

# Analysis of hydraulic unit operation stability according to its vibration monitoring results

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**Abstract**—During hydraulic unit steady - state operation it is necessary to support its functioning stability. The analysis of vibration results for correlated values is carried out with multivariate statistical control methods: the process average control is done based on Hotelling's algorithm, when multivariate dispersion control is done through the generalized variance algorithm. The article investigates the efficiency of generalized variance algorithm: how fast the generalized variance test chart reacts to a hydraulic unit vibration stability prone breakdown. The investigation revealed that the hydraulic unit operation stability versus multivariate dispersion is not always appropriately assessed through a standard generalized variance algorithm. To improve the monitoring sensitivity to a prone breakdown, it is reasonable to modify this algorithm with a search of non-random structures on the corresponding chart, with a warning limit and exponentially weighed moving average (EWMA) on a generalized variance.

**Keywords**—statistical process control, multivariate scattering, generalized dispersion, control chart

## I. INTRODUCTION

During hydraulic unit steady-state operation it is necessary to provide for its functioning stability. In fact, vibration instability might lead to emergencies and extraordinary cases with dramatic consequences. The example of Sayano-Shushenskaya hydraulic electro power station hydraulic unit destruction with multiple fatalities in result is the most vivid illustration hereto.

The vibration value data during motion monitoring, in online mode, are applied to hydraulic unit control stand, and if necessary, when the vibration data processing system predicts its significant increase, the load is reduced. The analysis of the data applied can be carried out in different ways [1-4].

One of the approaches, widely used in the technical process stability monitoring, is the statistical control method. The data monitoring is performed, the process non-random deviation is revealed in result: the monitored data are to be located within the limits of the corresponding confidence intervals. By deviation we mean the graphical location of one of the points on the chart beyond the limit. At the same time the physically monitored data are still within the limits, however, the statistics reveals the process instability [5-6]. Shewhart control charts are applied to monitor the independent values: both mean level and process dispersion are monitored simultaneously. The standards assume the application of average values and range charts or standard deviation, as well as individual observation and moving range charts. In the vibration monitoring of a hydraulic unit some readings of vibration pick-ups are not correlated with the others and this is the

case, when Shewhart control charts can be applied. It is not always, that usual Shewhart control charts are quick enough in revealing the stability violation. The various ways of their efficiency improvement are used. Such as: special form structures searching on the chart, warning limit introduction, process monitoring with memory charts application (cumulative sum and exponentially weighed moving average control charts), etc. The efficiency of this or that statistical tool application depends on the type of the most hazardous for the current process kind of breakdown. It might be a rapid rise of average or process dispersion, its trend, etc.

For the correlated values multivariate statistical monitoring the control methods are used: the monitoring of a process average is done based on Hotelling's algorithm, when multivariate dispersion control is done through the generalized variance algorithm. After certain time intervals the samples are taken, and for each sample there is an estimated Hotelling's value and generalized variance, i.e. controllable values covariance matrix determinant; the alternation of this parameter characterizes the scattering process stability [7-11]. This approach is applied in different domains [12-15].

The hydraulic unit vibration monitoring data were analyzed: there were 10 values to assess: the vibration of lower  $X_1$  and upper  $X_3$  generator set bearing, upstream and on the RH coast  $X_2$ ,  $X_4$ , hydraulic turbine shaft vibration downstream  $X_5$  and on the RH coast  $X_6$ , hydraulic generator shaft vibration  $X_7$ ,  $X_8$ , and also hydraulic turbine cover vibration  $X_9, X_{10}$ .

Figure 1 shows multivariate charts, plotted within Statistica [16] system by two correlated values  $X_6-X_8$  (the significant correlation is available between these two values, the significant correlation by Student criteria at significance level equal to 0.05; sample correlation coefficient equal to  $r = 0.61$ ). Both charts testify to vibration stability: Hotelling's value does not exceed the limit (13.756), generalized dispersion is also within the limits (limit is 14.514).

It is worth saying that the limits mentioned above are determined by means of statistics methods and are not the limits for vibration; these are the limits of the existing confidential interval (CI). Their violation means stability breakdown, though the limit values remain within the limits yet. Timely reaction to such breakdown incidents excludes the emergency situation.

However, not only beyond-the-limit controlled statistics testifies to the process failure, but different special form structures on the chart do. Along with it, the mentioned above methods do not always react effectively to the process

prone breakdowns. Hotelling's algorithm controlling multivariate level of average is well enough studied in this respect [5-9], which is not the case with multivariate dispersion control.

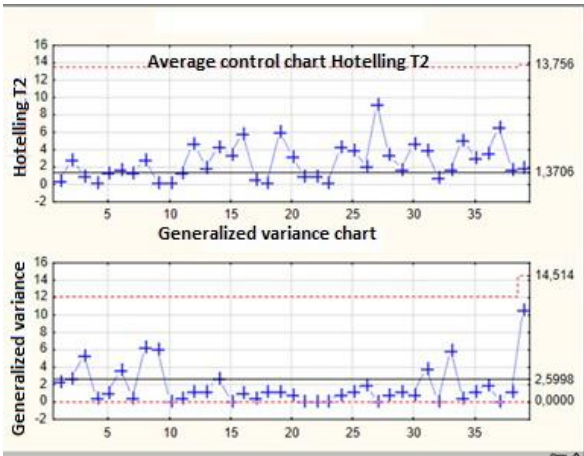


Fig. 1. Multivariate charts.

The aim of the investigation is to increase the efficiency of hydraulic unit vibration monitoring in its operation values multivariate dispersion criteria through the assessment of its generalized variance algorithm sensitivity: how fast the generalized variance test chart reacts to a hydraulic unit vibration stability prone breakdown.

## II. GENERALIZED VARIANCE ALGORITHM SENSITIVITY ASSESSMENT

Generalized variance algorithm is in fact the check for the hypothesis of covariance matrix equality of the vibration process  $\Sigma$  to the set value  $\Sigma_0$ . For each moment of time  $t$  a sample covariance matrix  $S_t$ , is formed, the elements of which are as following:

$$s_{jkt} = \frac{1}{n-1} \sum (x_{ijt} - \bar{x}_j)(x_{ikt} - \bar{x}_k), \quad (1)$$

$x_{ijt}$  is the result of observation  $i$  as per index  $j$  in sample  $t$  ( $i = 1, \dots, n$ ,  $n$  is the sample size,  $j, k = 1, \dots, p$ ,  $p$  is the quantity of the monitored values,  $t = 1, \dots, m$ ,  $m$  is the number of samples taken for the vibration analysis). The determinant  $|S_t|$  of matrix (1) is the generalized dispersion of instantaneous sampling  $t$ .

The estimated covariance average is also calculated as per the whole sample population :

$$\bar{s}_{jk} = \frac{1}{m} \sum_{t=1}^m s_{jkt}, \quad (2)$$

which forms the covariance matrix  $S$ ; its determinant  $|S|$  is used as the assessment of target generalized dispersion  $|\Sigma_0|$ . While plotting the control chart, sample values of generalized dispersion  $|S_t|$  for each sample  $t$  are taken.

The generalized dispersion chart limits are determined as per the following formula:

$$\left. \begin{matrix} UCL \\ LCL \end{matrix} \right\} = |\Sigma_0| (b_1 \pm u_{1-\alpha/2} \sqrt{b_2}), \quad (3)$$

where  $u_{1-\alpha/2}$  is the quintile of normal distribution policy  $1 - \alpha/2$ ,  $\alpha$  is the significance (probability of false alarm); the coefficients are calculated as per the following formulae:

$$b_1 = \frac{1}{(n-1)^p} \prod_{j=1}^p (n-j); \quad (4)$$

$$b_2 = \frac{1}{(n-1)^{2p}} \prod_{j=1}^p (n-j) \left[ \prod_{k=1}^p (n-k+2) - \prod_{k=1}^p (n-k) \right], \quad (5)$$

and the assessment of target generalized dispersion  $|\Sigma_0|$  is determined as per the learning sample. Provided the low control limit  $LCL$  as per the formula (3) turns out to be negative, zero value is taken.

Vibration stability break down is testified by the location of at least one point on the chart of the generalized dispersion beyond one of the limits, that means that the process is steady if the following inequality is true:

$$LCL < |S_t| < UCL,$$

where  $t$  means the number of the controlled sample.

For the quality rating of algorithm sensitivity to the process, prone breakdown average sample run length is applied, i.e. the number of observations done within the period of time between the moment of the initial breakdown occurrence and the moment of the breakdown finding.

For the experiment purpose a set of samples, similar to real ones in motion, were simulated. The bench-mark data are vector of mean values and correlated values covariance matrix. The algorithm of simulating multinomial random variables is used.

For the simulated samples different failures of process scattering are introduced, and the number of samples from the moment of the introduced failure till the moment of the process running beyond the warning limit on the plotted charts of the generalized variance is determined. Averaging these data for all the samples we will get an average run length.

Figure 2 shows the results of the carried out experiments with multivariate dispersion of two correlated values. There was simulated a dispersion abrupt increase by 1.25 times (sample value of the determinant of covariance matrix was multiplied by  $d = 1.25$ ), by 1.5, by 1.75, by 2 times. The corresponding values of  $d$  are plotted on the diagram on its horizontal axis. The vertical axis shows the values of average run length  $L(d)$ , estimated by 1000 simulated samples.

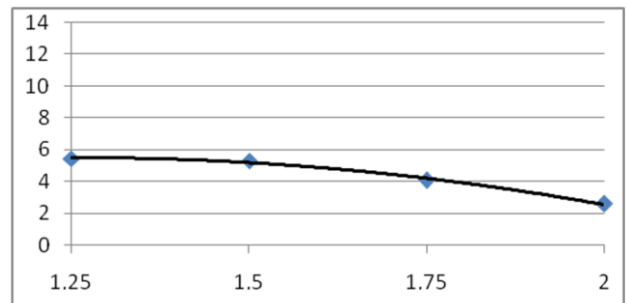


Fig. 2. Average run length in result of the experiments.

The results of the experiments (experimental results) were approximated by the regression parabola relation, built in the environment of Excel spread sheets (trend line):

$$L(d) = -5.36d^2 + 13.50d - 3.028$$

Determination factor  $R^2 = 0.993$  indicates the high quality of the plotted model. Using this relation and knowing which scattering increase value is jeopardizing (or critical) for the tested item, we can assess the quality of a generalized variance algorithm and make corrections in the process of multivariate dispersion control.

Similar results were achieved for other sets of correlated values.

Let us assume that for two vibration values monitoring the abrupt increase in dispersion by 1.6 times is hazardous. Then the mentioned formula means that the generalized variance chart will find this breakdown after  $L(1.6) = 4.8$  samples. Sometimes this value is inadmissible: within this period of time the vibration will cause unintended consequences. In this case it is necessary to change the control procedure in order to improve its sensitivity.

### III. GENERALIZED VARIANCE ALGORITHM SENSITIVITY IMPROVEMENT METHODS

To improve the control efficiency one may use several different approaches: to analyse the non-random structures on the chart of generalized variance, to introduce an additional warning limit, to apply exponentially weighed moving average (EWMA) on a generalized variance.

Analysing the non-random structures on the generalized variance we proceed from the assumption that generalized variance algorithm is based on the use of normal distribution (ND) (three-sigma rule), so to reveal the defect the same types of structures could be used as for Shewhart control charts [17-18]. The space between the central line and upper limit is divided into three; the width of each one is equal to one standard deviation. The non-random structures, whose probability is commensurable with the probability of a false warning, are (figure 3):

- a) at least one point runs beyond the limit,
  - b) at least two out of three consecutive points above the central line run beyond two sigma limit,
  - c) at least four out of five consecutive points above the central line run beyond one sigma limit,
  - d) six increasing or decreasing points in a raw (trend),
- etc.

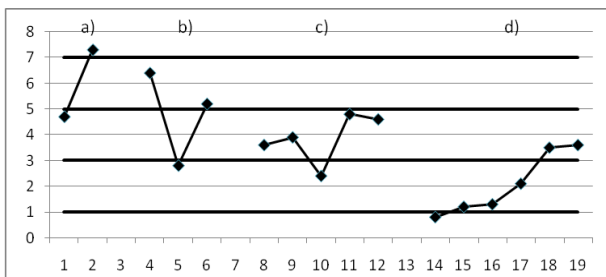


Fig. 3. Chart of non-random structures on the generalized variance.

The introduction of a warning limit increases the sensitivity of the generalized variance control chart (Fig. 4). The position of such a limit line is assessed according to the number of points between the warning and control limit lines, considered to be an abnormality (usually two, three, or four). The estimation of the warning limit position (upper

warning limit *UWL* and lower warning limit *LWL*) is done through Markovian chain similar to average charts limit estimation [19-21].

The calculation results can be presented as follows:

$$\left. \begin{matrix} UWL \\ LWL \end{matrix} \right\} = \Sigma_0 (b_1 \pm B \sqrt{b_2}), \quad (6)$$

*B* coefficient is determined from the tables [21] as per the number of points between the warning and control limits. It is reasonable to check all three variants in practice: 2, 3 or 4 points are between the limits.

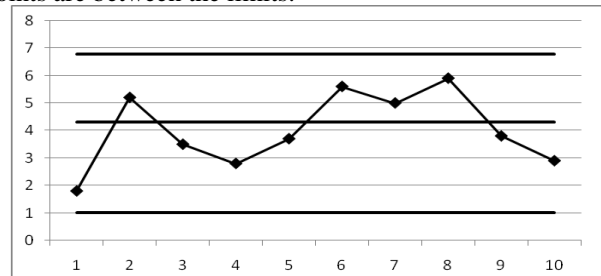


Fig. 4. Three consecutive points in a raw between the warning and control limits on the generalized variance chart.

One more approach, providing dispersion monitoring efficiency increase under certain conditions, is the use of exponentially weighed moving average on a generalized variance (figure 5). The tests revealed that this chart senses the abrupt increase of the dispersion faster than the usual chart of generalized variance.

The values of exponentially weighed moving average EWMA, plotted on the chart, is calculated as per the following formula:

$$E_t = (1 - \lambda)E_{t-1} + \lambda \Sigma_t, \quad (7)$$

where  $\lambda$  means the parameter of exponential smoothing ( $0 < \lambda < 1$ ).

The position of the control limits of the exponentially weighed moving average control chart for the generalized dispersion is determined as per the following formula:

$$\left. \begin{matrix} UCC \\ LCC \end{matrix} \right\} = \Sigma_0 \pm H \sigma_{Et}, \quad (8)$$

where *H* means the parameter, specifying the position of the limits (as a rule it is assumed that  $H = 3$ ); the standard deviation of exponentially weighed moving average can be found as per the formula:

$$\sigma_{Et}^2 = \frac{\sigma_{\Sigma}^2}{n} \frac{\lambda}{2 - \lambda} [1 - (1 - \lambda)^{2t}] \quad (9)$$

where  $\sigma_{\Sigma}$  means the assessment of the generalized dispersion standard deviation.

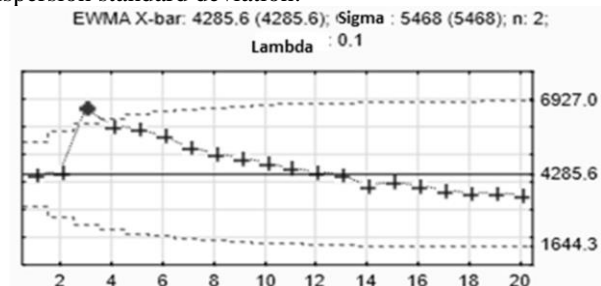


Fig. 5. Chart of exponentially weighed moving average on a generalized variance.

## IV. CONCLUSION

The conducted experiment revealed that hydraulic unit functioning stability monitored as per vibration monitoring multivariate dispersion criteria is not always appropriately assessed through the generalized variance standard algorithm. The dispersion increase is often found too late, when vibration may cause harmful circumstances. To increase the sensitivity of monitoring to prone breakdowns it is reasonable to modify this algorithm by the search of non-random structures on the corresponding chart, by introducing a warning limit, or by the use of exponentially weighed moving average on a generalized variance.

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