

Analysis of Monopolistic Competition in Consumer Goods Markets with Credit Sales

Michael Geraskin
 Department of mathematical
 methods in Economics
 Samara National Research University
 Samara, Russia
 innvation@mail.ru

Olga Kuznetsova
 Department of mathematical
 methods in Economics
 Samara National Research University
 Samara, Russia
 olga_5@list.ru

Abstract—The article considers the problem of the monopolistic competition in markets, which are interconnected within a vertically integrated system of retailers, banks and insurers. The system is organized to increase in the sale volumes of consumer goods by means of the credit tools, and it includes three levels. The retailers' level corresponds to the sale of goods, the banks' level is related to the lending transactions and the insurers' level credit corresponds to the insurance. There are a great number of competing firms (hereinafter, agents) at each level of the system. The formulas for calculating the maximum possible number of agents at each level are derived. The simulation of the competition is carried out on the basis of the household appliances market.

Keywords—integrated economic systems, retailer, bank, insurance, demand curves, interconnected markets

I. INTRODUCTION

Integrated economic systems [1, 2] are formed, when the buyer's need for one product is due to the fact of the another product need. The “retailer-bank-insurer” system is a typical example of such integration [3]. In this case, the integrated system is organized within the framework of the retailer's credit turnover. On the one hand, the demand for the expensive goods encourages the buyers to borrow loans in the banks. Then, the banks encourage the buyers to insure their solvency. On the other hand, the possibility of obtaining the credit resources expands the demand for the expensive goods. Thus, the desire to increase in the demand leads to an emergence of the integrated system [16]. Such integrated system arises in the process of selling the household appliances.

At the state level, we consider the interaction between the following markets: the household appliance retail market, the banking market and the insurance market. In the Russian Federation, the economic system consists of 451 banks, 232 insurance companies [17] and more than 20 retail chains of household appliances sellers [18]. For example, the Eldorado network consists of 328 branches [19], the M-Video network consists of more than 358 branches. The relationship between the retailers, the banks and the insurers is demonstrated in Fig. 1.

In Fig. 1, we introduce the following designations: N is the actual number of the retailers in the market, M is the actual number of the banks in the market, P is the actual number of the insurance companies in the market, N_{max} is the maximum number of the agents in the retail market, M_{max} is the maximum number of the agents in the banking services market, P_{max} is the maximum number of the agents in the insurance market, R_i is the i -th agent in the retail market, B_i is the i -th agent in the banking market, I_i is the i -th agent in the insurance market, indicates the agent's affiliation to a particular market. The maximum numbers of the agents are

calculated further during the simulation, and they are presented in this figure to illustrate the possible scale of the system.

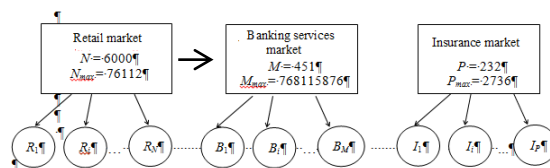


Fig. 1. Diagram of agents in the household appliances sale system.

We introduce the following definitions. The agent's environment includes the agents of the system excepting this agent [20, 10]. If the agent's utility (profit) function depends on his own action and on the environment's actions, then the system is strongly connected [21]. In particular, in the “retailer-bank-insurer” system, the agents' costs are interdependent (i.e., inseparable), therefore, the system stability is ensured by mutual payments (commissions, discounts, etc.). Agents' revenues can be interdependent, when the system has a mechanism for distributing the aggregate utility [4,13]. In this case, the utilities of the agents are transferable [5, 6]. The vertically integrated system that contains one agent at each level was considered in [14].

As a consequence of the agents heterogeneity in the terms of economic activity, the problem of coordinating the agents' interests in the integration process arises. If the agent's good initiates the demand for goods of other agents, he is characterized by predominant economic activity and he is named as a meta-agent. In addition, the meta-agent has information about the true utility functions of other agents or their utility values.

The meta-agent can choose the distribution mechanism of the aggregated integration effect in the system [7, 8]. In the “retailer-bank-insurer” system, the meta-agent is a retailer. The Pareto-efficient [11] algorithm for the distribution of the transferable utility for such strongly connected system [9] was developed in [15]. Our study considers the “retailer-bank-insurer” system, in which three levels correspond to the sale of goods (i.e., retailers), the transaction lending (i.e., banks) and the loan insurance (i.e., insurance companies), respectively. The initiator of integration in such system is the retailer, because he has the greatest amount of resources for distribution. Because the bank's sales volume depends on the retailer's sales volume, the banking system is the second level of the interaction. Additionally, the insurer's sales volume depends on the bank's sales volume, therefore, this is the third level of the interaction. There are great numbers of competing firms at each level of the system. In this case, a situation of the monopolistic competition arises at each level. The competition is monopolistic, because the firms' products differ in quality characteristics, that makes them different.

Consequently, the agent's sales volume depends on his own price and on the prices of the competitors.

Fig. 2 shows the interaction scheme in an integrated system with many agents at each level.

The strong integration relationship occurs when the i -th agent of the upper level interacts with the j -th agent of the lower level. If the i -th agent of the upper level interacts with several agents of the lower level or vice versa, the integration relationship is weak, because in this case the agent may choose the agents' set at other levels for the interaction.

If N is equal to 1, then the retail market is characterized as a monopoly of the retailer. If M is equal to 1, then the banking services market is characterized as the bank's monopoly. If P is equal to 1, then the insurance market is characterized as a monopoly of the insurance company. If N, M, P are greater than 1, then these markets are defined as the monopolistic competition, and the occupied markets shares are determined by the price ratio of the competitors.

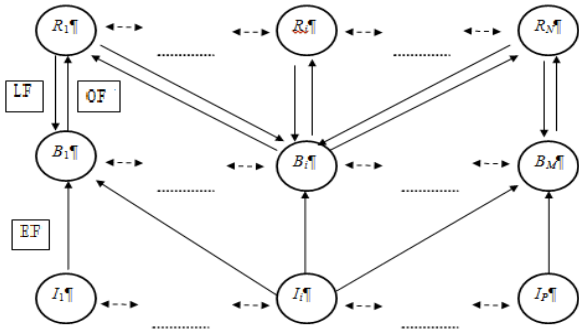


Fig. 2. Scheme of agents interaction in system.

\leftrightarrow monopolistic competition, \Leftrightarrow direction of vertical integration, R_i is the i -th retailer, B_i is the i -th bank, I_i is the i -th insurer

The following notation is used in Figure 2: LF (lending fee) is a premium that the retailer pays to the bank, if the bank's loans quantity corresponds to the retailer's need; OF (operating fee) is the rent that the bank pays to the retailer for the right to participate in the integration; EF (exposure fee) is the premium that the insurance company pays to the bank, if the bank allows the insurer to sell his product (i.e., to participate in the integration) by introducing the compulsory credit insurance conditions.

Thus, our contribution consists of the following items. First, we investigate the interconnected markets with great numbers of agents. Second, we calculate the quantitative estimates of these markets, i.e. the maximum numbers of agents.

II. METHODS AND MATERIALS

The market is described as a set of existing and potential consumers, producers, intermediaries, which enter into relationships for the purpose of purchase, sale and consumption of goods and services. The market capacity refers to the value of goods that consumers can purchase at the current price. The market capacity is a function of the product price. The market size is the value of goods that all firms can offer at the current price. The total sales volume is determined by the prices set in the market; it is less than or

equal to the market capacity. The utilities (profits) of the agents are calculated by using the following formulas:

$$\pi_{k_i}(Q_{k_i}) = a_{k_i} Q_{k_i}^{b_{k_i}+1} - C_{v_{k_i}} Q_{k_i} - C_{f_{k_i}} \quad k = \{R, B, I\} \quad (1)$$

where $\pi_{k_i}(Q_{k_i})$ is the agent's profit function; a_{k_i}, b_{k_i} are coefficients of the price function of the i -th agent in the k -th market; K is the set of agents; k are the elements of the set K , and $k \in \{R \cup B \cup I\}$; $k \in R$ is the retail market, $k \in B$ is the banking services market, $k \in I$ is the insurance services market; Q_k is the sales volume of the i -th agent in the k -th market; $C_{v_{k_i}}$ is the direct cost per unit of goods of the i -th agent in the k -th market, $C_{f_{k_i}}$ is the constant cost of the i -th agent in the k -th market.

We introduce the following assumptions.

1) The market capacity is defined as the total maximum sales volume of firms in the market.

2) The agents act in monopolistic competition markets, then the inverse demand functions are described by the power functions

$$p_{k_i} = a_{k_i} Q_{k_i}^{b_{k_i}}, a_{k_i} > 0, b_{k_i} < 0, |b_{k_i}| < 1, k \in K$$

where p_{k_i} is the price of the i -th agent's goods in the k -th market.

We consider the following problem: to search for the maximum number of agents $N_{max}, M_{max}, P_{max}$ that can operate in the retail market, the banking market and the insurance market, respectively, provided that non-negative profit is achieved, i.e., the following inequalities hold

$$\pi_{R_i}(Q_{R_i}) \geq 0 \quad \sum_{i=1}^N Q_{R_i} \leq Q_{R\Sigma} \quad (2)$$

$$\pi_{B_i}(Q_{B_i}) \geq 0 \quad \sum_{i=1}^M Q_{B_i} \leq Q_{B\Sigma} \quad (3)$$

$$\pi_{I_i}(Q_{I_i}) \geq 0 \quad \sum_{i=1}^P Q_{I_i} \leq Q_{I\Sigma} \quad (4)$$

where $\pi_{R_i}, \pi_{B_i}, \pi_{I_i}$ are the profits of companies in the retail market, the banking market and the insurance market, respectively; $Q_{R_i}, Q_{B_i}, Q_{I_i}$ are the sales volume of the i -th agent in these markets, respectively; $Q_{R\Sigma}, Q_{B\Sigma}, Q_{I\Sigma}$ are the capacity in these markets, respectively.

III. RESULTS

In each market, the agent is the i -th firm, therefore, we use the designation k_i where $i \in (1, \dots, N)$ for $k \in R$, $i \in (1, \dots, M)$, for $k \in B$, $i \in (1, \dots, P)$ and for $k \in I$.

Accordingly, the firm achieves a non-negative profit in the following range

$$\underline{Q}_{k_i} \leq Q_{k_i} \leq \overline{Q}_{k_i}$$

where $\underline{Q}_{k_i}, \overline{Q}_{k_i}$, are the minimum and the maximum sales at which the i -th firm in the k -th market obtains the non-negative profit. The boundaries of this interval are the sales volume in the firm's break-even point (i.e., the profit is zero).

$$\pi_{k_i}(\underline{Q}_{k_i}) = 0, \pi_{k_i}(\overline{Q}_{k_i}) = 0.$$

The profit function has two points, which correspond to this requirement, therefore, based on the conditions for the maximum number of firms in the market, we define

$$Q_{k_i}^0 = \min\{ \bar{Q}_{k_i}, \bar{Q}_{k_i} \} \quad (5)$$

where $Q_{k_i}^0$ is the minimum sales volume at which the i -th firm in the k -th market obtains the non-negative profit.

A substitution of (5) in (2), (3), (4) yields

$$\sum Q_{k_i}^0 \leq Q_{\Sigma k} \quad (6)$$

and restrictions (2) - (4) taking into account (1) have the following form:

$$a_{k_i} Q_{k_i}^{b_{k_i}+1} - C_{v_{k_i}} Q_{k_i} - C_{f_{k_i}} \geq 0 \quad (7)$$

We rewrite (6) as follows

$$\sum Q_{k_i}^0 \leq Q_{\Sigma k} (p_{\max}^0) \quad (8)$$

In this case, the price p_{\max}^0 is calculated as a maximum of all firms' prices in this market:

$$p_{\max}^0 = \max_{k \in K} p_{ki}(Q_{ki}^0)$$

The formula for calculating the minimum sales volume at a break-even point of the firm is obtained from the following equation

$$\pi_{k_i}(Q_{k_i}) = a_{k_i} Q_{k_i}^{b_{k_i}+1} - C_{v_{k_i}} Q_{k_i} - C_{f_{k_i}} = 0, \quad (9)$$

and this equation has only numerical solution.

If all firms in the k -th market have the different type parameters $C_{v_{ki}}, C_{f_{ki}}$, then the solution to problem (2) - (4) is calculated by cumulative summation of the values Q_{ki}^0 and calculating the number of firms, then restrictions (2) - (4) satisfy:

$$N_{\max} = \frac{Q_{\Sigma k}}{Q_{k_i}^0} \quad (10)$$

$$M_{\max} = \frac{Q_{\Sigma k}}{Q_{k_i}^0} \quad (11)$$

$$P_{\max} = \frac{Q_{\Sigma k}}{Q_{k_i}^0} \quad (12)$$

Thus, we make formulas (10)-(12) for calculating the maximum numbers of the firms in interconnected markets.

IV. NUMERICAL EXPERIMENT

The aggregate demand curve of the retailer market (Fig. 3) is derived on the basis of the statistical information about the firms' activities in the market in 2017-2019 [18, 19]. The parameters of the demand function are calculated similarly to the procedure [16].

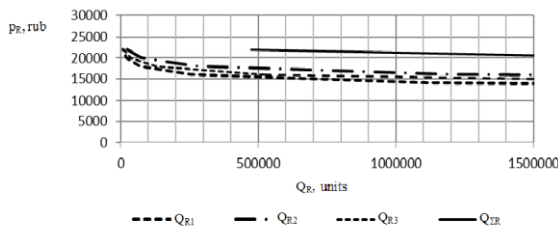


Fig. 3. Retailers' demand curves.

The aggregate demand curve of the retailer market is described by the following demand function

$$p_R(Q_R) = 52812Q_R^{-0.067}$$

According to formula (1), the retailer's profit function has the form:

$$\pi_{Ri}(Q_{Ri}) = 49000Q_{Ri}^{0.91} - 12500Q_{Ri} - 200000000$$

The capacity of the retail market is determined by rule (6), and it is equal to 40054097 units.

From Fig. 4, it is obvious that the retailer's profit is zero at two points. The retailer's profit function enables us to determine the retailer's break-even point, and, accordingly, the sales volume interval in which the firm makes the non-negative profit. A numerical solution of equation (9) for the retail market demonstrates that \bar{Q}_i is 28 thousand units. Based on the assumption of the firm identity in the retail market, according to (10), the maximum number of firms in the retail market N_{\max} is equal to 76112.

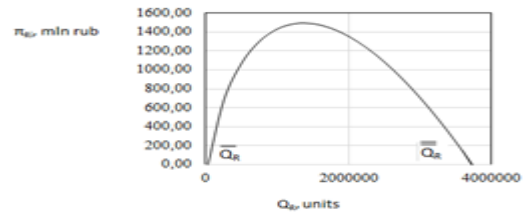


Fig. 4. Retailer's profit curves.

On the basis of the banking market data in 2017-2019 [17] the aggregate demand curve of the banking market is derived. Fig. 5 presents the statistics of three banks and the aggregate demand curve.

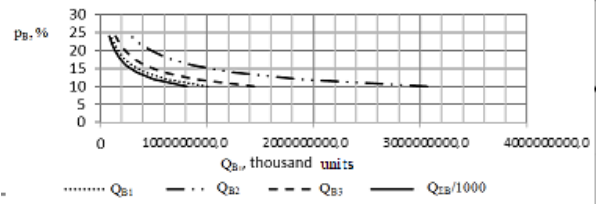


Fig. 5. Banks' demand curves.

The aggregate demand curve of the banking market is described by the demand function of the following form:

$$p_B(Q) = 4168Q_B^{-0.36}$$

The capacity of the banking market is determined by rule (6), and it is equal to 4147830 million contracts.

According to formula (1), the bank's profit function has the form:

$$\pi_{Bi}(Q) = 4168Q_{Bi}^{0.63} - 0.053Q_{Bi} - 1000000$$

From Fig. 6 it is obvious that the profit of the bank at two points is zero. The bank's profit function allows us to determine the bank's break-even point, and, accordingly, the sales volume interval of the non-negative profit. A numerical solution of equation (9) for the banking market shows that \bar{Q}_i is equal to 5.4 thousand loans.

Based on the assumption of the firms identity in the bank's market, from (11) it is possible to determine the maximum number of the firms under the non-negative profit condition. The maximum number of firms M_{\max} is equal to 768115876.

The aggregate demand curve of the insurance market is derived based on the insurance statistical data in 2017-2019

[17]. Fig. 7 presents the statistics of three insurance companies and the aggregate demand curve.

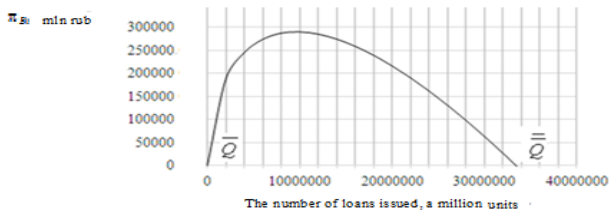


Fig. 6. Bank's profit curve.

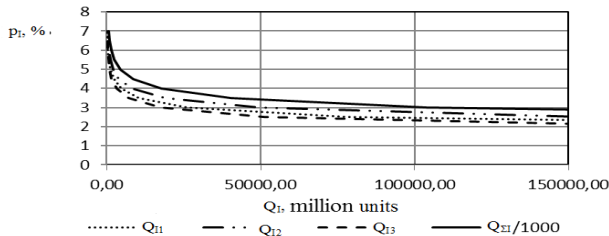


Fig. 7. Demand curves of insurers.

In the insurance market, the aggregate demand curve is described by the following demand function

$$p_i(Q) = 0.6079Q^{-0.163}$$

The capacity of the insurance market is determined by rule (6), and it is equal to 4524764 contracts.

According to formula (1), the insurer's profit function has the form:

$$\pi_{li}(Q) = 0.5107Q_i^{0.834} - 0.05Q_i - 2000$$

From fig. 8 it is obvious that the profit of the insurance company at two points is zero.

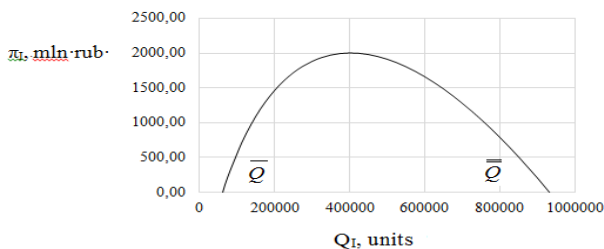


Fig. 8. Insurer's profit curve.

The profit function of the insurance company enables us to determine the sales volume, which satisfies the zero profit condition. A numerical solution of equation (9) for the insurance market shows that \bar{Q}_s is 174142262 units. Based on the assumption of the firms identity in the insurance market, according to (12), we determine the maximum number of firms that can be in the market, in the case of all firms obtain non-negative profits. The maximum number of companies in the insurance market P_{max} is 2736.

CONCLUSION

We investigate the interconnected markets with great numbers of agents, such as the retail market, the banking market and the insurance market. Based on the statistical analysis of these markets, we prove that the aggregate demand is described by a power function. As a result, we write in a similar form the profit functions of agents in these markets. The agent's profit function has a maximum point and two points with zero profit. Accordingly, the ranges of non-negative profits are determined. An analysis of the break-even point of the firm enable us to develop a technique

for calculating the quantitative estimates of these markets, i.e. the maximum numbers of agents.

In practice, this technique provides a guideline for firms, when they choose the market entry strategy. If in the market, the number of firms reaches the maximum, then the entry of a new firm into the market is disadvantageous, because it may not achieve the non-negative profit.

In addition, we calculated the following specific results for the analyzed markets. The retailer has the non-negative profit when selling a product in the range from 28800 to 3650000 units. The bank achieves the non-negative profit in the range from 5400 to 33411100 million loans. The insurer obtains the non-negative profit in the range from 63655 to 930000 units. The maximum number of firms in the retail market, in the banking services market and in the insurance market are 1390 units, 76811587 units and 2736 units, respectively.

REFERENCES

- [1] S. Paltsev, E. Monier, J. Scott, A. Sokolov and J. Reilly, "Integrated economic and climate projections for impact assessment," Climatic Change, pp. 21-33, 2015.
- [2] O. Ellabban and A. Alassi, "Integrated Economic Adoption Model for residential grid-connected photovoltaic systems: An Australian case study," Energy Reports, vol. 5, pp. 310-326, 2019.
- [3] P. Bolton and M. Dewatripont, "Contract Theory," Cambridge: MIT Press, 2005.
- [4] K.M. Ortman, "Fair allocation of capital growth," Operational Research, vol. 6, no. 2, pp. 181-196, 2016.
- [5] J.W. Hatfield, S.D. Kominers, A. Nichifor, "Stability and competitive equilibrium in trading networks," Journal of Political Economy, vol. 121, no. 5, pp. 966-1005, 2013.
- [6] M. Ostrovsky, "Information Aggregation in Dynamic Markets With Strategic Traders," Econometrica, vol. 80, no. 6, pp. 2595-2647, 2012
- [7] Z. Xu, Z. Peng, L. Yang and X. Chen, "An improved shapley value method for a green supply chain income distribution mechanism," International journal of environmental research and public health, vol. 15, no. 9, 2018.
- [8] N. Hayashi and M. Nagahara, "Distributed Proximal Minimization Algorithm for Constrained Convex Optimization over Strongly Connected Networks," IEICE Transactions on Fundamentals of Electronics, Communications and Computer Sciences, p. 351-358, 2019.
- [9] D. Wang, Z. Wang and W. Wang, "Distributed optimization for multiagent systems over general strongly connected digraph," 36th Chinese Control Conference (CCC), 2017.
- [10] R. Shao and L. Zhou, "Voting and the optimal provision of the public good," Journal of Public Economics, vol. 134, pp. 3-41, 2016.
- [11] T. Wakayama and T. Yamato, "Comparison of the voluntary contribution and Pareto-efficient mechanisms under voluntary participation," 2019, 143 p.
- [12] Continuity and incentive compatibility in cardinal voting mechanisms [Online]. URL: <https://papyrus.bib.umontreal.ca>.
- [13] H. Moulin, "Designing a single mechanism," Theoretical Economics, vol. 12, no. 2, pp. 587-619, 2017.
- [14] A. Ventura, K. Kaffero and M. Montibeller, "Pareto efficiency, Coase's theorem and external effects: a critical look," Economic Problems, vol. 50, no. 3, pp. 872-895, 2016.
- [15] M.I. Geraskin, "Optimal mechanism for the distribution of the effect in an integrated strongly coupled system of anonymous agents with a transferable utility," Problemy upravleniya, vol. 2, pp. 27-41, 2017.
- [16] M.I. Geraskin and V.V. Manakhov, "Optimization of interactions in a multi-agent, tightly linked "retailer-bank-insurer" system," Problemy upravleniya, vol. 4, pp. 9-18, 2015.
- [17] Rating of banks and insurance companies [Online]. URL: <https://www.banki.ru/insurance/companies/?page=29>.
- [18] Analytics on the market of household appliances and electronics for 2017 - from smartphones to refrigerators [Online]. URL:

- <https://www.shopolog.ru/metodichka/kompanii-i-rynki/analiz-rynka-bytovoy-tehniki-i-elektroniki-za-2017-god/>.
- [19] MAGAMAGNAT is portal about trade in Russia [Online]. URL: <http://megamagnat.ru/ts/86.html>.
- [20] D.A. Novikov, V.N. Burkov, M.V. Gubko and N.A. Korgin, "Theory of management of organizational systems and other sciences of organization management," *Problemy upravleniya*, no. 4, pp. 2-10, 2012.
- [21] V.N. Burkov, I.I. Gorgidze and D.A. Novikov, "Models and mechanisms for the distribution of costs and revenues in a market economy," Moscow: ICS RAS, 1997.
- [22] N.D. Morunov and D.L. Golovashkin, "Features of constructing block algorithms of the FDTD method when organizing computations on a GPU using the MATLAB language," *Computer Optics*, vol. 43, no. 4, pp. 671-676, 2019. DOI: 10.18287/2412-6179-2019-43-4-671-676.