OntoMath^{Edu}: Towards an Educational Mathematical Ontology

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Abstract

We present OntoMath^{Edu}, a new educational mathematical ontology. This ontology is intended to be a Linked Open Data hub for mathematical education, a linguistic resource for intelligent mathematical language processing and an end-user reference educational database. OntoMath^{Edu} is organized in three layers: a foundational ontology layer, a domain ontology layer and a linguistic layer. The domain ontology layer, in turn, consists of the following modules: a kind hierarchy, a hierarchy of reified relationships, a roles hierarchy, and a network of points of view. Currently, OntoMath^{Edu} covers Euclidean plane geometry only, but we plan to extend it to the whole secondary school mathematics curriculum. We consider our work as a part of long-established trend of using Linked Open Data and ontologies in educational environments.

1 Introduction

We present OntoMath^{Edu} (`https://github.com/CLLKazan/OntoMathEdu`), a new educational mathematical ontology. The ontology is intended to be:

- A Linked Open Data hub for mathematical education. In this respect, the ontology lies at the intersection of two long-established trends of using LOD for educational purposes [1-4] and for mathematical knowledge management [5, 6].
- A linguistic resource for common mathematical language processing. In this respect, the ontology can complement mathematical linguistic resources, such as SMGloM [7, 8], and serve as an interface between raw natural language texts and mathematical knowledge management applications.
- An end-user reference educational database, and play the same role in secondary school math, that PlanetMath or MathWorld play in professional mathematics.

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This ontology is a central component of the digital educational platform under development, which is intended for solving such tasks as: (1) automatic knowledge testing; (2) automatic recommendation of educational materials according to an individual study plan; (3) semantic annotation of educational materials.

In the development of OntoMath^{Edu} we would rely on our experience of the development of OntoMath^{PRO} (`http://ontomathpro.org/`) [9], an ontology of professional mathematics. This ontology underlies a semantic publishing platform [10-12], that takes as an input a collection of mathematical papers in LaTeX format and builds their ontology-based Linked Open Data representation. The semantic publishing platform, in turn, is a central component of OntoMath digital ecosystem [13, 14], an ecosystem of ontologies, text analytics tools, and applications for mathematical knowledge management, including semantic search for mathematical formulas [15] and a recommender system for mathematical papers [16].

Despite the fact that OntoMath^{PRO} has proved to be effective in several educational applications, such as assessment of the competence of students [9] and recommendation of educational materials in Virtual Learning Communities [17-20], its focus on professional mathematics rather than on education prevents it to be a strong foundation for the digital educational platform. The main differences between OntoMath^{PRO} and a required educational ontology are the following:

- Conceptualization. OntoMath^{PRO} ontology specifies a conceptualization of professional mathematics, whilst the required educational ontology must specify a conceptualization of school mathematics. These conceptualizations are noticeably different, for example, in school conceptualization, *Number* is a primitive notion, while in professional conceptualization it is defined as a subclass of *Set*.
- Selection of concepts. The required educational ontology must contain concepts from a school mathematics curriculum.
- Terminology. Concepts of OntoMath^{PRO} ontology are denoted by professional terms, whilst concepts of the required educational ontology must be denoted by school math terms. There isn't so much difference between professional and educational terminology in English, but this difference is more salient in such languages as Russian or Tatar. For example, the term 'многочлен' (the native word for 'polynom') should be used instead of the professional term 'полином' (the Greek loan word with the same meaning) in educational environment.
- Didactic relationships. The required educational ontology must contain didactic relationships between concepts. A didactic relationship between two concepts represents the fact, that the first concept can't be learned until the second one is learned. For example, comprehension of the *Addition* concept is required to grasp the concept of *Multiplication*, and, more interesting, to grasp the very concept of *Function*, even though, from the logical point of view the later concept is more fundamental and is used in the definitions of the first two. So, didactic relationships are orthogonal to the logical ones, and are similar to prerequisite relationships in MOOC or narrative relationships in OMDoc documents.
- Points of view. In addition to universal statements, the required educational ontology must contain statements relativized to particular points of view, such as different educational levels. For example, a concept can be defined differently on different educational stages; and a statement can be considered as an axiom according to one axiomatization, and as a theorem according to another.

With regard to the foregoing, there is a need for developing a new educational ontology OntoMath^{Edu}. In the rest of the paper we describe our experience of developing a part of this ontology, covering Euclidean plane geometry.

2 Ontology structure

According to the project, OntoMath^{Edu} ontology will be organized in three layers:

1. **Foundational ontology layer**, where a chosen foundational ontology is UFO [21].
2. **Domain ontology layer**, which contains language-independent math concepts from the secondary school mathematics curriculum. The concepts are grouped into several modules, including the general concepts module and modules for disciplines of mathematics, e.g. Arithmetic, Algebra and Plane Geometry. The concepts will be interlinked with external LOD resources, such as DBpedia [22], ScienceWISE [23] and OntoMath^{PRO}. Additionally, relaying on the MMT URIs scheme [24], the concepts can be aligned with MitM ontology [25], and through it with the concepts of several computer algebra systems.
3. **Linguistic layer**, containing multilingual lexicons, that provide linguistic grounding of the concepts from the domain ontology layer. The lexicons will be expressed in terms of Lemon [26, 27], LexInfo and OLiA

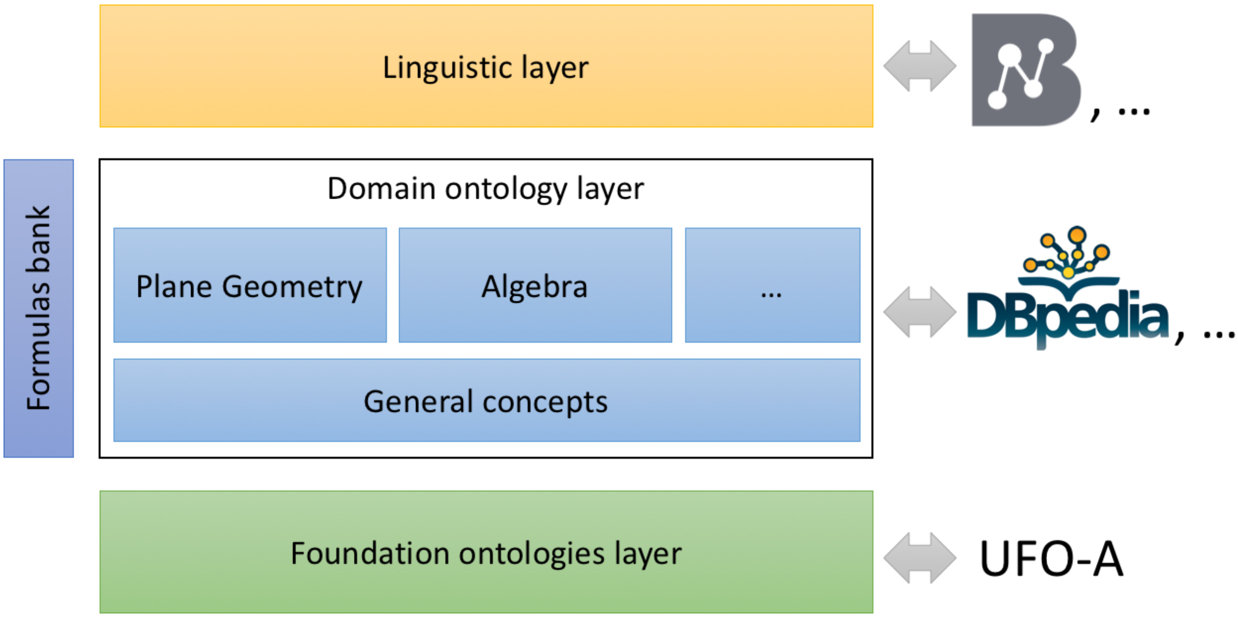

Figure 1: OntoMath^{Edu} ontology structure

[28] ontologies. A lexicon consists in (a) lexical entries, denoting mathematical concepts; (b) forms of lexical entries; (c) syntactic trees of multi-word lexical entries, (d) and syntactic frames. A syntactic frame contains a subcategorization model for a particular lexical entry and its mapping to parameters of a corresponding math concept (for example, in the "Riemann integral of f over x from a to b" lexical entry, the "from a" dependent constituent expresses the lower limit of integration). The lexicons will be interlinked with the external lexical resources from the Linguistic Linked Open Data (LLOD) cloud [29, 30], first of all in English [31, 32], Russian [33] and Tatar [34].

This three-layered structure is represented at Figure 1.

3 Domain ontology layer structure

The domain ontology layer of OntoMath^{Edu} is being developed according to the following modelling principles:

1. Common mathematical language conceptualization. OntoMath^{Edu} reflects the conceptualization of the Common mathematical language (CML) [35], not that of the language of fully formalized mathematics. These conceptualizations are very different. For example, according to the fully formalized mathematics conceptualization, the *Set* concept subsumes the *Vector* concept, but in the CML conceptualization *Vector* is constituted by *Set*, and is not subsumed by it. More important, in contrast to the fully formalized mathematics conceptualization, according to the CML conceptualization, mathematical objects are neither necessary nor timeless, and the domain of discourse can expand in a process of problem-solving.
2. Strict adherence to ontological distinctions provided by the foundational ontology. For example, we explicitly mark concepts as Kinds or Roles.
3. Reification of domain relations. Mathematical relations are represented as concepts, not as object properties. Thus, the mathematical relationships between concepts are first-order entities, and can be a subject of a statement.
4. Multilinguality. Concepts of ontology contains labels in English, Russian and Tatar.
5. Educational literature warrant. The ontology contains only those concepts, that are represented in actual education literature.

Current version of OntoMath^{Edu} contains approximately 600 concepts from the secondary school Euclidean plane geometry curriculum (5th–9th grades). Concepts are represented as ontology classes and have English, Russian and Tatar labels. The ontology consists in the following modules: (1) hierarchy of kinds, (2) hierarchy of roles, (3) hierarchy of reified relationships, and (4) network of points of view.

3.1 Hierarchy of kinds

The main hierarchy of the ontology is a hierarchy of kinds. A kind is a concept that is rigid and ontologically independent [21, 36]. So, for example, the concept of "Triangle" is a kind, because any triangle is always a triangle, regardless of its relationship with other figures.

The top level of the kind hierarchy consists of the following classes:

1. "Plane figure", with subclasses such as "Line", "Polygon", "Ellipse", or "Angle".
2. "Euclidean plane geometry axiom", with subclasses such as "Axiom of construction of a circle with a given center and radius".
3. "Euclidean plane geometry theorem" with subclasses such as "Angle sum theorem of triangle" or "Pythagorean theorem".
4. "Euclidean plane geometry problem" with subclasses such as "Problem of straightedge and compass construction" or "Heron's problem".
5. "Unit of measurement", with subclasses such as "Centimeter", "Radian", or "Square meter".
6. "Measurement and construction tool", with subclasses such as "Protractor", "Astrolabe", "'T-square", "Sliding T bevel", or "Marking gauge".

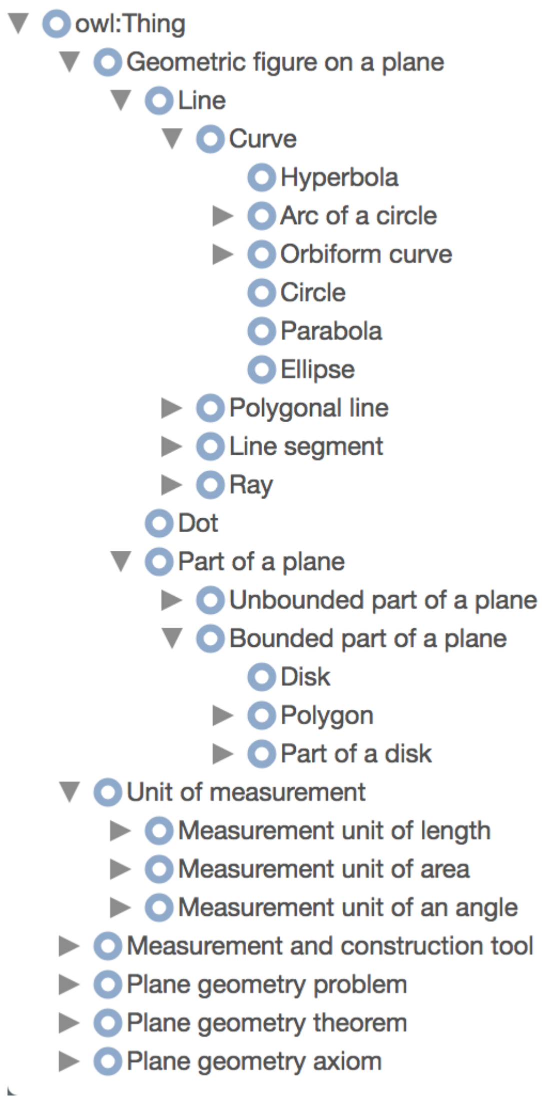

Figure 2: A fragment of the hierarchy of kinds

A fragment of the hierarchy of kinds is represented at the Figure 2.

3.2 Hierarchy of roles

A role is a concept that is anti-rigid and ontologically dependent [21, 36]. An object can be an instance of a role class only by virtue of its relationship with another object. So, for example, the concept "median" is a role, since a line segment is a median not by itself, but only in relation to a certain triangle (Figure 3).

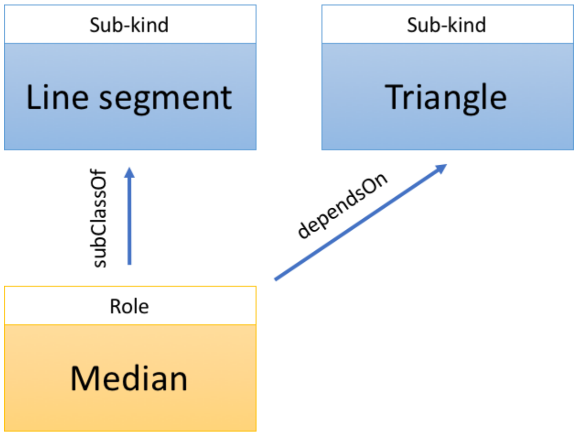

Figure 3: A role example

3.3 Hierarchy of reified relationships

Relations between concepts are represented in ontology in a reified form, i.e. as ontological concepts, not as ontological properties (such representation fits the standard ontological pattern for representing N-ary relation with no distinguished participant [37], but is applied to binary relations too). Thus, the relationships between concepts are first-order entities, and can be a subject of a statement.

The top level of the hierarchy of reified relationships consists of the following classes:

1. "Mutual arrangement of geometric figures on a plane", with subclasses such as "Inscribed polygon" or "Triangle with vertices at Euler points".
2. "Comparison relation between plane figures", with subclasses such as "Congruent triangles" or "Similar polygons".
3. "Plane transformation", with subclasses such as "Translation" or "Axial symmetry".
4. "Metric property of a plane figure", with subclasses such as "Length of a circle", "Tangent of acute angle in right triangle", or "Eccentricity of an ellipse".

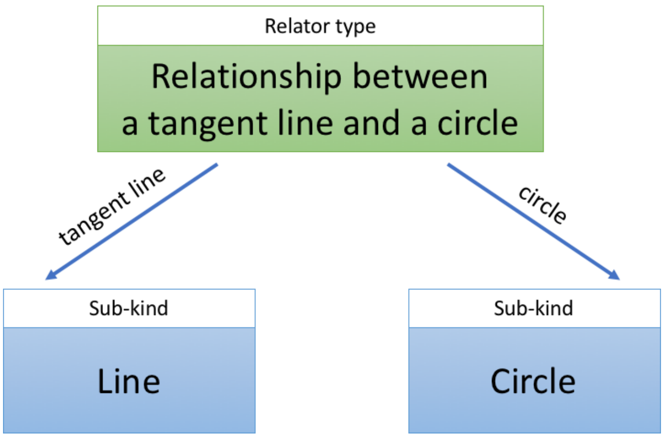

Figure 4: A relationship example

Reified relationships are linked to their participants by object properties (Figure 4).

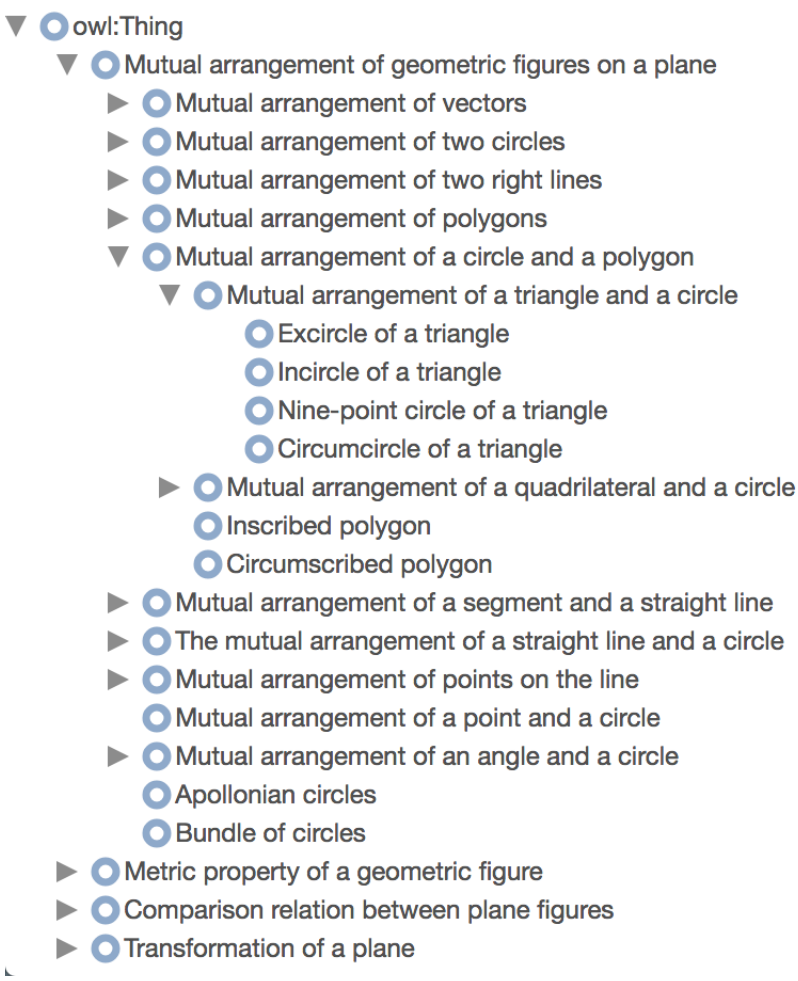

Figure 5: A fragment of the hierarchy of reified relationships

A fragment of the hierarchy of reified relationships is represented at the Figure 5.

3.4 Network of points of view

In addition to universal statements, an ontology contains statements relativized to particular points of view. Points of view are represented using the "Descriptions and Situations" design pattern, and are based on the top-level ontology DOLCE + DnS Ultralite [38-40].

At present, there are the following types of point of view:

1. *Definitions.* One and the same concept may be defined in different ways from different points of view. For example, according to one definition, a concept A is defined by a concept B, and according to another definition, the concept B is defined by the concept A.
2. *Educational levels.*

Currently, development of the network of points of view is at an early stage.

3.5 Relations

The ontology contains the following relations, represented by object properties:

1. Part-whole relation. For example, a "Vertex of a triangle" is a part of a "Triangle".
2. "Determined by" relation, that holds between a figure and an element, that determines this figure, but is not its part. For example, a "Circle" is determined by a "Centre of a circle" and a "Radius of a circle".

3. Relations of ontological dependence that bind a role concept to its depended concept. For example, the "Focus of an ellipse" concept depends on the "Ellipse" concept and the "Chord of a circle" concept depends on the "Circle" concept.
4. "Theorem-property" relation, that holds between a figure and a theorem stating a property of this figure. For example, a "Triangle" is related to the "Triangle inequality" and the "Theorem on the sum of acute angles of a right triangle" theorems.
5. "Theorem-criterion" relation, that holds between a figure and a theorem stating a criterion of this figure.
6. "Is expressed by the formula" relation, that holds between a metric property of a geometric figure and a formula for calculation of this property value. For example, an "Area of a polygon" is related to "Heron's formula".

3.6 External links

Currently, the OntoMath^{Edu} ontology has been interlinked only with DBpedia. The mapping was constructed semi-automatically on the base of the method proposed in [41] and then manually verified. This mapping contains 142 connections.

Now we are working on interlinking the concepts of OntoMath^{Edu} with the Kind and Role concepts defined by the UFO foundational ontology. The obtained links can be formalized by the MLT Ontology [42, 43] and then be used to verify the correctness of the OntoMath^{Edu} hierarchies according to the OntoClean methodology [44].

4 Conclusions

In this paper, we present a new educational mathematical ontology OntoMath^{Edu}, clarify the context of its development and describe its current version.

While there are many educational ontologies on the one hand, and several mathematical ontologies on the other, to our knowledge, OntoMath^{Edu} is the first general-purpose educational mathematical ontology. Additionally, it is the first Linked Open Data mathematical ontology, intended to: (1) respect ontological distinctions provided by a foundational ontology; (2) represent mathematical relationships as first-oder entities; and (3) provide strong linguistic grounding for the represented mathematical concepts.

The ontology is under development now, so our first priority is to release its stable version. After that, our future work will be undertaken in two directions: Firstly, we are going to further develop this ontology according to the above-mentioned plan. Secondly, we are going to apply the modeling principles, drafted on this project, in the development of the new revised version of the ontology of professional mathematics OntoMath^{PRO}.

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