Research of Pareto-Optimal Schemes of Control of Availability of the Information System for Critical Use

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Abstract. The relevance of the task of control the availability of information systems in general and of information systems for critical use (ISCUs) in particular is increasing simultaneously with the information needs of humanity. However given the importance of an information resources of ISCU introducing new schemes of control the availability without a simulation stage can lead to significant material and in particular human losses. These circumstances led to the expediency of research the scientific and applied results of which are presented in the article. In the article the new mathematical models for identification of the Pareto-optimal schemes of control of accessibility of ISCU with a parallel functioning set of input information ports are synthesized. The ISCU is described by the composition of single-line Markov or semi-Mark queuing systems in which unlike existing ones availability loss functions are formulated with a focus on the ability of quantitative or qualitative estimation of the scheme of control of availability and guarantee the processing of access requests with different priority degree which allows to set and solve the multicriteria optimization task for an identification of problem-oriented scheme of control of availability of ISCU taking into account its architectural features.

Keywords: information system for critical use, mathematical models, availability, access control, optimization, mathematical programming, operations research.

1 Introduction

Process modeling in modern information systems remains actual and evolves in sync with the development of the information society. Such attention is driven by the constant growth of both the material value of information and its direct impact on people's lives, which is especially evident in information systems for critical use (ISCUs). Applied mathematical models allow to predict the results of current and service operations with information in such systems, to optimize information processes, to rationally control information systems in conditions of active person-system interaction, to design mechanisms of information security etc. However, the complex nature of information systems makes it essential to limit the scope of mathematical models created to prevent the complexity of the latter. However, for a complete description of the information systems under study, the mathematical models created should be generalized by a certain interconnected hierarchical structure, the role of which the dependability concept [1-5] has played in critical systems for decades.

Naturally, the integration of applied mathematical models of information systems dependability attributes is simplified if the first are built in the paradigm of a single mathematical apparatus. In particular, the authors have already explored [6-9] mathematical models of confidentiality, accessibility, integrity, reliability of information systems for critical use in the paradigm of Markov process theory. If we concentrate on modeling the availability of ISCU, the access process to the information resources of the latter, taking into account its architectural features and the technologies used to ensure confidentiality, is described in [6, 7, 10], but the question of the synthesis of optimal availability control schemes remained open, which led to the thematic focus of this article. Accordingly, the *object* of the study will be considered the process of synthesis of the target optimal schemes for controlling the availability of ISCU, and the ***subject*** of the study will be methods of Markov theory, queuing theory and operations research theory.

2 Review of related works

Mathematical modeling of optimization tasks has its own specificity and generally includes the step of identifying a set of factors from which a subset of managed variables is distinguished, generalizations of the latter into the objective function and a constraint system whose expressions are compared with free members that are determined by boundary values of the unmanaged variables. For applied optimization tasks, which may include the synthesis and analysis of optimal ISCU availability schemes, the optimization task statement is implemented individually, within the chosen mathematical apparatus, the choice of which largely determines the adequacy of the model and the method of solution. For the tasks of optimization of processes of functioning of real technical, information, economic telecommunication network systems, in particular, mathematical apparatus of queuing theory is used [11-17]. A typical queuing system (QS) includes a finite number of servicing devices or service channels. In addition to the service channels, a specific QS is identified by the values of such characteristics as flow of requirements, function of distribution of duration of service, queue parameters etc. QS optimization is to identify a configuration scheme, the application of which provides the extreme value of a formalized optimality criterion.

In classical queuing theory, no intervention is assumed in the functioning of the system under study, that is, external control effects aren't taken into account. When describing QSs that control the impact on the performance of which can't be ignored, use the derived theory of managed queuing systems. Methods of managed QS theory allow us to set the tasks of optimal management of queue, service channels, duration of service, input flow of requirements etc. The flexibility of the mathematical apparatus of queuing theory allows us to describe the information flows of QS using a powerful mathematical apparatus of theory of Markov processes [18-21] and to identify the optimal control schemes, apply the appropriate methods of the theory of operations research [22, 23]. This arrangement allows to provide adequate mathematical description of large-scale real information systems [15, 17-19, 21]. However, it should be noted that in many models of description of applied information systems in the QS paradigm [12-17] due consideration isn't given to the simultaneous management of several characteristics, which is natural for ISCU in which, for example, it is necessary to combine a high degree of confidentiality with a high characteristic availability. Therefore, the ***aim*** of the study is to synthesize and analyze the problem-oriented optimal schemes of multicriteria accessibility management of the information system for critical use modelled in the QS paradigm, taking into account the architectural features of the latter.

3 Problem statement

Communication of multiple authorized persons (APs) $I = \{i\}$, $i = \overline{1,n}$, with the information environment of ISCU is implemented through a set of open information ports $J = \{j\}$, $j = \overline{1,m}$. Technologically person-system interaction is organized in the form of closed sessions. If at a certain time moment the j th information port is occupied by the session of the i th AP then until its completion this port is closed to the rest of the APs and their requests addressed to this port are ordered in a queue. We'll consider queries sent from the i th AP to the j th information port of ISCU as an independent Poisson process with intensity λ_i and characterize by the probability of satisfaction of the request x_{ij}. The duration of the session during which the j th port is occupied by the i th AP will be considered by stochastic value with exponential distribution with indicator μ_j. Given the described input data to model and study the person-system interaction we use the mathematical apparatus of queuing theory [11-17] in the paradigm of which we'll set the problems of identification of optimal schemes of control of availability of ISCU, which will differ in the formalization of the personified indicator of efficiency of control of availability of ISCU – loss function $L_i(x)$ which characterizes the average number of denials of access to the i th AP for a certain period of time. Taking into account the competitive nature of access to the information environment of ISCU we formalize the process of choosing the most optimal scheme from a set of ISCU accessibility management schemes in the paradigm of the mathematical apparatus of game theory [22, 23]. We present a generalized objective function of such an optimization task with a tuple

$$\Gamma = \left\langle I, \left\{ X_i \right\}_{i \in I}, \left\{ L_i \right\}_{i \in I} \right\rangle, \tag{1}$$

where I is a set of APs indexes, $X_i = \left\{ x_i = \left(x_{i1}, \dots, x_{im} \right) : x_{ij} \in [0,1], \sum_{j=1}^{m} x_{ij} = 1 \right\}$ is a set of an access organization schemes for an i th AP, $L_i\left(x\right)$ is a loss function for an i th AP, $x = \left(x_1, \dots, x_n \right) \in X_i$. We classify the optimization task generalized by tuple (1) as a multi-coalition game, the optimal solution of which is the equilibrium point $x^* = \left(x_1^*, \dots, x_n^* \right)$ for which such an inequality holds:

$$L_i\left(x_1^*, \dots, x_i^*, \dots x_n^* \right) \le L_i\left(x_1, \dots, x_i, \dots, x_n \right), \tag{2}$$

$\forall i \in A, x_i \in X_i$ and functions $L_i\left(x_i\right)$ are continuous and differentiated by x_i. Therefore, the ***objectives*** of the study are to formulate rational variants of loss functions $L_i\left(x\right)$, to formalize for them the optimization tasks of the form (1), synthesis of methods for solving sach the optimization tasks and interpretation of the obtained optimal solutions – equilibrium points for the organization of schemes of the availability to the target ISCU.

4 Models and methods

The mathematical apparatus of game theory for the study of the task of finding the optimal schemes for the availability of ISCU is chosen in particular for the potential property of its optimal solution as Pareto optimality [24-26] which is that the equilibrium point found for the input data indicate an optimal scheme of control of availability of ISCU which not only improves the access conditions for the part of the APs, but also ensures that the access conditions of the other APs don't deteriorate which is especially important given the industry purpose of ISCU. Therefore, we formulate loss functions so that the solutions synthesized on the basis of their optimization tasks are Pareto optimal. With a look at the mathematical apparatus of queuing theory we formalize the loss function $L_i\left(x\right)$ by an expression

$$L_i\left(x\right) = \lambda_i \sum_{j=1}^{m} x_{ij} P_j\left(x\right), \tag{3}$$

where $x \in X$, $i = \overline{1,n}$, $x_i = \left(x_{i1}, \dots, x_{im} \right)$, $P_j\left(x\right)$ is the stationary probabilities that the j th port is busy at the time of accessing the i th AP. Applying to the analyzed model of the availability of ISCU with a parallel operating set of input information ports (1) the method of polynomial approximation [12-16] we decompose it into m one-line Markov QS [17] whose stationary probabilities $P_j\left(x\right)$ are described by an expression

$$P_j(x) = \Lambda_j \left(\mu_j + \Lambda_j\right)^{-1}, \tag{4}$$

where $\Lambda_j = \sum_{j=1}^{m} \lambda_i x_{ij}$, $i = \overline{1,n}$. Let's generalize expressions (3), (4) by analytically formalizing the task of finding a Pareto-optimal scheme of availability of the ISCU with the objective function of the form

$$\min_{x \in X} \sum_{i=1}^{n} \lambda_i L_i(x) = \min_{x \in X} \sum_{i=1}^{n} \lambda_i \sum_{j=1}^{m} x_{ij} \Lambda_j \left(\mu_j + \Lambda_j\right)^{-1} = \sum_{i=1}^{n} \lambda_i L_i\left(x^*\right), \tag{5}$$

where $x^* = \left(x_1^*, \ldots, x_n^*\right) \in X$ is a Pareto-optimal scheme of the control of availability of the ISCU for which the following conditions are fulfilled:

- for all $i = \overline{1,n}$, $j = \overline{1,m}$, the equality $\left.\frac{\partial L_i\left(x_1^*, \ldots, x_{i-1}^*, x_i^*, x_{i+1}^*, \ldots, x_n^*\right)}{\partial x_{ij}}\right|_{x_j^*} = 0$ holds;

- for all $x_{ij} \geq 0$, $i = \overline{1,n}$, $j = \overline{1,m}$, the equality $\sum_{j=1}^{m} x_{ij} = 1$ holds;

- the Hessian matrix of functions $L_i\left(x_1^*, \ldots, x_{i-1}^*, x_i^*, x_{i+1}^*, \ldots, x_n^*\right)$ is positive defined whereby the derivatives $\frac{\partial L_i\left(x_1^*, \ldots, x_{i-1}^*, x_i^*, x_{i+1}^*, \ldots, x_n^*\right)}{\partial x_{ij} \partial x_{ik}}$, $i = \overline{1,n}$, $j,k = \overline{1,m}$, $j \neq k$, that form the matrix are calculated provided that $\sum_{j=1}^{m} x_{ij} = 1$.

We formalize the process of solving the optimization task described above with the objective function (5) and the corresponding constraint conditions using the Lagrange multiplier method [26].

Let's formalize the process of solving the optimization task described above with the objective function (5) and the corresponding constraint conditions using the Lagrange multiplier method [26]. We formulate Lagrange functions of the form

$$Z_i\left(x_1, \ldots, x_n\right) = L_i\left(x_1, \ldots, x_n\right) - \beta_i \left(\sum_j x_{ij} - 1\right), \; i = \overline{1,n}, \; j = \overline{1,m}, \tag{6}$$

where β_i are Lagrange multipliers. Then, given (6) the constraint conditions of the optimization task (5) are generalized by the system of equations of the form

$$\begin{cases} \frac{\partial Z_i\left(x_1^*, \ldots, x_{i-1}^*, x_i^*, x_{i+1}^*, \ldots, x_n^*\right)}{\partial x_{ij}} = 0, \\ \frac{\partial Z_i\left(x_1^*, \ldots, x_{i-1}^*, x_i^*, x_{i+1}^*, \ldots, x_n^*\right)}{\partial \beta_i} = 0. \end{cases} \tag{7}$$

For the optimization task under study with objective function (5) the system (7) with controllable variables β_i, x_{ij} is formed by nonlinear equations of the form

$$\left(\Lambda_j^2+\mu_j\Lambda_j+x_{ij}\lambda_i\mu_j\right)\left(\mu_j+\Lambda_j\right)^{-2}=\beta_i\lambda_i^{-1}, \tag{8}$$

where $i=\overline{1,n}$, $j=\overline{1,m}$, and for values of x_{ij} equality $\sum_{j=1}^{m}x_{ij}-1=0$ must be satisfied.

Simplify the system (7) formed by the equations of the form (8) by summing the latter by $i=\overline{1,n}$. As a result system (7) will be formed by the equations of the form

$$\left(n\Lambda_j^2+(n+1)\Lambda_j\mu_j\right)\left(\Lambda_j+\mu_j\right)^{-2}=\sum_{i=1}^{n}\beta_i\lambda_i^{-1}, \ j=\overline{1,m}. \tag{9}$$

Given that the left-hand side of the equations of form (9) doesn't contain control variables it can be represented as

$$\left(n\Lambda_1^2+(n+1)\Lambda_1\mu_1\right)\left(\Lambda_1+\mu_1\right)^{-2}=\left(n\Lambda_j^2+(n+1)\Lambda_j\mu_j\right)\left(\Lambda_j+\mu_j\right)^{-2}, \ j=\overline{2,m}. \tag{10}$$

Developing the idea underlying expression (10) we present equation (9) like

$$C_{0j}\Lambda_j^2+C_{1j}\Lambda_j+C_{2j}=0, \ j=\overline{2,m}, \tag{11}$$

where $C_{0j}=n\mu_1^2+(n-1)\Lambda_1\mu_1$, $C_{1j}=(n+1)\mu_1^2\mu_j-(n-1)\Lambda_1^2\mu_j$, $C_{1j}=(n+1)\Lambda_1\mu_1-$ $-n\Lambda_1^2\mu_j^2$ are the corresponding coefficients. The analytic solution of system (7) formed by the equations of the form (11) for $\Lambda_j\geq 0\forall j$ will be an expression:

$$\sum_{i=1}^{n}\lambda_i=\sum_{j=1}^{m}\Lambda_j=\Lambda_1-0.5\sum_{j=2}^{m}C_{0j}^{-1}\left(C_{1j}-\sqrt{C_{1j}^2-4C_{0j}C_{2j}}\right)=\Lambda_1+\Lambda_1\mu_1^{-1}\sum_{j=2}^{m}\mu_j,$$

from which we get

$$\Lambda_1=\Lambda_2=\ldots=\Lambda_m=\mu_1\sum_{i=1}^{n}\lambda_i\left(\sum_{j=1}^{m}\mu_j\right)^{-1}. \tag{12}$$

Given that $\sum_{j=1}^{m}x_{ij}-1=0$ we express x_{ij} from expression (8) and obtain a system of equations of the form

$$\beta_i\lambda_i^{-2}\sum_{j=1}^{m}\mu_j^{-1}\left(\mu_j+\Lambda_j\right)^2-\lambda_i^{-1}\sum_{j=1}^{m}\mu_j^{-1}\left(\mu_j+\Lambda_j\right)\Lambda_j=1, \ i=\overline{1,n}, \tag{13}$$

Analytical solution of equation (13) with respect to expression (12) is formalized as

$$\beta_i = \lambda_i^2 \mu (\lambda + \mu)^{-2} + \lambda_i \lambda (\lambda + \mu),\ i = \overline{1,n}, \tag{14}$$

where $\lambda = \sum_{i=1}^{n} \lambda_i$, $\mu = \sum_{j=1}^{m} \mu_j$. Expressions (12) and (14) can be generalized by substituting them into expression (13). As a result we obtain an expression to identify the equilibrium point for the optimization task with the objective function (5):

$$x_{ij}^* = \mu_j \mu^{-1},\ i = \overline{1,n},\ j = \overline{1,m}. \tag{15}$$

The values of the objective functions $L_i(x)$ at the point x_{ij}^* are calculated by the expression

$$L_i(x_1^*, \ldots, x_n^*) = \lambda_i \lambda (\mu + \lambda)^{-1},\ i = \overline{1,n}. \tag{16}$$

Assume that the constraint conditions of the optimization task with the objective function (5) for the equilibrium point (15) are satisfied. Substitute in the second derivative of the functions $L_i(x)$ the coordinates of the equilibrium point x_{ij}^* calculated by the expression (15): $\frac{\partial^2 L_i(x)}{\partial x_{ij}^2} = \mu_j \Lambda_j (\mu_j + \Lambda_j - \lambda_i x_{ij})(\mu_j + \Lambda_j)^{-3} =$ $= \lambda \mu^{-1} (\mu + \lambda - \lambda_i)(\mu + \lambda)^{-1} > 0$. Accordingly for other points: $\frac{\partial^2 L_i(x)}{\partial x_{ij} \partial x_{ik}} = 0$, $i = \overline{1,n}$, $j,k = \overline{1,m}$, $j \neq k$. The Hessian matrix $H(L_i(x))$ is positive defined and the rest of the constraint conditions formulated in the setting of the optimization task with the objective function (5) are satisfied. We also prove that the optimal solution obtained in the form (15) is also Pareto optimal. According to the statement of task (5): $x_{i1} = 1 - \sum_{j=2}^{m} x_{ij}$, $i = \overline{1,n}$, then $\Lambda_1 = \sum_{n=1}^{n} \left(1 - \sum_{j=2}^{m} x_{ij}\right) \lambda_i$, $\Lambda_j = \sum_{i=1}^{n} x_{ij} \lambda_i$, $j = \overline{2,m}$; $L_i(x_1, \ldots, x_n) = \lambda_i \left(\Lambda_1 (\mu_1 + \Lambda_1)^{-1} \left(1 - \sum_{j=2}^{m} x_{ij}\right) + \sum_{j=2}^{m} \Lambda_j (\mu_j + \Lambda_j)^{-1} x_{ij} \right)$, $i = \overline{1,n}$, respectively, $L(x) = \sum \beta L_i(x) = \beta \sum_{i=1}^{n} \lambda_i \left(\Lambda_1 (\mu_1 + \Lambda_1)^{-1} \left(1 - \sum_{j=2}^{m} x_{ij}\right)\right) +$ $+ \beta \sum_{i=1}^{n} \left(\lambda_i \sum_{j=2}^{m} \Lambda_j (\mu_j + \Lambda_j)^{-1} x_{ij} \right)$. We identify the extremum of the function $L(x)$: $\frac{\partial L(x)}{\partial x_{ij}} = -\beta \lambda_i \left(\Lambda_1 (2\mu_1 + \Lambda_1)(\mu_1 + \Lambda_1)^{-2} - \Lambda_j (2\mu_j + \Lambda_j)(\mu_j + \Lambda_j)^{-2} \right) = 0$, $i = \overline{1,n}$, $j = \overline{2,m}$, why simplify the latter: $\mu_1^2 \Lambda_1^2 + 2\mu_1^2 \mu_j \Lambda_j - \mu_j^2 \Lambda_1 (2\mu_1 + \Lambda_1) = 0$ and solve the resulting expression with respect to Λ_j: $\Lambda_j = -\mu_j +$

$+\sqrt{\mu_j^2 + \mu_1^{-2}\mu_j^2\left(2\mu_1 + \Lambda_1\right)\Lambda_1} = \mu_j \mu_1^{-1}\Lambda_1$, where do we express Λ_1: $\Lambda_1 = \mu_1 \sum_{i=1}^{n} \lambda_i \left(\sum_{j=1}^{m} \mu_j \right)^{-1} = \mu_1 \lambda \mu^{-1}$. On the basis of the expression for Λ_1 we determine Λ_j: $\Lambda_j = \mu_j \Lambda_1 \mu_1^{-1} = \mu_j \lambda \mu^{-1}$ and form a system of linear equations $\sum_{i=1}^{n} \lambda_i x_{ij} = \Lambda_j$, an analytic view of the solution whose, $x_{ij} = \mu_j \mu^{-1}$, $i = \overline{1,n}$, $j = \overline{1,m}$, is identical to the expression (15). Therefore formalized by expression (15) the optimal solution of the optimization task with the objective function (5) is Pareto-optimal.

Applying to the analyzed model of the availability of ISCU with a parallel operating set of input information ports (1) the method of polynomial approximation we decompose it into m one-line semi-Markov QS. In this case the stationary probabilities $P_j(x)$ defined by expression (4) will be redefined as:

$$P_j(x) = \Lambda_j \tau_j \left(1 + \Lambda_j \tau_j\right)^{-1}, \tag{17}$$

where $\Lambda_j = \sum_{j=1}^{m} \lambda_i x_{ij}$ are the intensity of incoming requests from the APs, $i = \overline{1,n}$, τ_j is the instantaneous value of the duration of the incoming request service for the j th open port of ISCU - a stochastic variable with arbitrary distribution. Accordingly the analytic appearance of the objective function (5) will change:

$$\min_{x \in X} \sum_{i=1}^{n} \lambda_i L_i(x) = \min_{x \in X} \Lambda_j \tau_j \left(1 + \Lambda_j \tau_j\right)^{-1} = \sum_{i=1}^{n} \lambda_i L_i\left(x^{**}\right). \tag{18}$$

The constraint conditions formulated for the optimization task with the objective function (5) remain relevant also for the optimization task with the objective function (18). Let's carry out the analytical formalization of the process of identification of the equilibrium point $x^{**} = \left(x_1^{**}, \ldots, x_n^{**}\right) \in X$ which is the Pareto-optimal scheme for the control of the availability of ISCU whose loss function $L_i(x)$ given (17) is described by the expression

$$L_i(x) = \lambda_i \sum_{j=1}^{m} x_{ij} \Lambda_j \tau_j \left(1 + \Lambda_j \tau_j\right)^{-1}, \; i = \overline{1,n}. \tag{19}$$

As in the previous study we use the Lagrange multiplier method to describe the process of solving the optimization task with the objective function (18). On the basis of the Lagrange function (6) we generalize the constraint conditions for the optimization task with the objective function (18) with respect to the controlled variables β_i, x_{ij} by a system of nonlinear equations of the form

$$\left(\Lambda_j^2\tau_j^2+\Lambda_j\tau_j+\lambda_i\tau_j x_{ij}\right)\left(1+\Lambda_j\tau_j\right)^{-2}=\lambda_i^{-1}\beta_i \tag{20}$$

at $\sum_{j=1}^{m}x_{ij}-1=0$, $i=\overline{1,n}$. Simplifying the system formed by the equations of the form (20) we consistently obtain analytical expressions to calculate the values of Λ_1, x_{ij}^*:

$$\Lambda_1=\left(\tau_1\sum_{j=1}^{m}\tau_j^{-1}\right)^{-1}\sum_{i=1}^{n}\lambda_i , \tag{21}$$

$$x_{ij}^*=\left(\tau_j\sum_{j=1}^{m}\tau_j^{-1}\right)^{-1}. \tag{22}$$

Assume that for the optimization result obtained in the form of expression (22) the constraint conditions are satisfied: $\frac{\partial^2 L_i\left(x^*\right)}{\partial x_{ij}^2}=\Lambda_j\tau_j\times$ $\times\left(1+\Lambda_j\tau_j-\lambda_i\tau_j x_{ij}\right)\left(1+\Lambda_j\tau_j\right)^{-3}>0$, $\frac{\partial^2 L_i\left(x^{**}\right)}{\partial x_{ij}\partial x_{ik}}=0$, $j,k=\overline{1,m}$, $j\neq k$. The Hessian matrix $H\left(L_i\left(x\right)\right)$ at the point x_{ij}^{**} is positive defined. Therefore, the point x_{ij}^{**} calculated by expression (22) is an equilibrium point and the values of the loss functions at this point are determined by the expression $L_i\left(x^{**}\right)= \ =\lambda_i\sum_{i=1}^{n}\lambda_i\left(\sum_{i=1}^{n}\lambda_i+\sum_{j=1}^{m}\tau_j\right)$, $i=\overline{1,n}$. The procedure for proving the Pareto-optimality of the equilibrium point x_{ij}^* calculated by expression (15) is also valid for the point x_{ij}^{**} calculated by expression (22).

In addition to the variants formalized by expressions (3), (19) the formulation of loss functions $L_i\left(x\right)$ with an orientation on a quantitative estimation of the optimal scheme for the control of the availability of ISCU is of practical importance. We formalize the loss functions in the form of such an expression:

$$L_i\left(x\right)=\lambda_i\sum_{j=1}^{m}x_{ij}K_j\nu_j , \tag{23}$$

where K_j is the cost per unit of waiting time for the release of j th information port of the ISCU and a parameter

$$\nu_j=\Lambda_j\left(\mu_j\left(\mu_j-\Lambda_j\right)\right)^{-1} \tag{24}$$

describes the average waiting time for the release of this information port. For functions $L_i(x)$ defined by expression (23) expression (8) will look like this:

$$\left(K_j\left(\mu_j\left(\Lambda_j+\lambda_i x_{ij}\right)-\Lambda_j^2\right)\right)\left(\mu_j\left(\mu_j-\Lambda_j\right)^2\right)^{-1}=\beta_i\lambda_i^{-1}, \tag{25}$$

where $\sum_{j=1}^{m} x_{ij}=1$, $i=\overline{1,n}$, $j=\overline{1,m}$. Provided that

$$K_j\mu_j^{-1}=K_m\mu_m^{-1},\ j=\overline{1,m-1}, \tag{26}$$

we sum up equation (25) by $i=\overline{1,n}$ and obtain quadratic equation with respect to Λ_j:

$$\left(n+C_m\right)\Lambda_j^2-\mu_j\left(n+2C_m+1\right)\Lambda_j+C_m\mu_j^2=0, \tag{27}$$

where $C_m=\left(\mu_m\Lambda_m\left(n+1\right)-\Lambda_m^2 n\right)\left(\mu_m-\Lambda_m\right)^{-2}$. Solving equation (27) we obtain the value of Λ_j substituting which in equation (25) we get the expression to calculate the coordinates of the equilibrium point x_{ij}^*:

$$x_{ij}^*=\mu_j\mu^{-1},\ i=\overline{1,n},\ j=\overline{1,m}. \tag{28}$$

The value of the loss functions at the equilibrium point x_{ij}^* is analytically formalized by the expression $L_i\left(x_{ij}^*\right)=\lambda_i\lambda\mu^{-1}\sum_{j=1}^{m}K_j\left(\mu-\lambda\right)^{-1}=\lambda_i\lambda\left(\mu\left(\mu-\lambda\right)\right)^{-1}\sum_{j=1}^{m}K_j$, where $\lambda=\sum_{i=1}^{n}\lambda_i$, $\mu=\sum_{j=1}^{m}\mu_j$. The procedure for proving the Pareto-optimality of the equilibrium point for the task of finding the optimal scheme for the control of accessibility of ISCU with the loss functions of the species (23) and the objective function of the species (5) is methodologically identical to that described for expression (15) and allows us to claim that the equilibrium point (28) is a Pareto-optimal if condition (26) is satisfied. Condition (26) is rational since it governs the proportional relationship between the cost of waiting for the AP to gain access and the speed of service of incoming requests of the ISCU.

Finally in order to handle emergencies we mathematically describe the precedent that a i th AU with a probability ω_i or $1-\omega_i$ can qualify its request as a priority or ordinary respectively. Consider this information in the corresponding loss function:

$$L_i\left(\omega_1,\ldots,\omega_n\right)=K_{i1}\lambda_i M\left\{\gamma_1\right\}\omega_i+K_{i2}\lambda_i M\left\{\gamma_2\right\}\left(1-\omega_i\right), \tag{29}$$

where $M\left\{\gamma_1\right\}=M\left\{\gamma_1\left(\omega_1,\ldots,\omega_n\right)\right\}$ and $M\left\{\gamma_2\right\}=M\left\{\gamma_2\left(\omega_1,\ldots,\omega_n\right)\right\}$ are the average delay in satisfaction of priority and ordinary requests for access, arranged in the corresponding queues γ_1 and γ_2 the unit cost of the request in which for the i th AP are K_{i1} and K_{i2} respectively. Therefore, two queues $\Lambda_1 = \sum \lambda_i \omega_i$ and $\Lambda_2 = \sum \lambda_i (1-\omega_i)$ are formed on the open information port of ISCU for priority and ordinary requests respectively. We extend this concept to m open ports of ISCU the parameter of session duration for which is a stochastic quantity with exponential distribution with indicator μ and the queues γ_1 and γ_2 are characterized by the inequalities $(\Lambda_1+\Lambda_2)(m\mu)^{-1} < 1$ and $\Lambda_1 (m\mu)^{-1} < 1$ respectively. As a result we define the parameters $M\{\gamma_1\}$ and $M\{\gamma_2\}$ by expressions $M\{\gamma_1\} = C(m)\left(1-m^{-1}\sum_{i=1}^{n}\rho_i\omega_i\right)^{-1}$ and $M\{\gamma_2\} = C(m)\left(\left(1-\rho m^{-1}\right)\times \times\left(1-m^{-1}\sum_{i=1}^{n}\rho_i\omega_i\right)\right)^{-1}$, where $\rho_i = \lambda_i \mu^{-1}$, $C(m) = \left(m\mu\left(1+(m-1)!(m-\rho)\rho^{-m}\sum_{k=0}^{m-1}\left((k!)^{-1}\rho^k\right)\right)\right)^{-1}$, $\rho = \sum_{i=1}^{n}\rho_i = \mu^{-1}(\Lambda_1+\Lambda_2)$.

Let's redefine the loss function (29) taking into account the above:

$$L_i(\omega_1,\dots,\omega_n) = \lambda_i C(m)\left(K_{i1}\omega_i + K_{i2}(1-\omega_i)\right)\times$$

$$\times\left(1-m^{-1}\sum_{i=1}^{n}\rho_i\omega_i\right)\left(1-m^{-1}\rho\right)^{-1},\ i=\overline{1,n}. \tag{30}$$

The optimization task is to identify the point $\omega^* = \left(\omega_1^*,\dots,\omega_i^*,\dots,\omega_n^*\right)$ for which condition $L_i(\omega^*) \le L_i(\omega)\forall i\in I, \omega\in\Omega$ is satisfied. To find the coordinates of a point ω^* we use the necessary conditions for finding the extremum of a convex function:

$$\begin{cases} \dfrac{\partial L_i(\omega_1,\dots,\omega_i,\dots,\omega_n)}{\partial\omega_i} = 0, \\ \dfrac{\partial^2 L_i(\omega_1,\dots,\omega_i,\dots,\omega_n)}{\partial\omega_i^2} > 0, \end{cases} \quad i=\overline{1,n}. \tag{31}$$

The first condition (31) for the task of identifying the optimal scheme for control of availability of ISCU with loss functions (30) and the objective function (5) is generalized by a system of linear equations with respect to ω_i:

$$m^{-1}\sum_{\substack{k=1\\k\ne i}}^{n}\rho_k\omega_k = \left(K_{i1}\left(1-m^{-1}\rho\right)-K_{i2}\left(1-m^{-1}\rho_i\right)\right)\left(K_{i1}\left(1-m^{-1}\rho\right)-K_{i2}\right),\ i=\overline{1,n}, \tag{32}$$

solving which we obtain the set of admissible schemes for control of availability of ISCU, investigating which using the second condition of (31) we identify the optimal scheme:

$$x_i^* = m\left(\left(n-1\right)\rho_i\right)^{-1}\sum_{\substack{k=1,\\k\neq i}}^{n}\left(r_k-\left(n-2\right)r_i\right) = m\left(\left(n-1\right)\rho_i\right)^{-1}\sum_{k=1}^{n}\left(r_k-\left(n-1\right)r_i\right),\ i=\overline{1,n}, \quad (33)$$

where $r_i = \left(K_{i1}\left(1-m^{-1}\rho\right)-K_{i2}\left(1-m^{-1}\rho_i\right)\right)\left(K_{i1}\left(1-m^{-1}\rho\right)-K_{i2}\right)$ and the values of the coefficients K_{i1} and K_{i2} satisfy the inequalities $K_{i1}K_{i2}^{-1} > \left(1-m^{-1}\rho_i\right)\left(1-m^{-1}\rho\right)^{-1}$.

Thus the section synthesizes mathematical models for the identification of Pareto-optimal schemes for control of accessibility of ISCU with a parallel functioning set of input information ports described by a composition of single-line Markov or semi-Markov QS. The optimization tasks formulated differ in the view of the loss functions of APs which allow one to calculate a quantitative or cost estimate of the identified Pareto-optimal scheme for control of availability of ISCU which takes into account the processing of emergency access requests indicated by the corresponding priority.

5 Experiment and results

Approbation of the models proposed in the article for Pareto-optimal control of availability of ISCU was carried out on a real object - ISCU of the Situation Center of the Information Technology Department of Vinnytsia City Council. The Situation Center has been operational since 2018. Its main function is to capture and store video events in the city, generalizing the incoming video stream from more than 550 camcorders. Employees of the Vinnitsa City Council, the National Police of Ukraine, the Security Service of Ukraine, and municipal enterprises have access to a distributed database of video recordings. In the investigated ISCU B open information ports are allocated for ordinary requests servicing and $C-B$ open information ports are allocated for priority requests, where C is the total number of open information ports available for the APs. Ordinary and priority requests are assumed to arrive at a set of open ISCU ports with intensities λ_O and λ_P respectively. New requests from the APs at the time of receipt of which less than B open information ports are free in ISCU are forwarded to queue Q_1 with capacity N_O. Priority requests at the time of receipt of which all the C open information ports of ISCU are occupied are forwarded to the queue Q_2 with capacity N_P. Both queues are served by information environment of ISCU in accordance with the FCFS algorithm. If a priority access request is in the queue Q_2 it will be satisfied with the ISCU as soon as at least one of the C open information ports is released. If an ordinary access request is registered in queue Q_1 then it will be satisfied with the ISCU information environment if queue Q_2 is empty and the number of free open information ports exceeds B. If at some time moment both queues of ISCU are filled then when a new request is received it is rejected. An arbitrarily accepted by ISCU input request is satisfied with intensity μ_+ or rejected by a system with intensity μ_-. In general an arbitrary query is analyzed by ISCU over a time interval whose value is a stochastic variable with exponential distribution and mathematical expectation $\mu^{-1} = (\mu_+ + \mu_-)^{-1}$. The AP whose request is pending in the queue Q_1 can cancel it with the intensity $\mu_- = \mu_{O-}$. Similarly the AP whose request is pending in queue Q_2 can cancel it with intensity μ_{P-}. The processes that determine the availability of the investigated ISCU are described in the structural scheme shown in Fig. 1.

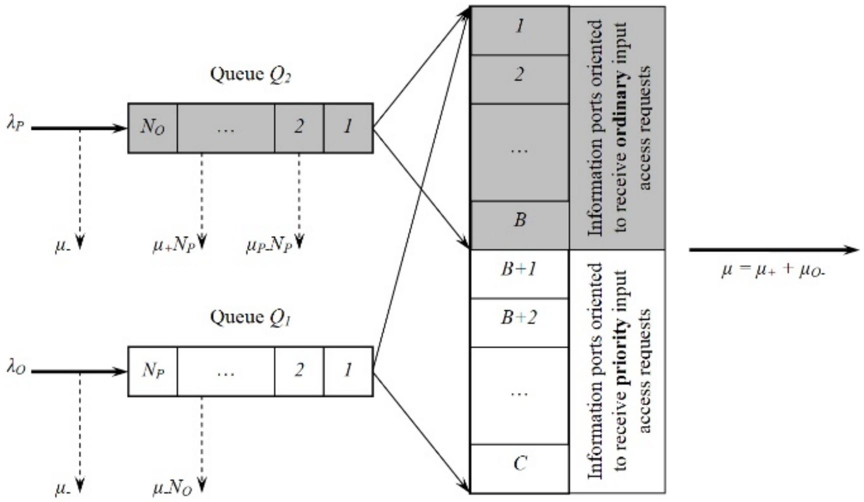

Fig. 1. The structural scheme of the organization of processes that determine the availability of the investigated ISCU.

Within the framework of the above mathematical apparatus we characterize the availability of the investigated ISCU by a two-dimensional random process $X(t) = \{(L_1(t), L_2(t)), t \geq 0\}$ which is realized at a continuous time and can be in discrete states defined by the set $X = \{(i, j) : B \leq i \leq C + N_P, 0 \leq j \leq N_O\}$ $\bigcup \{(i,0) : 0 \leq i \leq B-1\}$. In this case the functions $L_1(t)$ and $L_2(t)$ will describe the total number of incoming requests received by the ISCU and at the time moment $t \geq 0$ waiting for satisfaction in the queues Q_1 and Q_2 respectively. We classify the formalized random process $X(t)$ as a Markov process over the state space X and visualize by the transition intensity diagram shown in the form of the UML state diagram in Fig. 2.

Applying the mathematical apparatus described above to numerically solve the optimization task of control the availability of the investigated ISCU described by QS we find the Pareto-optimal equilibrium point $x^* = \{p(i,j)\}$ which identifies the stationary probabilities $p(i,j)$, $(i,j) \in X$ of states of a Markov process $X(t)$.

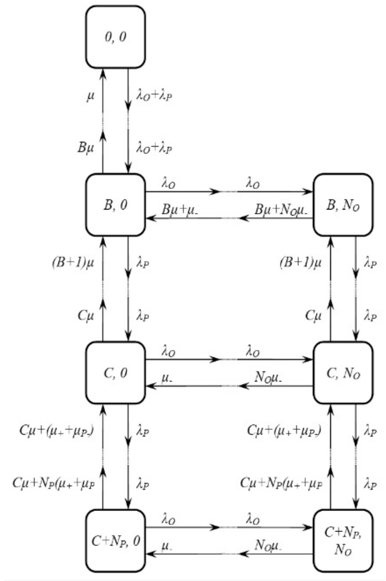

Fig. 2. UML state diagram of the Markov process of control the availability of the investigated ISCU.

At the identified equilibrium point x^* it is possible to analytically formalize such applied probabilistic characteristics of the investigated ISCU as:

- the probability of an incoming request being rejected as a result of the queue Q_1 overflow: $R_O = \sum_{i=B}^{C+N_P} p(i, N_O)$;
- the probability of an incoming request being rejected as a result of the queue Q_2 overflow: $R_P = \sum_{i=0}^{N_O} p(C+N_P, j)$;
- the probability that an incoming request received by the system will be queued: $Z_O = \sum_{i=C}^{C+N_P} \sum_{j=1}^{N_O} p(i,j)$;
- the intensity of the flow of new incoming requests rejected by the ISCU: $A_- = \mu_- Z_O$;
- the intensity of the flow of priority requests received by the ISCU: $A_P = (1 - R_P)\lambda_P$;

- the probability that a priority request will be received after a priority input request to the ISCU: $P_{PP} = A_{-} A_P^{-1}$;
- the probability of premature termination of an active session sanctioned by the investigated ISCU: $P_f = R_P + \left(1 - R_P\right) P_{PP}$.

In Fig. 3 shows the results of a numerical experiment to identify the dependence of the probability of rejection of new incoming requests from the intensity of their receipt calculated for the investigated ISCU with parameters $B = 80$, $C - B = 20$, $N_O = 3 \cdot 10^3$, $N_P = 2 \cdot 10^3$, $\mu_+ = 0.5$, $\mu_- = 0.5$, $\mu_{P-} = 8$.

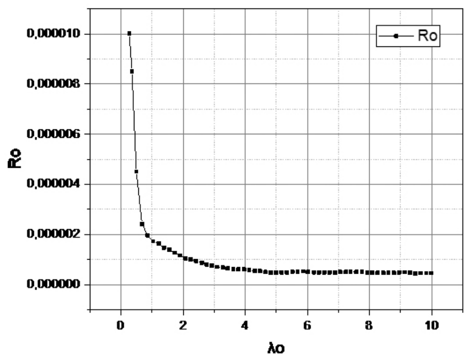

Fig. 3. The dependence of the probability of rejection of new incoming requests on the intensity of their receipt.

Parameters the probability of rejection of new incoming requests and the intensity of their receipt are the main ones that determine the availability of ISCU. Studies have shown that with increasing queue Q_1 capacity the probability of rejection of new incoming requests is significantly reduced. By varying the value of the queue Q_2 capacity with unlimited increase of the intensity of new input requests it is possible to decompose the model of availability of the investigated ISCU into two independent models of QS of the form $M/M/B/Q_O$ and $M/M/C-B/Q_P$ because the high value of the probability of rejection of new input requests due to the increase of λ_O the value of intensity λ_P stabilizes and ceases to depend on the value of λ_O. However, increasing the capacity of queue Q_2 contributes to the positive dynamics of the probability of rejection of received priority requests but increases the probability of rejection of new incoming requests.

Further investigations of the effectiveness of the optimal schemes of control the availability of ISCU was performed while varying the number of open information ports in the range $C = \{30, 50, 100\}$ and the value of the reduced intensity of the input requests which for queue Q_1 would be defined as $\rho_1 = \mu\left(\lambda_O + \lambda_P\right)$ and for queue Q_2 as $\rho_2 = \mu\lambda_P$. In further investigations we will take $\rho = \rho_1$ at $\rho_2 = const$ which will allow us to represent the results of experiments in the two-dimensional space.

Determine the empirical dependence of P_W – the probability that the incoming request received by the investigated ISCU will be allocated into the queue, from the values of parameters ρ and C. Taking into account the model of the investigated ISCU shown in Fig. 1 and 2 we describe analytically functional dependence $P_W = f(\rho, C)$ by the expression $P_W = \rho_1^C \left((C-1)!(C-\rho_2)\right)^{-1} P_0$ where the probability $P_0 = \left(1+\rho_1(1!)^{-1}+\rho_1^2(2!)^{-1}+\ldots+\rho_1^{C-1}\left((C-1)!\right)^{-1}+ \quad +\rho_1^C\left((C-1)!(C-\rho_2)\right)^{-1}\right)^{-1}$. The empirical values of P_W for the optimal scheme of control the availability of investigated ISCU for $C=\{30,50,100\}$ and $\rho = 0, 0.1, \ldots, 10$ are shown in Fig. 4.

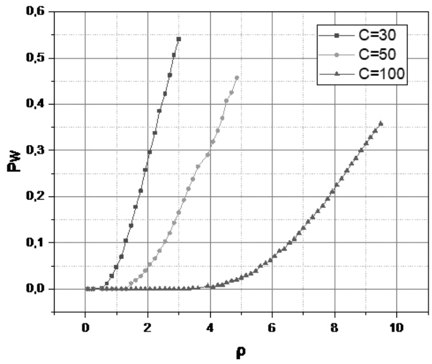

Fig. 4. The dependence of the probability that the incoming request received by the investigated ISCU will be allocated into the queue on the values of the parameters ρ and C.

From the results shown in Fig. 4 it can be seen that when the value of $\rho = 1$ when the intensity of the analysis of the incoming requests by ISCU and the intensity of the receipt of new requests coincide there are enough $30 < C < 50 < 100$ active input information ports to prevent the incoming requests from being rejected by the investigated system. It is also obvious that the dependence of $P_W = f(\rho, C)$ is nonlinear but as the value of parameter C increases the steepness of the value of parameter P_W decreases.

Determine the empirical dependence of $\overline{L}_Q$ – the average length of the queue of received incoming requests in the investigated ISCU on the parameters ρ and C. Taking into account the model of the investigated ISCU shown in Fig. 1 and 2 we describe analytically the functional dependence of $\overline{L}_Q = f(\rho, C)$ by the expression $\overline{L}_Q = (C-\rho_2)^{-1}\rho_2 P_W$. The empirical values of $\overline{L}_Q$ for the optimal scheme of control the availability of investigated ISCU for $C=\{30,50,100\}$ and $\rho = 0, 0.1, \ldots, 10$ are shown in Fig. 5.

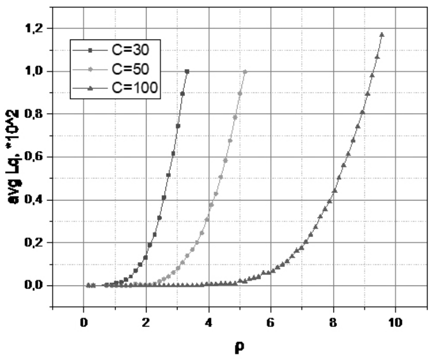

Fig. 5. The dependence of the average length of the queue in the investigated ISCU on the parameters ρ and C

From the results shown in Fig. 5 it can be seen that for optimal scheme of control the availability of investigated ISCU the queue begins to form when the value of the intensity of the new requests is exceeded the intensity of the analysis of the incoming requests by the ISCU is more than: - 3 times when $C = 30$; - 5 times when $C = 50$; - 9 times when $C = 100$. Therefore the Pareto-optimal scheme of control the availability of investigated ISCU is effective both with a limited number of input information ports and demonstrates a nonlinear increase in the efficiency of use of the input of the information system as it expands.

Determine the empirical dependence of $\overline{t}_{wq}$ – the mean time of the received input request staying in the queue waiting for access on the parameters ρ and C. Taking into account the model of the investigated ISCU shown in Fig. 1 and 2 we describe analytically the functional dependence of $\overline{t}_{wq} = f\left(\rho, C\right)$ by the expression $\overline{t}_{wq} = \left(\mu\left(C - \rho_2\right)\right)^{-1} P_W$. The empirical values of $\overline{t}_{wq}$ for the optimal scheme of control the availability of investigated ISCU for $C = \left\{30, 50, 100\right\}$ and $\rho = 0, 0.1, \ldots, 10$ are shown in Fig. 6.

From the results shown in Fig. 6 it can be seen that for optimal scheme of control the availability of investigated ISCU the mean time of the received input request staying in the queue waiting for access is non-zero at: - $\rho > 0.3$ when $C = 30$; - $\rho > 1.9$ when $C = 50$; - $\rho > 5$ when $C = 100$. The steep rise in the value of the parameter $\overline{t}_{wq}$ with the increase of the value of the parameter ρ decreases markedly with the increase of the value of the parameter C which confirms the effectiveness of the Pareto-optimal scheme of control the availability of investigated ISCU.

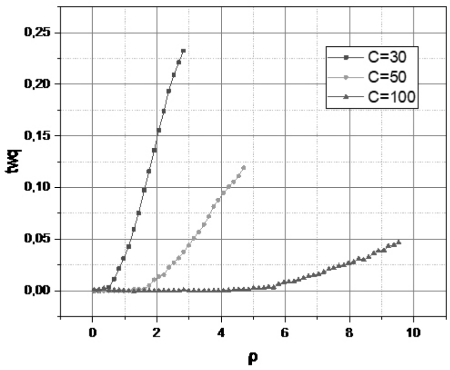

Fig. 6. The dependence of the mean time of the received input request staying in the queue waiting for access on the parameters ρ and C

6 Conclusions

The relevance of the task of control the availability of information systems in general and of ISCU in particular is increasing simultaneously with the information needs of humanity. However given the importance of an information resources of ISCU introducing new schemes of control the availability without a simulation stage can lead to significant material and in particular human losses. These circumstances led to the expediency of research the scientific and applied results of which are presented in the article.

In the article the new mathematical models for identification of the Pareto-optimal schemes of control of accessibility of ISCU with a parallel functioning set of input information ports are synthesized. The ISCU is described by the composition of single-line Markov or semi-Mark queuing systems in which unlike existing ones availability loss functions are formulated with a focus on the ability of quantitative or qualitative estimation of the scheme of control of availability and guarantee the processing of access requests with different priority degree which allows to set and solve the multicriteria optimization task for an identification of problem-oriented scheme of control of availability of ISCU taking into account architectural features of the latter. In the article on the basis of the identified Pareto-optimal scheme of control of availability of ISCU which represented by the equilibrium point analytically formalized such applied probabilistic characteristics of an investigated system as: the probability of an incoming request being rejected as a result of the queue of ordinary requests overflow; the probability of an incoming request being rejected as a result of the queue of priority requests overflow; the probability that an incoming request received by the system will be queued; the intensity of the flow of new incoming requests rejected by the ISCU; the intensity of the flow of priority requests received by the ISCU; the probability that a priority request will be received after a priority input request to the ISCU; the probability of premature termination of an active session sanctioned by the investigated ISCU.

To verify the models synthesized in the article the Pareto-optimal scheme of control of availability of information system of the Situation Center of the Information Technology Department of the Vinnytsia City Council was implemented. It turned out that parameters the probability of rejection of new incoming requests and the intensity of their receipt are the main ones that determine the availability of ISCU.

Further research is planned to be devoted to the analysis of the conformity of the obtained Pareto-optimal scheme of control of availability of ISCU with current information security standards.

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