Modification of the Software System for the Automated Determination of Morphological and Rhythmic Diagnostic Signs by Electrocardio Signals

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Abstract. In this paper we consider a modernized of the software system for the automated determination of morphological and rhythmic diagnostic signs of electrocardio signal. The modification of the system is to create a new method and appropriate software that involves processing electrocardio signals by reducing the discrete cyclic random process, as a model of electrocardio signal to isomorphic random periodic sequence. The use of a new mathematical model of electrocardio signals in the form of conditional discrete cyclic random process allowed to take into account and carry out automatic determination of both morphological and rhythmic diagnostic sings of electrocardio signal within the same mathematical model. The use of a new method of statistical processing based on the new model, allowed to obtain statistical characteristics that are infomative diagnostic signs (morphological and signs of rhythm) of the electrocardio signal. The application of the method of reducing a discrete cyclic random process to isomorphic random periodic sequence before the procedures of statistical processing of the electrocardio signal, in particular before obtaining morphological and rhythmic sings allowed to increase the speed of automated processing of the electrocardio signal in comparison with the previously developed methods which were based on it model in the form of a cyclic random process and did not account for the double stochastic model.In the structure of the modified software system, after the evaluation of the rhythmic structure of the electrocardio signal and the procedure of reduction to a random periodic sequence of a discrete cyclic random process, its processing is branched into two parallel stages. The first stage carries out the morphological analysis, which involves the statistical processing of the electrocardio signal, normalization of statistical estimates and their distribution in the Chebyshev base and decision making on the obtained morphological sings.

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Keywords: Software System, Electrocardio Signal, Analysis Of Heart Rhythm, Morphological Analysis Of Electrocardio Signals.

1 Introduction

Known processes and phenomena of reality are those that reflect over a repeating structure in time. Electrocardio signals are known and well-studied among such processes, signals. The current state of information technology development allows to approach the processing of such signals in a new way and solve the advanced tasks of modern medicine efficiently in the construction of diagnostic apparatus by creating new effective mathematical means which make it possible to increase the accuracy and informativeness of the processing of cyclic signals, in particular electrocardio signals [1-12]. Therefore, the development of diagnostic software systems for automated diagnosing the human heart condition by registered electrocardio signals is a relevant scientific and technical task, the solution of which will enable to improve the quality and efficiency of diagnosing of the functional state of the heart and cardiovascular system of the human body as a whole.

2 Related works

New mathematical models and methods of processing cyclic electrocardio signals have been developed and substantiated in papers [13, 14]. The use of new mathematical models has permitted to improve the accuracy and reliability of diagnosing the functional state of the heart by increasing the informativeness of morphological analysis and heart rhythm analysis. The software system for modeling and conducting morphological analysis, electrocardio signal analysis of heart rhythm on the basis of the developed mathematical models is presented in paper [15]. This system has been modernized and embodied newly developed methods for the electrocardio signals processing.

3 Overview of the Research

Increasing the speed of methods of statistical processing of electrocardio signals, along with increasing the informativeness of their automated analysis is achieved through the use of a new mathematical model that reflects the double stochasticity of the investigated electrocardio signals (stochastic morphological and rhythmic structures) and applying a new method of reducing a discrete cyclic random process to an isomorphic random periodic sequence.

This paper is devoted to the improvement of the software system, where in contrast to the previous development [13, 14], the system includes the new method that provides the processing of electrocardio signals by reducing to an isomorphic random periodic sequence of a discrete cyclic random process, as a model of electrocardio signal, which allowed to increase the speed of its processing in comparison with the previously developed methods by reducing the computational complexity of known statistical methods of estimating the probabilistic characteristics of cyclic random processes of a discrete argument.

4 Proposed model

The purpose of the research is to modernize the software system for morphological analysis and heart rhythm analysis with increased informativeness, on the account of using a new mathematical model of heart signals, in the form of a conditional discrete circular random process, and a new statistical processing method, namely the method of reducing a discrete cyclic random process to an isomorphic random periodic sequence, that by reducing the computational complexity of the electrocardiographic signal processing method, made it possible to speed up their processing in computer cardiac diagnostic systems compared to previously developed methods.

5 Results & Discussion

Here are presented the basic mathematical relations that underlie mathematical support in the modified software system. Still, we will focus on the developed part of the complex, which relates to the reduction to an isomorphic random periodic sequence of a discrete cyclic random process, as a model of the electrocardio signal. The software system enables the automated analysis of the electrocardio signal, particularly its morphological and rhythmic signs.

The mathematical model of electrocardio signal in the form of a cyclic random process and conditional cyclic random process.

It is known from [13] that a discrete random process: $\{\xi(\omega, t_{ml}), \omega \in \Omega, t_{ml} \in \mathbf{D}\}$ is called a cyclic discrete random process if there is such a discrete function $T(t_{ml}, n)$ (rhythm function) that satisfies the conditions: 1) $T(t_{ml}, n) > 0$, if n > 0; 2) $T(t_{ml}, n) = 0$, if n = 0; 3) $T(t_{ml}, n) < 0$, if n < 0; 4) for any $t_{m_l l_1} \in \mathbf{D}$ and $t_{m_2 l_2} \in \mathbf{D}$, for which $t_{m_2 l_2} > t_{m_l l_1}$, for function $T(t_{ml}, n)$ inequality $t_{m_l l_1} + T(t_{m_l l_1}, n) < t_{m_2 l_2} + T(t_{m_2 l_2}, n), \forall n \in \mathbf{Z}$ is applied; that finite-dimensional vectors ($\xi(\omega, t_{m_l l_1}), \xi(\omega, t_{m_2 l_2}), \dots, \xi(\omega, t_{m_k l_k})$) and $(\xi(\omega, t_{m_l l_1} + T(t_{m_l l_1}, n)), \xi(\omega, t_{m_k l_k}), t_{m_k l_k})$, $n \in \mathbf{Z}$, at all integer $k \ge 1$ is stochastically equivalent in a broad sense.

The method of reducing the statistical processing (estimation, analysis, prediction) of a cyclic random process of a discrete argument to the corresponding statistical processing of an isomorphic periodic random sequence consists in gradual execution of the following steps:

1) transformation of a ω -realization $\xi_{1_{\omega}}(t_{ml}), \omega \in \Omega, t_{ml} \in \mathbf{D}$ of a cyclic random process $\xi_1(\omega, t_{ml}), \omega \in \Omega, t_{ml} \in \mathbf{D}$ in a ω -realization $\xi_{2_{\omega}}(i), i \in \mathbf{Z}$ of an isomorphic

(for the process) in relation to order and values of a *L*-periodic sequence $\xi_2(\omega, i), \omega \in \Omega, i \in \mathbb{Z}$, by the action of a scale transformation operator $\mathbf{G}_{y(t_{ml})}$ with a scale transformation function $y(t_{ml}) = L \cdot (m-1) + l$;

2) application of known methods of processing periodic random sequences and obtaining their results (statistical point and interval estimates of certain probabilistic characteristics);

3) obtaining statistical estimates of the probabilistic characteristics of a cyclic random process $\xi_1(\omega, t_{ml}), \omega \in \Omega, t_{ml} \in \mathbf{D}$, by the application inversion operator of the scale convension to previously obtained appropriate statistical estimations for *L*periodic random sequence.

We will assume, that is recorded M cycles by L counts in each cycle of the investigated cyclic signal whose mathematical model is a cyclic random process $\left\{\xi_1(\omega, t_{ml}), \omega \in \Omega, t_{ml} \in \mathbf{R}, m = \overline{1, M}, l = \overline{1, L}\right\}$ (for the simplified further marking $\xi_1(\omega, t_{ml})$ will be written). Relatively, mathematical model of a cyclic signal registrogram will be a ω -realization $\left\{\xi_{1_{\omega}}(t_{ml}), t_{ml} \in \mathbf{R}, m = \overline{1, M}, l = \overline{1, L}\right\}$ (for the simplifying of further marking $\xi_{1_{\omega}}(t_{ml})$ will be written) of this cyclic random process of a discrete argument. The isomorphic for the investigated discrete process in relation to order and values of a L-periodic sequence $\left\{\xi_2(\omega, i), \omega \in \Omega, i = \overline{1, M \cdot L}\right\}$ (for the simplifying of further marking $\xi_2(\omega, i)$ will be written) is obtained, by the action of a scale transformation operator with a scale transformation function $y(t_{ml}) = L \cdot (m-1) + l$ to initial random process $\xi_1(\omega, t_{ml})$, namely:

$$\xi_2(\omega, i) = \mathbf{G}_{v(t_{ml})} \{\xi_1(\omega, t_{ml})\}$$
(1)

which is equivalent to a such system of equations:

$$\begin{cases} i = y(t_{ml}) = L \cdot (m-1) + l, \ m = \overline{1, M}, \ l = \overline{1, L}, \\ \xi_2(\omega, i) = \xi_1(\omega, t_{ml}), \ i = \overline{1, M \cdot L}, \ t_{ml} \in \mathbf{R}. \end{cases}$$
(2)

The same scale transformation operator is related M -cyclic ω -realization cyclic random process $\xi_1(\omega, t_{ml})$ and M -cyclic ω -realization $\left\{\xi_{2_{\omega}}(i), i = \overline{1, M \cdot L}\right\}$ (for the simplifying of further marking $\xi_{2_{\omega}}(i)$ will be written).

The analytical formula for calculating the value of statistical estimation of the initial moment function of the first order (mathematical expectation) L-periodic sequence $\xi_2(\omega, i)$, which is isomorphic in relation to the order and values of the cyclic process $\xi_1(\omega, t_m)$, has the form of

$$\hat{m}_{\xi_2}(l) = \frac{1}{M} \sum_{n=0}^{M-1} \xi_{2_{\omega}}(l+L \cdot n), \ l = \overline{1, L}$$
(3)

The analytical formula for the calculating the value of statistical estimation of central moment function of the second order (dispersion) *L*-periodic sequence $\xi_2(\omega, i)$, which is isomorphic in relation to the order and values of the cyclic process $\xi_1(\omega, t_{ml})$, has the form of

$$\hat{d}_{\xi_2}(l) = \frac{1}{M-1} \sum_{n=0}^{M-1} (\xi_{2_{\omega}}(l+L \cdot n) - \hat{m}_{\xi_2}(l))^2, \ l = \overline{1,L}$$
(4)

Due to the statistical procedure for the electrocardio signal processing normalization received statistical estimation and their reduction in Chebyshev basis, that is investigated in paper [13].

A mathematical model of electrocardio signal will be considered below. A model takes into account their double stochasticity, namely, morphological structure stochasticity and stochasticity of the rhythmic structures of electrocardio signal. The conditional cyclic random process is called a process $\{\xi(\omega, \omega', t), \omega \in \Omega, \omega' \in \Omega', t \in \mathbf{R}\}$, that is set on the Cartesian product of two stochasticly independent probabilistic spaces with the sample sets Ω and Ω' , and on the set of real numbers \mathbf{R} , and for which such conditions are satisfied:

- 1. a such random function exists $T(\omega', t, n)$, $\omega' \in \Omega'$, $t \in \mathbb{R}$, $n \in \mathbb{Z}$, that for each ω' , relatively ω' -realization $T_{\omega'}(t, n)$ of this function, satisfies the conditions of rhythm function;
- 2. for each ω' from Ω' finite-dimensional vectors $(\xi_{\omega'}(\omega, t_1), \xi_{\omega'}(\omega, t_2), ..., \xi_{\omega'}(\omega, t_k))$ and $(\xi_{\omega'}(\omega, t_1 + T_{\omega'}(t_1, n)), \xi_{\omega'}(\omega, t_2 + T_{\omega'}(t_2, n))), ..., \xi_{\omega'}(\omega, t_k + T_{\omega'}(t_k, n))$, $n \in \mathbb{Z}$, where $\{t_1, t_2, ..., t_k\}$ separable set of the process $\xi_{\omega'}(\omega, t), \omega' \in \Omega', \omega \in \Omega, t \in \mathbb{R}$, for all integer $k \in \mathbb{N}$ is a stochastic equivalent in a broad sense;
- 3. for any different $\omega'_1 \in \mathbf{\Omega}'$ and $\omega'_2 \in \mathbf{\Omega}'$ random processes $\xi_{\omega'_1}(\omega, t)$ and $\xi_{\omega'_2}(\omega, t)$ are isomorphic in relation to the order and values of the cyclic random process.

A mathematical model of a rhythm cardio signal with increased resolution, according to the paper [14], is a discrete random process $\left\{T(\omega', t_{ml}, n), \omega' \in \Omega', t_{ml} \in \mathbf{R}, m \in \mathbf{Z}, l = \overline{1, L}, L \ge 2, n \in \mathbf{Z}\right\}$, which is embedded in a random rhythm function $T(\omega', t, n), \omega' \in \Omega', t \in \mathbf{R}, n \in \mathbf{Z}$ of a conditional cyclic ran-

dom process $\{\xi(\omega, \omega', t), \omega \in \Omega, \omega' \in \Omega', t \in \mathbf{R}\}$. The first stage of a heart rhythm analysis on the basis of a rhythm cardio signal with increased resolution is a formation of a vector of random stationary and stationary connected sequences $\mathbf{\Xi}_L(\omega', m) = \{T_l(\omega', m), \omega' \in \Omega', l = \overline{1, L}, m = \overline{1, M}\}$. Then the statistical processing of a vector component is conducted. At the same time a mathematical expectation, dispersion, the type of distribution (checking it for normality) is evaluated, by the building a histogram and the use of Pearson's agreement criterion χ^2 . We present the basic mathematical relations for estimating the probabilistic characteristics of the components of this vector of random sequences.

An expression for calculating a realization of a statistical estimation of $\hat{c}_{1_{T_l}}$ of the relative vector component of the first-order initial moment $c_{1_{T_l}}$ (mathematical expectation) of a stationary random sequence $T_l(\omega', m)$, namely:

$$\hat{c}_{1_{T_{l}}} = \frac{1}{M} \sum_{k=1}^{M} T_{l_{o'}}(k), l \in \left\{\overline{1, L}\right\}$$
(5)

where M - the number of cycles of registered realization of an electrocardio signal, $T_{l_{of}}(k) - l$ vector component rhythmic cardio signal.

Statistic estimation of autocorrelation function will have the form of:

$$\hat{r}_{2_{T_{l_{1}}T_{l_{2}}}}(u) = \hat{r}_{2_{T_{l_{1}}T_{l_{2}}}}(m_{1} - m_{2}) = \frac{1}{M - M_{1} + 1} \sum_{k=0}^{M - M} \left[T_{l_{1_{\omega'}}}(k) - \hat{c}_{1_{T_{l_{1}}}} \right] \cdot \left(T_{l_{2_{\omega'}}}(k + u) - \hat{c}_{1_{T_{l_{2}}}} \right),$$
$$u = \overline{0, M_{1} - 1}, m_{1}, m_{2} \in \left\{ \overline{1, M_{1}} \right\}, l_{1}, l_{2} \in \left\{ \overline{1, L} \right\}$$
(6)

where M_1 - number of counts of correlation function, depth of correlation.

The decreasing of the number of diagnostic signs in an information system of heart rhythm analysis on the basis of the main vector of a rhythm cardio signal with increased informativeness is achieved by the use of a spectral decompositions of triangular matrix elements $\hat{\mathbf{R}}_T = \left[\hat{r}_{2\tau_{l_1}\tau_{l_2}}(u), l_1 = \overline{1, L}, l_2 = \overline{l_1, L}\right]$, in particular, by the using of a discrete Fourier transform for the estimation of the autocorrelation and intercorrelation functions of this matrix. Namely, instead of a triangular matrix $\hat{\mathbf{R}}_T = \left[\hat{r}_{2\tau_{l_1}\tau_{l_2}}(u), l_1 = \overline{1, L}, l_2 = \overline{l_1, L}\right]$ of correlation functions, a triangular matrix

 $\hat{\mathbf{S}}_{T} = \left[\hat{S}_{2\tau_{l_{1}}\tau_{l_{2}}}(v), l_{1} = \overline{\mathbf{1}, L}, l_{2} = \overline{l_{1}, L}\right] \text{ could be used, elements of which are the Fourier transform of the image of relative estimations of a correlation function from the matrix <math>\hat{\mathbf{R}}_{T}$. Namely, the Fourier transform of the image from the matrix $\hat{\mathbf{S}}_{T}$ are calculated as:

$$\hat{S}_{2_{T_{l_1}T_{l_2}}}(\nu) = \sum_{u=0}^{M_1-1} \hat{r}_{2_{T_{l_1}T_{l_2}}}(u) \cdot e^{\frac{-j2\pi u \nu}{M_1}}, \nu = \overline{0, M_1 - 1},$$

$$l_1 = \overline{1, L}, l_2 = \overline{l_1, L}, j = \sqrt{-1}$$
(7)

The structural and functional scheme of the modified software system is presented on the fig. 1. The system of programs is implemented in a programming language Object Pascal.

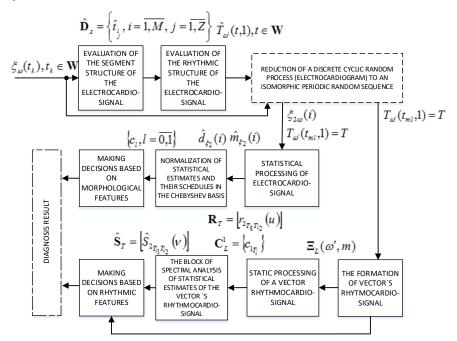


Fig. 1. Structural-functional scheme of the software system for the analysis of morphological and rhythmic diagnostic of the electrocardio signal signs

The processing procedure of the investigated electrocardio signal involves the estimation of the segmental by means of the segmentation methods. The estimation of rhythm function is conducted by interpolation of rhythmic structure (discrete rhythm function). After the evaluation of a rhythmic structure and the procedure of reduction to a random sequence of a discrete cyclic random process of processing branches into two parallel stages. The first stage conducts a morphological analysis which according to the given structure provides a statistical processing of the electro cardio signal, normalization of statistical estimations and their reduction in Chebyshev basis and making decision due to the obtained morphological sings. The second stage conducts the rhythm analysis and consists in the formation of the vector rhythm cardio signal, the statistical vector processing and spectral analysis of the obtained statistical estimations [13].

As an example, the Fig. 2 shows the general view of the program interface is given for the statistical processing of the electrocardio signal, which provides the use of the method of reducing the cyclic random process of a discrete argument to isomorphic random sequence.

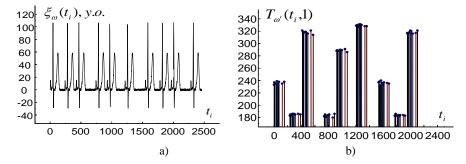
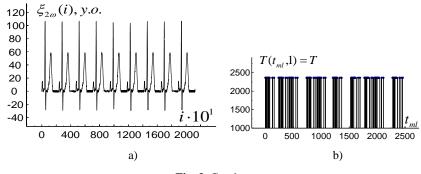


Fig. 2. Results of the processing: a) a few cycles of the examined electro cardio signal; b) rhythmic structure of the electro cardio signal, the red color defines the samples corresponding to R-R - intervals

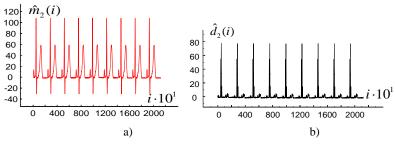
Figures 3-6 show the graphs for the explanation of the stages of the processing of electrocardio signals by the software system.





a) a few cycles of the realization of the L-periodic random sequence, which is obtained from the electro cardiogram, the action of a scale transformation operator;

b) the estimations of rhythm function $T(t_{ml}, n)$ (at n = 1) of a L-periodic random sequence





a) a few estimation cycles $\hat{m}_{\xi_2}(i)$ of the initial moment function of the first order of the *L*-periodic random sequence $\xi_2(\omega, i)$;

b) a few estimation cycles $\hat{d}_{\xi_2}(i)$ of the central moment function of the second order of the *L* - periodic random sequence $\xi_2(\omega, i)$

A separate vector component of the rhythm cardio signal and the results of its statistical processing is depicted on the figures 4-6.

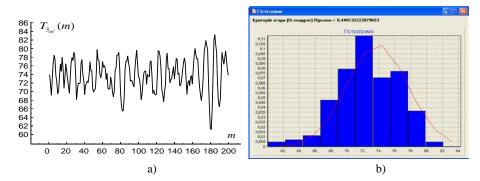
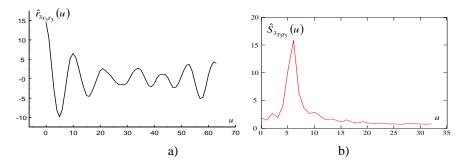


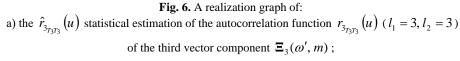
Fig. 5. Graphs of:

a) a realization $T_{3_{\omega'}}(m)$ of the third vector component of rhythmic cardio signal $T_3(\omega',m)$,

which describes the duration of the T-intervals in the electro cardio signal; b) histogram for the third component $T_3(\omega', m)$, which describes the duration of the T-intervals in the electro cardio signal

In this paper, the mathematical support of the software system for increasing the speed of electrocardio signal processing in comparison with previously known methods is substantiated; the structural and functional scheme of the modernized software system is developed; developed a program that implements a new method of reducing a discrete random process to an isomorphic random periodic sequence, which will achieve faster processing of electrocardio signals in computer cardiodiagnostic systems; the modernization of the modernized software system on real electrocardio signals was carried out.





b) $\hat{S}_{3_{T_3T_3}}(v)$ statistical estimations of a spectral power densities $S_{3_{T_3T_3}}(v)$ ($l_1 = 3, l_2 = 3$) of the third vector component $\Xi_3(\omega', m)$

6 Conclusion

Modernized software system due to the extension of its mathematical software, which is based on a new approach to the processing of electrocardiograms based on a mathematical model in the form of a conditional cyclic random process and a meth-od of reducing their mathematical model in the form of a discrete cyclic random process to an isomorphic random sequence morphological analysis and analysis of the rhythm of the cardio signals with increased informativeness, which made it possible to increase the speed of their processing and increase the accuracy and reliability of diagnosis of the cardiovascular system of the human body.

The created program system can be used as a component of specialized software in automated diagnostic systems for system morphoanalysis and heart rhythm analysis

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