A Short Note on the Chisholm Paradox

Guido Governatori

Data61, CSIRO, Australia

Abstract. We advance an alternative version of the Chisholm Paradox and we argue that the alternative version (while logically equivalent to the original version), in its manifestation in the natural language, is not intuitively consistent. The alternative version of the paradox suggests some requirements for deontic logics designed for legal reasoning.

Keywords: Chisholm Paradox, Deontic Disjunctive Syllogism

The Chisholm paradox [2] introduced the topic of the so-called contrary-to-duty obligations and the problems related to their formalisation in Deontic Logic. A cornucopia of research sparkled from the seminal paper and a multitude of logical systems have been proposed to address the formalisation of CTDs. Chisholm proposed a set of four statements that seem logically independent from each other when formulated in natural language but whose formal representation in (Standard) Deontic Logic either leads to an inconsistency or the statements are no longer logically independent.

The formulation of the paradox reads as follows:

S1 A person ought to help his neighbour.
S2 It ought to be that if the person helps the neighbour he has to tell he is going to help.
S3 If the person does not help the neighbour then he ought not to tell that he is going to help.
S4 the person does not help the neighbour.

Literature understood the above set of statements to be intuitively consistent.

> There is a widespread *agreement* in the literature that, form the intuitive point of view, this set is consistent, and its members are logically independent of each other; [1, p. 294]

However, when the four statements are encoded in (standard) Deontic Logic by the following formulae

L1 O$help$
L2 O$(help \rightarrow tell)$

L3 $\neg help \rightarrow \mathrm{O}\neg tell$
L4 $\neg help$

we obtain a contradiction given that from L1 and L2 we derive O$tell$ and from L3 and L4 we conclude O$\neg tell$. Moreover, when we replace L2 with $help \rightarrow \mathrm{O}tell$, then the alternative version of L2 is a logical consequence of L4, and the four statements are no longer independent of each other.

Chisholm implicitly implies that the four statements in their natural language manifestation are consistent, though there is no real argument supporting the conclusion that they must be consistent. Similarly, Åqvist [5, pp. 365–6] proposed a set of three requirements that (r1) the statements are mutually consistent, (r2) none of the statements logically follows from the other three statements and (r3) the obligation of the contrary-to-duty statement should be obtained. Åqvist does not provide a justification for the first requirement. Similarly, Tomberlin [4, p. 359], who otherwise challenged the other requirements, accepted, with no discussion, that the four statements are, plainly, mutually consistent. However, to the best of our knowledge, no real discussion or empirical evaluation of the claimed consistency has been proposed or carried out.

In this short note, we introduce an alternative formulation of the Chisholm set that is logically equivalent to the original set when represented in (Standard) Deontic logic. Nevertheless, there is a fundamental difference: the set of statements is no longer consistent. Before giving the alternative version and the justification of why that version is not consistent, we return on the analysis provided by Chisholm. Chisholm [2, p. 34] suggested that $\mathrm{O}(a \rightarrow b)$ is not *adequate* for the expression of contrary-to-duty imperatives, and (ii) advanced the following reading: one should refrain from the joint action of doing a and not doing b. Hence, Chisholm proposed to read the obligation as $\mathrm{O}(\neg(a \wedge \neg b))$. Let us take an extra step, and let us apply the De Morgan Law to the content of the obligation. This gives us $\mathrm{O}(\neg a \vee b)$, where the intuitive reading is that one is obliged to choose between refraining from doing a or doing b. According to what we have just discussed, the alternative version of the set of statements replaces the second statement with the following one

S2' it ought to be that the person chooses between not helping the neighbour or to tell the neighbour that he is going to help.

The statement can be naturally represented by

$$\mathrm{O}(\neg help \vee tell) \tag{L2'}$$

Clearly, given the logical equivalence of the logical representations, we formally conclude that the alternative formal representation is logically inconsistent. What it remains to do is to investigate whether the alternative version in natural language is intuitively consistent or not. This leads us to consider the plausibility of what we can call *Deontic Disjunctive Syllogism*. The Deontic Disjunctive Syllogism is the inference pattern that from the obligation of a conjunction derives the obligation of one of the disjuncts when the other disjunct is forbidden (or when the other disjunct leads to a violation).

Consider the rules of sudoku (9×9) that state that

1. for every cell, for every row, column or block the cell must contain one digit from 1 to 9; and

2. for every row, column or block, if a cell contains a digit, no other cell in the same row, column or block can contain that digit.

9	5	8	7	6	4	2	3	
3				8		7		5
	1	7		3	5	8	4	
6	7	9	4	2	8	5	1	3
	8		5	7	3	6	2	9
5	2	3	6			4	8	7
	9	2	8	5		3		
		5	3			1		2
	3		2		7	9	5	

Fig. 1. A Partially Solved Sudoku Puzzle

Consider now a situation where two cells in a column have not been filled (see the 4th column in the sudoku diagram in Figure 1, where the missing digits are 1 and 9). This means that only the remaining two digits can occur in these cells. Accordingly, we can say, that it the cell in the 3rd row, and 4th column must contain either the digit 1 or the digit 9. The same applies to the cell in column 4 and row 2. However, in one of the rows for the empty cells (namely the cells in the third row in the diagram), one cell contains one of the two digits (i.e., 1). Hence, to obtain a legal solution, the other digit must be in the cell. Alternatively, we can say that, to get a valid solution, it is forbidden to put the digit 1 in the fourth cells in the third row. Thus, the digit that must occur in that cell is 9.

Based on the discussion we had so far, the Deontic Disjunctive Syllogism appears to be a reasonable, sound and intuitive inference pattern for reasoning with (disjunctive) obligations. This view is also shared by Horty [3, p. 430–431] who proposed the example of two norms "fight in the army or perform alternative service" and "don't fight in the army" (possibly from two difference sources), where he claims that the obligation to perform the alternative service follows, from an intuitive standpoint, from the two (partially) conflicting norms. Before going back to the alternative version of the Chisholm paradox, we quickly investigate the plausibility of disjunctive obligations from a formal point of view. To this end, we study how to model the sudoku rules in (Standard) Deontic Logic, using disjunctive obligations as the main means for the formalisation. First of all we assume the following set of atomic propositions $\{d_{cr} | d, c, r \in \{1, \ldots 9\}\}$ where the meaning of d_{cr} is that digit d appears in the cell whose coordinates are c (for the column) and r (for the row); in addition, we partition the 81 coordinates cr into nine blocks (each as a set of 9 coordinates forming a 3×3 square), and we use $cr \in B$ to indicate that the cell with coordinate cr is in block B. Then, the two sudoku rules

are encoded as follows (where d, c and r range from 1 to 9):

$$\mathrm{O}\Big(\bigvee_{1\leq d\leq 9} d_{cr}\Big) \tag{1}$$

$$d_{cr} \rightarrow \bigwedge_{r\neq r'} \mathrm{O}\neg d_{cr'} \wedge \bigwedge_{c\neq c'} \mathrm{O}\neg d_{c'r} \wedge \bigwedge_{\substack{cr\neq c'r'\\ cr,c'r'\in B}} \mathrm{O}\neg d_{c'r'} \tag{2}$$

$$\mathrm{O}d_{cr} \rightarrow \bigwedge_{r\neq r'} \mathrm{O}\neg d_{cr'} \wedge \bigwedge_{c\neq c'} \mathrm{O}\neg d_{c'r} \wedge \bigwedge_{\substack{cr\neq c'r'\\ cr,c'r'\in B}} \mathrm{O}\neg d_{c'r'} \tag{3}$$

$$d_{cr} \rightarrow \bigwedge_{d\neq d'} \left(\neg d'_{cr} \wedge \mathrm{O}\neg d'_{cr}\right) \tag{4}$$

A valid sudoku is a set of initial clues (providing the placement of some digits in the grid) that has a unique solution that satisfies the two rules above. The clues are a set of propositions of the form d_{cr} that generates a set of 81 obligations of the from $\mathrm{O}d_{cr}$, derived from the initial placement and the formulas above. A valid (or legal) solution is a set of 81 (atomic) propositions, such that all the 81 obligations have been fulfilled. Alternatively, we can define a legal/valid solution as a set of 81 (atomic) propositions containing the initial clues and satisfying the two rules/formulas above.

Let us examine the diagram in Figure 1: in the fourth column the digits 1 and 9 are missing; hence either 1 or 9 should be in the cell with coordinate 42 or 43. This means, that with repeated use of (2) and the Deontic Disjunctive Syllogism we can derive (i) $\mathrm{O}(1_{42} \vee 1_{43})$ and (ii) $\mathrm{O}(9_{42} \vee 9_{43})$. Moreover, the digit 1 appears in the cell with coordinate 23, 1_{23} and from (2) we conclude $\mathrm{O}\neg 1_{43}$, which, together with (i) implies $\mathrm{O}1_{42}$

Harmed with the discussion we had about the reasonableness of the Deontic Disjunctive Syllogism, we can address the issue if the alternative version of the Chisholm Paradox we have proposed is consistent or not. Clearly not helping the neighbour is the opposite of helping, thus we can use the Deontic Disjunctive Syllogism on 1. and 2'. to conclude that the person ought to tell the neighbour that he is going to help. On the other hand, from 3. and 4., by Modus Ponens, we conclude that the person ought to refrain from telling that he is going to help. Consequently, in a situation where the person does not comply with the obligation to help, the person ought to tell and ought not to tell, thus no matter what the person does, the person cannot comply with the requirements about informing the neighbour about his intention (or lack of it) of helping him. Finally, when we assume, as it is typically the case in Deontic Logic, that a set of norms is consistent (encoded by the axiom $\mathrm{O}a \rightarrow \neg\mathrm{O}\neg a$, we conclude that the statements in the alternative version of the paradox are not mutually consistent.

What are the consequences of the alternative version?

1. We have argued that the set of statements in the alternative version of the Chisholm paradox appears to be, from an intuitive analysis, not consistent. Accordingly, we can argue that the (original) Chisholm set is not necessarily consistent, or at least its consistency requires the use of mechanisms to handle conflicting obligations, in particular under the reading proposed by Chisholm for $\mathrm{O}(a \rightarrow b)$.

2. S2' and S3 require different representations, and then, since L2' and L2 are logically equivalent S2 and S3 require different representations as well.
3. L2' cannot be used to justify the switch to a deontic conditional or a dyadic obligation to maintain that the four statements are logically independent. To achieve the same result one can replace S4, the non-compliant behaviour, with the behaviour complying with S1: "the person helps the neighbour", formalised as
   L4' $help$
   which then makes L3 derivable from it using material implication.
4. A deontic logic should accept both factual detachment for a normative conditional (or a dyadic obligation operator), to be able to derive $\mathrm{O}b$ from a and $a \Rightarrow_{\mathrm{O}} b$ (or, in case of a dyadic obligation $\mathrm{O}(b/a)$, and deontic detachment for material implication (or deontic disjunctive syllogism) to handle disjunctive obligations.
5. In addition to the previous remark, Independently from whether one admits the deontic disjunctive syllogism, the set of $\mathrm{O}\neg a$, $\mathrm{O}\neg b$ and $\mathrm{O}(a \vee b)$ cannot be consistently complied with; and it will require a mechanism to solve the conflict among the three obligations (e.g., by considering the three obligations as *prima facie* obligations)

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References

1. José Carmo and Andrew J. I. Jones. *Deontic Logic and Contrary-to-Duties*, pages 265–343. Springer Netherlands, Dordrecht, 2002.
2. Rodrick M. Chisholm. Contrary-to-Duty Imperatives and Deontic Logic. *Analysis*, 24(2):33–36, 1963.
3. John Horty. Deontic modals: Why abbandon the classical semantics. *Pacific Philosophical Quarterly*, 95:424–460, 2014.
4. James E. Tomberlin. Contrary-to-duty imperatives and conditional obligation. *Noûs*, 15(3):357–375, 1981.
5. Lennart Åqvist. Good Samaritan, contrary-to-duty imperatives, and epistemic obligations. *Noûs*, 1(4):361–379, 1967.

A The Initial Configuration of the Sudoku Puzzle

For the readers who enjoy solving a sudoku puzzle, here is the initial configuration of the puzzle discussed in the previous pages.

	5	8	7		4			
3								
		7		3	5		4	
6					8	5	1	
	8			7			2	
	2	3	6					7
	9		8	5		3		
								2
			2		7	9	5	

The solution is on the next page.

B The Solution of the Sudoku Puzzle

9	5	8	7	6	4	2	3	1
3	6	4	1	8	2	7	9	5
2	1	7	9	3	5	8	4	6
6	7	9	4	2	8	5	1	3
4	8	1	5	7	3	6	2	9
5	2	3	6	1	9	4	8	7
7	9	2	8	5	1	3	6	4
8	4	5	3	9	6	1	7	2
1	3	6	2	4	7	9	5	8