Penalty Method for Reliable Allocations of Wireless Network Resources

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Abstract

In the paper we consider the problem of allocation of network resources in telecommunication networks using both utility and reliability. We suggest a scalar utility maximization problem subject to capacity constraints and within the pre-defined reliability level. This problem is proposed to be solved by a penalty method. We present the results of numerical results on various test problems for the method.

1 Introduction

The current development of information technologies and telecommunications gives rise to new control problems related to efficient transmission of information and allocation of limited network resources. All these problems are determined on distributed systems where the spatial location of elements is taken into account. Due to strong variability and increasing demand of different wireless telecommunication services, fixed allocation rules usually lead to serious congestion effects and inefficient utilization of network resources despite the presence of very powerful processing and transmission devices. Hence, one has to find more flexible allocation mechanisms instead of the fixed allocation ones. These mechanisms are based on proper mathematical models; see e.g. [CW03, SWB06, WNH10]. For example, solution methods for network resource allocation based on optimization formulations of network manager problems and decomposition techniques were presented in [KKL18, KK19].

In addition, wireless networks should be reliable with respect to various attacks. The most commonly seen attack in wireless networks are eavesdropping in which attackers aim at acquiring important/private information of users, jamming and distributed DoS attacks which attempt to interfere and disrupt network operations by exhausting the resources available to legitimate systems and users. These attacks may lead to degrading the network performance and quality of service (QoS) as well as losing important data, reputations, and revenue; see e.g. [ZJT13, MZA13, ZJT13, LHW17].

In this paper we consider a problem of telecommunication network links allocation among users under reliability control of network connections. For this problem we suggest a penalty method with respect to links capacity and reliability level constraints. We present the results of numerical results on test problems for the method.

2 Problem Description

We begin our description from the basic optimal flow distribution problem in wireless telecommunication networks from [KMT98]. For a fixed time period we are given a network that contains a set L of transmission links (arcs)

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and accomplishes some submitted data volume transmission requirements from a set I of selected pairs of origin-destination vertices. Denote by x_i and α_i the current and maximal value of data transmission for pair demand $i \in I$, respectively, and by c_l the capacity of link $l \in L$. For the sake of simplicity we suppose that each pair demand is associated with a unique data transmission path, hence each link l is associated uniquely with the set N_l of pairs of origin-destination vertices, whose data transmission paths contain this link. For each pair demand i we denote by $u_i(x_i)$ the utility value at the data transmission volume x_i. Then the problem of the total utility maximization of the network is written as follows:

$$\max \to \sum_{i \in I} u_i(x_i)$$

subject to

$$\begin{array}{l} \sum\limits_{i \in N_l} x_i \leq c_l, \ l \in L, \\ 0 \leq x_i \leq \alpha_i, \ i \in I. \end{array}$$

If the functions $u_i(x_i)$ are concave, this is a convex optimization problem over a polyhedron.

In order to extend the model and take into account the reliability of connections we now suppose that the reliability depends on arc flow volumes. More precisely, let $\mu_l(f_l)$ denote the non-reliability of arc l at its flow volume f_l. Hence, we have to add the second goal:

$$\min \to \sum_{l \in L} \mu_l(f_l)$$

where

$$f_l = \sum_{i \in N_l} x_i, \ l \in L,$$

and obtain a vector optimization problem. The scalarized goal version with some weights will take the form

$$\min \to \sigma_1 \sum_{l \in L} \mu_l(f_l) - \sigma_2 \sum_{i \in I} u_i(x_i),$$

but the choice of right weights σ_1 and σ_2 for so different goals seems too difficult here. For this reason, it is better to determine the pre-defined non-reliability level β_i for each connection $i \in I$ and to maximize utility under all these constraints. That is, the origin-destination vertices require some desired level of their reliability to work. This problem is now formulated as follows:

$$\max \to \sum_{i \in I} u_i(x_i) \tag{1}$$

subject to

$$\sum_{i \in N_l} x_i = f_l, \ l \in L, \tag{2}$$

$$\sum_{l \in L_i} \mu_l(f_l) \leq \beta_i, \ i \in I, \tag{3}$$

$$f_l \leq c_l, \ l \in L, \tag{4}$$

$$0 \leq x_i \leq \alpha_i, \ i \in I. \tag{5}$$

If the functions $\mu_l(f_l)$ are convex, this is a convex optimization problem.

3 Solution Method

Usually, the functions $u_i(x_i)$ and $\mu_l(f_l)$ in problem (1)–(5) are smooth, hence we can apply a number of well known smooth optimization methods; see e.g. [DR68, PD78]. However, these problems have large dimensionality and inexact data, hence their solution methods should be rather simple and provide some desired accuracy within an acceptable time interval. Therefore, the penalty based methods are suitable here; see e.g. [FM72, GK81]. We impose penalties only on binding constraints in (2)–(3). Set

$$\begin{aligned} X &= \{x \mid 0 \le x_i \le \alpha_i,\ i \in I\}, \\ F &= \{f \mid 0 \le f_l \le c_l,\ l \in L\}, \\ W &= X \times F, \end{aligned}$$

and define the penalty functions

$$\Phi_1(x, f) = \sum_{l \in L} \left[\sum_{i \in N_l} x_i - f_l \right]^2$$

and

$$\Phi_2(x, f) = \sum_{i \in I} \left[\sum_{l \in L_i} \mu_l(f_l) - \beta_i \right]_+^2 .$$

We take positive penalty parameters τ_1 and τ_2 and define the penalized problem

$$\max_{(x,f) \in W} \to \Psi(x, f, \tau), \tag{6}$$

where $\tau = (\tau_1, \tau_2)$,

$$\Psi(x, f, \tau) = \sum_{i \in I} u_i(x_i) - \tau_1 \Phi_1(x, f) - \tau_2 \Phi_2(x, f). \tag{7}$$

Denote by $(x^*(\tau), f^*(\tau))$ a solution of problem (6)–(7). If each τ_i is positive, increasing, and tending to $+\infty$, then the corresponding points $(x^*(\tau), f^*(\tau))$ will tend to a solution of problem (1)–(5). Moreover, this is the case for some approximations of points $(x^*(\tau), f^*(\tau))$. In order to find these approximate solutions of problem (6)–(7) we propose to apply the gradient projection method; see e.g. [DR68, PD78].

Let $g(x, f) = (g_x(x, f), g_f(x, f))$ denote the gradient of $\Psi(x, f, \tau)$ where τ is fixed. At the current point (x, f) we find the points

$$\widetilde{x} = \pi_X[x + \lambda' g_x(x, f)], \tag{8}$$

$$\widetilde{f} = \pi_F[f + \lambda'' g_f(x, f)], \tag{9}$$

where $\lambda' > 0$, $\lambda'' > 0$. The next iterate (x^{new}, f^{new}) can be found from $(\widetilde{x}, \widetilde{f})$ after inserting a proper line-search if necessary. That is, we set

$$\begin{aligned} x^{new} &= \eta \widetilde{x} + (1 - \eta) x, \\ f^{new} &= \eta \widetilde{f} + (1 - \eta) f, \text{ for } \eta \in (0, 1]. \end{aligned}$$

The partial derivatives of $\Psi(x, f, \tau)$ are written as follows:

$$g_{x_i}(x, f) = u_i'(x_i) - 2\tau_1 \sum_{l \in L} \left(\sum_{j \in N_l} x_j - f_l \right) \alpha_{il},$$

where

$$\alpha_{il} = \begin{cases} 1, & \text{if } i \in N_l, \\ 0, & \text{otherwise,} \end{cases}$$

and

$$g_{f_l}(x, f) = 2\tau_1 \left(\sum_{i \in N_l} x_i - f_l \right) - 2\tau_2 \sum_{i \in I} \left[\sum_{s \in L_i} \mu_s(f_s) - \beta_i \right]_+ \mu_l'(f_l) \gamma_{il},$$

where

$$\gamma_{il} = \begin{cases} 1, & \text{if } i \in N_l, \\ 0, & \text{otherwise;} \end{cases}$$

for all $i \in I$ and $l \in L$. By using these formulas the main steps in (8)–(9) decompose into single-dimensional problems:

$$\max_{0 \le y_i \le \alpha_i} \to \left\{ g_{x_i}(x, f)(y_i - x_i) - \frac{1}{2\lambda'} (y_i - x_i)^2 \right\},\ i \in I,$$

and

$$\max_{0 \le v_l \le c_l} \to \left\{ g_{f_l}(x, f)(v_l - f_l) - \frac{1}{2\lambda''} (v_l - f_l)^2 \right\},\ l \in L,$$

respectively. Their solutions can be found by explicit formulas.

4 Numerical Experiments

As part of the work, a numerical study of the considered method was carried out on test examples. The method was implemented in C++ with a PC with the following facilities: Intel(R) Core(TM) i7-4500, CPU 1.80 GHz, RAM 6 Gb.

We took the linear functions of arc non-reliability:

$$\mu_l(f_l) = \mu_{0,l} f_l,\ \mu_{0,l} > 0,\ l \in L,$$

and logarithmic functions of connection utilities

$$u_i(x_i) = u_{2,i} \log(u_{0,i} + u_{1,i} x_i),\ u_{j,i} > 0,\ j = 0, 1, 2,\ i \in I.$$

The coefficients $\mu_{0,l}$, $u_{0,i}$, $u_{1,i}$, and $u_{2,i}$ were determined on the basis of trigonometric functions:

$$\begin{aligned} \mu_{0,l} &= |\cos(l)| + 1, \\ u_{0,i} &= 2 * |\sin(2i)| + 1, \\ u_{1,i} &= |\sin(i+1)| + 1, \\ u_{2,i} &= 3 * |\sin(2i)| + 1. \end{aligned}$$

The maximal flow c_l along arc l was selected in the segment [1, 10] depending on the arc number as follows:

$$c_l = 10 * |\cos(l+2)| + 1.$$

The maximal flow α_i for connection i was selected in the segment [1, 7] depending on the connection number as follows:

$$\alpha_i = 7 * |\sin(i-1)| + 1.$$

The upper non-reliability bound β_i was selected in the segment [1, 5] depending on the connection number as follows:

$$\beta_i = 3 * |\cos(i-1)| + 1.$$

The parameters τ_1, τ_2, λ', and λ'' were fixed as follows:

$$\tau_1 = 0.9, \tau_2 = 0.9, \lambda' = 0.009, \lambda'' = 0.009.$$

The distribution of the available arcs across the connections was chosen either uniformly or according to the normal distribution law. We took two versions of the gradient projection method. The first does not involve any line-search, the second involves an Armijo type line-search.

Let us introduce the additional notations:

ε is the accuracy of a solution of the problem;

T_ε is the total solution time (in seconds) of the penalty method containing the gradient projection method without line-search;

$T_{\varepsilon,ls}$ is the total solution time (in seconds) of the penalty method containing the gradient projection method with line-search;

I_ε is the number of iterations of the penalty method containing the gradient projection method without line-search;

$I_{\varepsilon,ls}$ is the number of iterations of the penalty method containing the gradient projection method with line-search.

Table 1: Calculations for $|I| = 620$, $|L| = 310$ and different ε

ε	T_ε	I_ε	$T_{\varepsilon,ls}$	$I_{\varepsilon,ls}$
10^{-1}	0.031	280	0.063	400
10^{-2}	0.094	914	0.219	1250
10^{-3}	0.156	1310	0.313	1780
10^{-4}	0.219	1729	0.422	2377

The penalty method was stopped if the norm difference between two sequential iterates appeared less than the accuracy. In Table 1, the numerical results are given for the case where $|I| = 620$, $|L| = 310$ and for different values of the accuracy ε.

Note that the working time was less than one second. We can also observe that similar results were obtained for fixed ε and $|L|$ and different $|I|$ (see 2), and for fixed ε and$|I|$ and different $|L|$ (see 3).

Table 2: Calculations for $\varepsilon = 10^{-2}$, $|L| = 310$ and different $|I|$

$\|I\|$	T_ε	I_ε	$T_{\varepsilon,ls}$	$I_{\varepsilon,ls}$
310	0.032	626	0.079	863
620	0.094	914	0.219	1250
930	0.157	869	0.328	1192
1240	0.203	809	0.406	1110

Table 3: Calculations for $\varepsilon = 10^{-2}$, $|I| = 310$ and different $|L|$

$\|L\|$	T_ε	I_ε	$T_{\varepsilon,ls}$	$I_{\varepsilon,ls}$
310	0.032	626	0.079	863
620	0.047	559	0.109	775
930	0.109	538	0.203	746
1240	0.109	519	0.235	719

From the experiments we conclude that the version of the penalty method containing the gradient projection method without line-search appeared somewhat better than that with the line-search. In general, the method attained a solution with low accuracy quickly enough, but the additional analysis and selection of parameters for more effective implementation of the method are necessary.

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