,1)$ for $\mathcal{C}_t$ with $\mathcal{R} = \{(a, \{b, bcc\}), (b, \{a\})\}$ and
$(ab)^+ \subset GE(\mathcal{C}_t), \quad b^+ \subset LF(\mathcal{C}_t), \quad DD(\mathcal{C}_t) = \emptyset, \quad c^+ \subset NL(\mathcal{C}_t).$

$\chi(\mathcal{C}_t) = (1,1,1,0)$ for $\mathcal{C}_t$ with $\mathcal{R} = \{(a, \{b\}), (b, \{c\})\}$ and
$a^+ \subset GE(\mathcal{C}_t), \quad b^+ \subset LF(\mathcal{C}_t), \quad c^+ \subset DD(\mathcal{C}_t), \quad NL(\mathcal{C}_t) = \emptyset.$ □

4 Conclusions

Structures of the environment of the b-mode colonies and t-mode colonies differ in some aspects. According to the presented results we have

1) $LF(\mathcal{C}_b)$ is infinite for every $\mathcal{C}_b$ and $GE(\mathcal{C}_t)$ is infinite for every $\mathcal{C}_t$.

2) Nonempty $DD(\mathcal{C}_b)$ implies that $DD(\mathcal{C}_b)$ is infinite, while nonempty $GE(\mathcal{C}_b)$ and $NL(\mathcal{C}_b)$ can be either finite or infinite.

Nonempty $LF(\mathcal{C}_t)$ and nonempty $DD(\mathcal{C}_t)$ imply that these languages are infinite, while nonempty $NL(\mathcal{C}_b)$ can be either finite or infinite.

3) The set of characteristic vectors of COL_b consists of 9 vectors, while the set of characteristic vectors of COL_t consists of 6 vectors.

The topic studied in this paper is applicable to all other variants of colonies [6] as well as for the other types of rewriting systems.

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