Optimal orthogonal bases in optical applications

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***Abstract*—The tools of diffraction optics allow to implement in optics a wide range of mathematical functions useful for various applications. The orthogonal bases are of particular interest as they are optimal in terms of representation and transmission of optical information. The scientific school of Professor Viktor A. Soifer, Academician of the Russian Academy of Sciences, pays considerable attention to addressing the problems in this area. The following problems have been solved successfully: optical multiplexing - demultiplexing of various laser beams for modal compaction of communication channels, numerical and optical implementation of the Karhunen-Loeve expansion for the investigation of the stability of vortex beams propagation in a medium with random fluctuations, and the use of eigenfunctions of bounded optical systems for signal transmission with less distortion. The results achieved in the development of new optical devices can serve as the basis for the advanced information technologies.**

Keywords—diffraction optics, mathematical functions, orthogonal bases, optical information, scientific school, laser beam, Karhunen-Loeve expansion, vortex beams

I. INTRODUCTION

In information theory, the optimal representation of a certain signal [1–2] means choosing an orthogonal basis with the minimal number of coefficients of expansion by the basis functions. In optical applications, special attention is paid to the bases representing the solution of a differential or integral operator of propagation through a specific optical medium or system. As a rule, these are laser radiation modes. In addition, the bases that are optimal in terms of the presentation and transmission of optical information are of particular interest. For example, the Karhunen-Loeve basis, which provides the minimum number of expansion terms in the representation of a random signal, as well as eigenfunctions of bounded optical systems, the matching with which ensures the transmission of a signal with less distortion. Such complex basis functions, which sometimes even have no analytical representation, can be implemented in optics only by using the tools of diffraction optics. The scientific school of Professor Viktor A. Soifer, Academician of the Russian Academy of Sciences made a great contribution to the development of theoretical foundations and methods of diffraction optics. This article provides a brief overview of the achievements of the scientific school related to the formation and analysis of optical signals based on optimal orthogonal bases.

II. LASER RADIATION MODES

The plane wave basis is well known in optics, its spectrum can be generated in the focal plane of a lens. Along with the plane waves, expansion in conical waves is used often, these waves also correspond to the eigenfunctions of optical fibers with a constant refractive index, i.e. to Bessel modes. However, it is not so easy to perform optical expansion by the basis of conical waves. A zero-order Bessel beam can be formed using a glass cone (refractive axicon) [3], but the generation of high-order Bessel modes required the development of fundamentally different optical elements, which can be referred to with the concept of "Bessel optics" [4]. Hermite-Gaussian modes and Laguerre-Gaussian modes, which are the eigenfunctions of gradient media, are used widely in the theory of resonators, gradient waveguides, and paraxial optical systems [5]. When analyzing wavefront aberrations, the Zernike basis is used [6]. Generation, as well as optical decomposition by such bases, became possible only after the development of diffractive optical elements (DOEs). In the works of A.M. Prokhorov, I.N. Sisakyan, V.A. Soifer et al. [4, 7–10] it was proposed to synthesize optical elements - "modans" that generate and select individual laser radiation modes. A similar statement of the problem was contained in the article A.W. Lohmann, G.K. Grau et al. [11] published a year after the publication of M.A. Golub, A.M. Prokhorov, I.N. Sisakyan and V.A. Soifer [6]. These pioneering works were developed further at the scientific school of Professor Viktor A. Soifer, Academician of the Russian Academy of Sciences [12].

The group of Prof. V.V. Kotlyar calculated, and then produced in collaboration with Prof. S.N. Khonina and the group of Prof. J. Turunen (University of Joensuu, Finland) the DOEs that enable the formation of multimode laser beams with the pre-defined self-reproduction properties [13–18].

III. KARHUNEN-LOEVE BASIS

In addition to the bases listed above, other optimal bases are known that have no analytical representation. They are usually associated with additional conditions or restrictions imposed on optical systems or the optical signal.

In the statistical approach to the description of signals, the optimal basis for representing particular realizations of random signals is the Karhunen-Loeve basis (KL) [19], in which the error rate averaged over the ensemble of implementations is minimal. That is, the KL expansion provides the minimum number of terms among all possible expansions in the representation of a random signal for a given mean square error [20]. This property is relevant for various applications: from recognition problems to the problem of increasing the stability of optical signal transmission under atmospheric turbulence [21–26].

At the beginning of the 1990s, the problem of calculating the KL basis for the exponential cosine correlation function [27] was successfully solved at the Image Processing

Systems Institute of Russian Academy of Sciences (IPSI RAS), and then the possibility of its optical realization was studied [28, 29] in order to form decorrelated criteria of optical signals. In recognition of these results V.A. Soifer and S.N. Khonina received the First Prize of the German Society for the Advancement of Applied Informatics for the best scientific work in the field of image processing and pattern recognition in 1993.

It becomes more and more urgent to tackle the issues related to the transmission of an optical signal over significant distances in free space, use of optical radiation for sensing the Earth's surface, determining environmental parameters, location and navigation [30–32]. The use of optical radiation for these applications requires to take into account the effect of atmospheric turbulence [33–35]. Therefore, a lot of efforts are aimed at finding the possibility to overcome the negative impact of turbulence of the medium. An overview of the current situation in this area can be found in the joint publication of the researchers from IPSI RAS, Samara University and University of Miami [36]. Also, at the initiative of Academician V.A. Soifer, numerical and experimental studies were performed on the resistance of vortex beams to random fluctuations of the optical medium [37–42].

In order to analyze the ability of certain beams to maintain information stability (for example, the orbital angular momentum) under the influence of random fluctuations of the optical medium, numerical simulation or laboratory experiments with turbulence simulators are used, including diffusers, scattering screens and turbulence cells [43–44]. The synthesis of such a simulator of turbulence can be implemented using the KL expansion for the given correlation operators based on a search for the eigenfunctions of these operators [45–46].

IV. THE BASIS OF PROLATE SPHEROIDAL WAVE FUNCTIONS

When analyzing and compensating the atmospheric distortions, not only the KL expansion, but also the basis of prolate spheroidal wave functions (PSWFs) is used [47]. According to the Fourier transform theory, a signal cannot be sharply bounded both in the object domain and in the spatial frequency band, but by using the PSWFs it is possible to provide the best field concentration in the object and spatial-frequency domains simultaneously [48–49]. PSWFs are also used in various applications: in the theory of antenna synthesis, in image-based reconstruction of objects, for superresolution, in the theory of resonators, in digital filtering [50–55]. At the beginning of 2000s, under the direction of V.A. Soifer a new stable method was developed at IPSI RAS for calculating the eigenvalues of the zero-order PSWFs for arbitrary parameter values [56], as well as for approximating the eigenfunctions by finite series [57–58]. Later, on the basis of the developed algorithms, the possibilities of applying the PSWF basis to the problems of forming non-diffraction beams [59–60] and increasing the resolution of imaging systems [61] were investigated.

V. COMMUNICATION MODES

The basis of spheroidal functions is closely related to the concept of communication modes [62–63], which are the eigenfunctions of some optical propagation operator. In particular, the communication modes in the Cartesian coordinate system for a finite (space-limited) Fourier transform correspond to prolate angular spheroidal functions. Spheroidal functions are also eigenfunctions for a two-lens system, in which an additional restriction appears in the plane of the spatial spectrum [48–49].

Another attractive feature of the communication mode method is that it simplifies free space diffraction to ordinary mathematical multiplication, thereby making it an interesting tool for propagating waves and synthesizing fields [64]. To implement this approach, methods of calculating DOEs correlated with PSWFs [65] were used at IPSI RAS. The possibility of optical generation of an arbitrary superposition of spheroidal functions allows to form optical fields passing through the corresponding optical systems without distortion [66–68].

The theory of communication modes (or eigenfunctions of optical operators) is applicable to arbitrary optical systems and electromagnetic waves [69–74].

A particular type of optical system is optical fiber. The current level of use of optical fiber for transmitting information over time and frequency channels tends to the limit of bandwidth. An additional increase in the number of information channels is possible on the mode division multiplexing (MDM) [10, 75]. This technology includes the transmission of information in various transverse modes on a single physical medium - optical fiber. The transmitted information can be contained in the mode structure and in the energy component carried by each mode in the laser beam individually. Moreover, multiplexing based on vortex beams associated with the orbital angular momentum is of the greatest interest [76–78]. For mode channel multiplexing based on the orbital angular momentum in real (bounded) fibers, it becomes necessary to calculate vortex eigenfunctions [78]. The propagation of an optical signal through multi-lens optical systems and gradient waveguides is well described by the fractional Fourier transform [79–84]. Spatial constraint inevitably leads to the necessity to consider spatially bounded propagation operators and calculate the corresponding eigenfunctions to simulate the propagation of an optical signal [85–86]. This approach allows both to understand the nature of optical signal distortions, and to form an approximation of the initial signal through decomposition by eigenfunctions of the lens system by analogy with the PSWFs. When forming such an approximation, a compromise can be observed between the accuracy of the approximation and the ability to transmit signal without distortion.

VI. CONCLUSION

Modern computing resources provide the possibility to calculate the eigenfunctions of fairly complex operators, including those describing near-field optics and scanning optical systems [87–88], thus the diffraction optics tools allow to implement these complex expansions in optics. In this area, the academic school of Academician V.A. Soifer has been at the level of world priorities for several decades [10, 89–93], creating new optical devices and forming advanced information technologies on this basis [94–97].

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