On the Telecommunication Traffic Forecasting in a Fractional Gaussian Noise Model

Vyacheslav Gorev[1[0000-0002-9528-9497]], Alexander Gusev[2[0000-0002-0548-728X]] and Valerii Korniienko[3[0000-0002-0800-3359]]

Dnipro University of Technology, Dnipro, Ukraine

[1]lordjainor@gmail.com
[2]gusev1950@ukr.net
[3]vikor7@ukr.net

Abstract. The paper is devoted to the investigation of the weight function of the Kolmogorov–Wiener filter for forecasting of continuous fractional Gaussian noise with a Hurst exponent H>0.5. We use a truncated polynomial expansion method for obtaining an approximate solution for the weight function under consideration. The method is based on the Chebyshev polynomials of the second kind. Approximations formed of different numbers of polynomials up to nineteen are investigated. The kernel of the Wiener–Hopf integral equation is a positively defined function in the case of a continuous fractional Gaussian noise, so the method is convergent. In a simple model telecommunication traffic may be described as a continuous fractional Gaussian noise, so the results of the paper may be applied to the telecommunication traffic forecasting.

Keywords: Kolmogorov–Wiener filter weight function, continuous fractional Gaussian noise, truncated polynomial expansion method, Chebyshev polynomials of the second kind, method convergence, telecommunication traffic.

1 Introduction and related works

The problem of telecommunication traffic forecast is an important problem for telecommunications. For example, in [1] it is stressed that this problem may be important for the detection of the defects which take place because of attacks. So this problem is important for information security. This problem may also be important for the development of intelligent systems, for example, intelligent traffic forecasting engines [2].

The telecommunication traffic in systems with data burst transfer is considered to be a fractal process (see, for example, [3, 4]). We consider the traffic as a continuous random fractal process. Such a consideration is reasonable in the case of a large amount of data [5].

There are plenty of telecommunication traffic models, for example: a fractional Gaussian noise model, a fractional Brownian motion model, wavelet models, etc.

(see, for example, [3, 4]). There are a lot of different approaches to the forecasting of fractal telecommunication traffic: the ARIMA approach, the FARIMA approach, approaches based on artificial neural networks, wavelet-based approaches, etc. [1, 6].

One of the simplest telecommunication traffic models is the model where the telecommunication traffic is described [4, 7] as a fractional Gausian noise which is a stationary random process. The Kolmogorov–Wiener filter may be used for the forecasting of stationary processes [8]. This filter is a rather simple linear stationary filter. So, in our opinion, it is naturally enough to use the corresponding filter for the forecasting of telecommunication traffic in simple models where the traffic is considered to be a stationary random process.

However, we know few works where the Kolmogorov–Wiener filter is used for telecommunication traffic forecasting, and we don't know any works where the corresponding filter is used for the forecasting of telecommunication traffic in the fractional Gaussian noise model.

The Kolmogorov–Wiener filter was proposed in [5] in order to make the traffic forecast in a model where the traffic is considered as a stationary random process with a power-law structure function. In [5] a simplified Volterra integral equation was proposed in order to obtain the filter weight function which is necessary for the calculation of the filter output. Our previous papers [8–10] were also devoted to the model proposed in [5].

However, the filter weight function should be obtained on the basis of the Fredholm integral equation rather than the Volterra one [8]. The papers [8–10] were devoted to the obtaining of the corresponding weight function based on the Fredholm integral equation of the first kind. A review of the methods of solving the corresponding integral equation is given in [11]. We investigated approximate solutions of the corresponding Fredholm integral equation with the help of a truncated polynomial expansion method (TPEM), which is a special case of the Galerkin method [11]. This method is rather simple and allows one to obtain analytical approximate solutions to the corresponding integral equation. The TPEM is rather popular in different fields of knowledge (for example, see its applications to statistical physics [12, 13]).

In paper [8] we used polynomials orthogonal without weight, and in papers [9, 10] we used the Chebyshev polynomials (CPs) of the second and first kind, respectively. The behavior of solutions is similar for different sets of polynomials, and the method is not necessarily convergent for processes with a power-law structure function. In our opinion, the reason is the following. The correlation function of the process under consideration, which is the kernel of the integral equation, is not a positively defined function. The convergence of a TPEM is guaranteed if the kernel of the corresponding integral equation is a positively defined function (see [14]).

In this paper we consider the TPEM in the model where the traffic is described as continuous fractional Gaussian noise with a Hurst exponent $H > 0.5$. The correlation function of the corresponding random process is a positively defined function [15], so the TPEM should be convergent for the model of continuous fractional Gaussian noise. The exact analytical solution of the corresponding integral equation can hardly be obtained, so we use the TPEM which is based on the CPs of the second kind. The goal of the work is to obtain the Kolmogorov–Wiener filter weight function for the forecasting of continuous fractional Gaussian noise with the help of the TPEM and to illustrate its convergence.

The importance of the problem of the telecommunication traffic forecast is described in what precedes. This paper is devoted only to the description of the theoretical fundamentals of the development of the Kolmogorov–Wiener filter for traffic forecasting in the model where the traffic is described as a fractional Gaussian noise. The practical use of the obtained results may the subject of another paper.

2 The Wiener–Hopf integral equation and the truncated polynomial expansion method

The Kolmogorov–Wiener filter weight function $h(t)$ obeys the following Wiener–Hopf integral equation [8], which is a special case of the Fredholm integral equation of the first kind:

$$\int_0^T d\tau h(\tau) R(t-\tau) = R(t+z) \tag{1}$$

where $R(t)$ is the correlation function of the stationary random process for which the forecast is made and $z \ll T$ is the time interval for which the forecast is made. The data for the input signal are given for $t \in [0,T]$, the non-noisy case is investigated. The correlation function of continuous fractional Gaussian noise in the case $H > 0.5$ is as follows [15]:

$$R(t) = 2H(2H-1)\sigma^2 |t|^{2H-2} \tag{2}$$

where σ^2 is the process variance and H is the Hurst exponent. The substitution of (2) into (1) leads to the following integral equation

$$\int_0^T d\tau h(\tau) |t-\tau|^{2H-2} = (t+z)^{2H-2} . \tag{3}$$

A search for an exact analytical solution of the integral equation (3) meets difficulties, so we use the TPEM for obtaining an approximate solution.

In the framework of the TPEM the unknown function $h(t)$ is sought as a truncated series in polynomials which are orthogonal on the time interval on which the integral on the left-hand side of (3) is taken. So a polynomial set which is orthogonal on the time interval $t \in [0,T]$ is needed. The paper is based on the CPs of the second kind [16]

$$U_n(x) = \sum_{k=0}^{[n/2]} C_{n+1}^{2k+1} x^{n-2k} \left(x^2-1\right)^k , \; C_n^k = \frac{n!}{k!(n-k)!}, \tag{4}$$

where $[n/2]$ is the integer part of $n/2$. But the CPs of the second kind are orthogonal on $x \in [-1,1]$, and a polynomial set which is orthogonal on the interval $[0,T]$ rather than on the interval $[-1,1]$ is needed. In paper [9] it is shown that the following orthogonality condition takes place

$$\int_0^T S_n(y) S_m(y) w(y) dy = \frac{T\pi}{4}\delta_{mn}, \; \delta_{mn} = \begin{cases} 1, m = n \\ 0, m \neq n \end{cases} \tag{5}$$

where

$$S_n(y) = U_n\left(\frac{2y}{T} - 1\right), \; S_m(y) = U_m\left(\frac{2y}{T} - 1\right), \; w(y) = \sqrt{1 - \left(\frac{2y}{T} - 1\right)^2}. \tag{6}$$

The polynomials $S_n(y)$ are orthogonal on $y \in [0,T]$ with the weight $w(y)$ and the unknown weight function $h(t)$ may be sought as a truncated series

$$h(\tau) = \sum_{n \geq 0}^{l-1} g_n S_n(\tau) \tag{7}$$

where g_n are the unknown coefficients multiplying the polynomials. Expression (7) is the expression for the Kolmogorov–Wiener weight function $h(\tau)$ in the l - polynomial approximation. The coefficients g_n are found as follows. First of all we substitute (7) into (3):

$$\sum_{n \geq 0}^{l-1} g_n \int_0^T d\tau S_n(\tau) |t - \tau|^{2H-2} = (t + z)^{2H-2}. \tag{8}$$

Then we multiply the left-hand side and the right-hand side of (8) by $S_k(t)$, $k = \overline{0, l-1}$, after which both the left-hand side and the right-hand side are integrated over t on the time interval $t \in [0,T]$. As a result we obtain the following system of linear equations in the coefficients g_n:

$$\sum_{n \geq 0}^{l-1} g_n G_{nk} = B_k, \; k = \overline{0, l-1} \tag{9}$$

where the following denotation is used:

$$G_{nk} = \int_0^T \int_0^T dt d\tau S_n(\tau) S_k(t) |t - \tau|^{2H-2}, \; B_k = \int_0^T dt (t + z)^{2H-2} S_k(t), \tag{10}$$

the quantities G_{nk} are called the integral brackets. In paper [9] it is shown that the integral brackets obey the following properties:

$$G_{nk} = G_{kn}\,;\; G_{nk} = 0 \text{ if } n,\ k \text{ are of different parity.} \tag{11}$$

The properties (11) are obtained in [9] for the correlation function of a fractal process with a power-law structure function rather than for the correlation function (2). But the only property of the correlation function which was used in [9] for the derivation of (11) is the fact that the correlation function $R(t)$ is an even one. So expressions (11) are valid for the problem under consideration. The first property in (11) takes place in general case, the second one takes place due to the choice of CPs of the second kind.

The properties (11) significantly reduce the computation time. The calculation of the integral brackets takes most of the computation time. With the help of (11) one can conclude that the number of the integral brackets for which a straightforward calculation is needed is equal to

$$f_1(l) = l + (l-2) + (l-4) + \ldots = \sum_{k=0}^{[l/2]} (l-2k) = \frac{1}{2}\left(\left[\frac{l}{2}\right]+1\right)\left(2l - 2\left[\frac{l}{2}\right]\right), \tag{12}$$

and in the general case a straightforward calculation is needed for the number of the integral brackets equal to

$$f_2(l) = l + (l-1) + (l-2) + \ldots + 1 = l(l+1)/2\,. \tag{13}$$

The ratio of $f_2(l)$ to $f_1(l)$ for different values of l is given in Table 1.

Table 1. The ratio of $f_2(l)$ to $f_1(l)$

l	$f_2(l)/f_1(l)$	l	$f_2(l)/f_1(l)$	l	$f_2(l)/f_1(l)$	l	$f_2(l)/f_1(l)$
1	1.000	6	1.750	11	1.833	16	1.889
2	1.500	7	1.750	12	1.857	17	1.889
3	1.500	8	1.800	13	1.857	18	1.900
4	1.667	9	1.800	14	1.875	19	1.900
5	1.667	10	1.833	15	1.875		

Table 1 illustrates the fact that the choice of the CPs of the second kind decreases the number of brackets for which a straightforward calculation is needed approximately by 2 times for a quite large number of polynomials.

It is possible to obtain analytical results for the integrals (10). They can be obtained with account for (4), (6), the binomial theorem and the following change of variables for the integral brackets:

$$x = t + \tau\,,\; y = t - \tau\,. \tag{14}$$

A straightforward calculation leads to the following results:

$$G_{nk} = \frac{1}{2}\sum_{b_1=0}^{[n/2]}\sum_{j_1=0}^{n-2b_1}\sum_{l_1=0}^{b_1}\sum_{a_1=0}^{j_1+l_1+b_1}\sum_{b_2=0}^{[k/2]}\sum_{j_2=0}^{k-2b_2}\sum_{l_2=0}^{b_2}\sum_{a_2=0}^{j_2+l_2+b_2}\left(C_{k+1}^{2b_2+1}C_{k-2b_2}^{j_2}C_{b_2}^{l_2}C_{j_2+l_2+b_2}^{a_2}C_{n+1}^{2b_1+1}C_{n-2b_1}^{j_1}\times\right.$$

$$\times C_{b_1}^{l_1}C_{j_1+l_1+b_1}^{a_1}\frac{T^{2H}\left(1+(-1)^{j_1+l_1+b_1-a_1+j_2+l_2+b_2-a_2}\right)(-1)^{n+k-a_1-b_2-j_2-l_2}\,2^{b_2+b_1-l_2-l_1}}{j_1+l_1+b_1-a_1+j_2+l_2+b_2-a_2+2H-1}\times$$

$$\left.\times\left[\frac{1}{j_1+l_1+b_1+j_2+l_2+b_2+2H}+\sum_{\beta=0}^{a_1+a_2}\frac{C_{a_1+a_2}^{\beta}\,2^{\beta}T^{-\beta}(-1)^{a_1+a_2-\beta}}{j_1+l_1+b_1+j_2+l_2+b_2+2H-\beta}\right]\right), \tag{15}$$

$$B_n = \sum_{k=0}^{[n/2]}\sum_{j=0}^{n-2k}\sum_{l=0}^{k}\sum_{a=0}^{j+l+k}\left(C_{n+1}^{2k+1}C_{n-2k}^{j}C_k^l\,C_{j+l+k}^{a}\times\right.$$

$$\left.\times\frac{(-1)^{n-a}\,2^{j+2k}\,z^{j+l+k-a}}{T^{j+k+l}}\frac{(T+z)^{a+2H-1}-z^{a+2H-1}}{a+2H-1}\right).$$

Expressions (15) are rather cumbersome. The coefficients g_n multiplying the polynomials in (7) are found on the basis of the system of linear equations (9) in matrix form

$$g = G^{-1}B \tag{16}$$

where

$$G = \begin{pmatrix} G_{00} & G_{01} & \cdots & G_{0,l-1} \\ G_{10} & G_{11} & \cdots & G_{1,l-1} \\ \vdots & \vdots & \ddots & \vdots \\ G_{l-1,0} & G_{l-1,1} & \cdots & G_{l-1,l-1} \end{pmatrix},\quad g = \begin{pmatrix} g_0 \\ g_1 \\ \vdots \\ g_{l-1} \end{pmatrix},\quad B = \begin{pmatrix} B_0 \\ B_1 \\ \vdots \\ B_{l-1} \end{pmatrix}. \tag{17}$$

3 Numerical solutions

The numerical solutions are obtained for the parameters

$$T = 100,\ z = 3,\ H = 0.8. \tag{18}$$

The following numerical values for the coefficients multiplying polynomials in the l-polynomial approximation are obtained, see Table 2. The calculations are made with the help of the Wolfram Mathematica package. Approximations up to the 19-polynomial one are investigated. The integrals (10) are calculated with the help of the standard function NIntegrate built in the package. The calculation of the corresponding integrals by expressions (15) is not faster than the calculation by the NIntegrate function; moreover, the calculation by expressions (15) becomes inadequate for the number of polynomials more than 10.

Table 2. Values for the coefficients multiplying the polynomials

l	Values of $g_0 \cdot 10^3$, $g_1 \cdot 10^3$, …, $g_{l-1} \cdot 10^3$
1	7.17
2	7.17, –5.98
3	5.67, –5.98, 5.64
4	5.67, –4.88, 5.64, –4.53
5	5.29, –4.88, 4.68, –4.53, 4.00
6	5.29, –4.58, 4.68, –3.79, 4.00, –3.21
7	5.15, –4.58, 4.40, –3.79, 3.35, –3.21, 2.82
8	5.15, –4.45, 4.40, –3.56, 3.35, –2.70, 2.82, –2.29
9	5.08, –4.45, 4.28, –3.56, 3.16, –2.70, 2.37, –2.29, 2.04
10	5.08, –4.39, 4.28, –3.47, 3.16, –2.54, 2.37, –1.92, 2.04, –1.68
11	5.05, –4.39, 4.22, –3.47, 3.07, –2.54, 2.23, –1.92, 1.71, –1.68, 1.53
12	5.05, –4.36, 4.22, –3.42, 3.07, –2.46, 2.23, –1.80, 1.71, –1.40, 1.53, –1.28
13	5.03, –4.36, 4.19, –3.42, 3.02, –2.46, 2.16, –1.80, 1.60, –1.40, 1.27, –1.28, 1.18
14	5.03, –4.34, 4.19, –3.39, 3.02, –2.42, 2.16, –1.75, 1.60, –1.31, 1.27, –1.06, 1.18, –1.01
15	5.02, –4.34, 4.17, –3.39, 2.99, –2.42, 2.12, –1.75, 1.54, –1.31, 1.18, –1.06, 0.980, –1.01, 0.950
16	5.02, –4.33, 4.17, –3.37, 2.99, –2.40, 2.12, –1.71, 1.54, –1.27, 1.18, –0.988, 0.980, –0.832, 0.950, –0.819,
17	5.01, –4.33, 4.15, –3.37, 2.97, –2.40, 2.09, –1.71, 1.51, –1.27, 1.14, –0.988, 0.910, –0.832, 0.783, –0.819, 0.787
18	5.01, –4.32, 4.15, –3.35, 2.97, –2.38, 2.09, –1.69, 1.51, –1.24, 1.14, –0.950, 0.910, –0.771, 0.783, –0.674, 0.787, –0.687
19	5.00, –4.32, 4.14, –3.35, 2.96, –2.38, 2.08, –1.69, 1.49, –1.24, 1.11, –0.950, 0.873, –0.771, 0.725, –0.674, 0.646, –0.687, 0.669

The values in Table 2 are rounded off to 3 significant digits. The left-hand side and the right-hand side of (3) are calculated in the Wolfram Mathematica as

$$\mathrm{Left}(t) = \int_0^t d\tau h(\tau)(t-\tau)^{2H-2} + \int_t^T d\tau h(\tau)(\tau - t)^{2H-2}, \quad \mathrm{Right}(t) = (t+z)^{2H-2}. \qquad (19)$$

The coincidence of the corresponding left-hand and right-hand sides is illustrated by the calculation of the mean absolute percentage error (MAPE)

$$\mathrm{MAPE} = \frac{1}{T}\int_0^T dt \left| \frac{\mathrm{Left}(t) - \mathrm{Right}(t)}{\mathrm{Right}(t)} \right| \cdot 100\% \ , \qquad (20)$$

the corresponding results are presented in Table 3

Table 3. The results for the MAPE for approximations of different numbers of polynomials

l	MAPE, %	l	MAPE, %	l	MAPE, %	l	MAPE, %
1	28	6	5.3	11	1.7	16	0.81
2	19	7	4.0	12	1.5	17	0.71
3	13	8	3.2	13	1.2	18	0.64
4	9.3	9	2.5	14	1.1	19	0.57
5	6.7	10	2.1	15	0.91		

The results in Table 3 are rounded off to 2 significant digits. Table 3 illustrates that the approximations of small numbers of polynomials are not accurate, but the approximations of rather large numbers of polynomials are accurate. Fig. 1 shows graphs of the functions (19) for the 19-polynomial approximation.

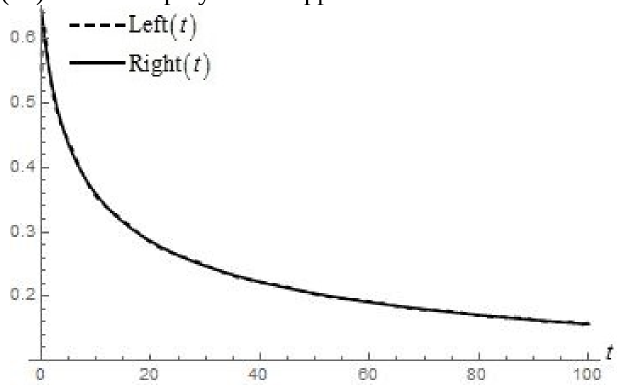

Fig. 1. Graphs of $\mathrm{Left}(t)$ and $\mathrm{Right}(t)$ for the nineteen-polynomial approximation.

Fig. 1 illustrates that the corresponding functions (19) almost coincide for the nineteen-polynomial approximation. Approximations of more than 19 polynomials are not investigated because the Wolfram Mathematica package has not enough recourses to build the corresponding graphs adequately.

Table 3 illustrates that accuracy of the approximations increases if the polynomial number increases. So one can conclude that the method is convergent, but one should not use a small number of polynomials for obtaining a rather accurate approximate solution for the weight function under consideration.

4 Results and conclusion

We investigate the Kolmogorov–Wiener filter for the forecasting of continuous fractional Gaussian noise. The subject of the investigation is the weight function of the corresponding filter. For simplicity, we restrict ourselves only to the case where the

Hurst exponent $H > 0.5$. The weight function for the filter under consideration obeys the integral equation (3). A search for an exact analytical solution of (3) meets difficulties, so we use the truncated polynomial expansion method (TPEM) in order to obtain an approximate solution for the unknown weight function. The paper is based on the Chebyshev polynomials (CPs) of the second kind orthogonal on the time interval $t \in [0, T]$, on which the filter input signal is given. The kernel of the integral equation (3) is a positively defined function, so the TPEM is convergent for the problem under consideration (see the corresponding discussion in [14]).

It is shown that the choice of the CPs of the second kind is convenient because the number of the integral brackets for which a straightforward calculation should be made is less than the corresponding number in the general case. It is shown that for the approximations of a rather large number of polynomials the number of brackets is approximately two times smaller than that in the general case.

Numerical approximate solutions are obtained for the parameters (18). The investigation is made up to the nineteen-polynomial approximation. The coefficients multiplying the polynomials and the mean absolute percentage error are calculated; the latter illustrates the accuracy of coincidence of the left-hand and the right-hand sides of the integral equation under consideration. The convergence of the method is illustrated, the accuracy of the approximations increases if the polynomial number increases. However, it should be stressed that one should use a rather large number of polynomials in order to obtain a rather accurate solution; approximations of small number of polynomials are not accurate. The kernel of the Wiener–Hopf integral equation is a positively defined function, so the proposed method should be convergent not only for the parameters (18), but also for other parameters.

The problem of traffic forecasting is an urgent problem for telecommunication systems. This problem may be important both for the information security and for the development of the intelligent systems. The description of the problem importance for information security is given in [1]. For example, in [2] it is stressed that the problem may be important for the development of the intelligent traffic forecasting engines. This paper is devoted to the development of theoretical fundamentals of the construction of the Kolmogorov–Wiener filter for forecasting of continuous fractional Gaussian noise. In a simple model [4, 7] telecommunication traffic for systems with data burst transfer can be described as fractional Gaussian noise. In [5] it is stressed that in the case of a large amount of data it is reasonable to investigate the traffic as a continuous random process. So, the results of the paper may be applied to the telecommunication traffic forecasting for systems with data burst transfer. A practical application of the obtained results to traffic forecasting in telecommunication systems is our plan for future research.

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