A Problem of Managing the Reserve of Capacity for the Arcs of a Communication Network

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Abstract. The article considers the problem of managing the reserve of capacity of arcs, which is relevant for the distribution of flows and designing reliable communication networks with discrete parameters and a constraint on flows delay time or average load factor of the network arcs. An algorithm for the approximate solution of the problem for the case of linear functions for the cost of arcs is proposed and the results of its experimental study on a network containing 1000 nodes and 4000 arcs are presented. The results of the experiment showed the sufficient accuracy and speed of the proposed algorithm, which allows us to assert of its practical applicability for engineering calculations on the large-dimensional networks.

Keywords: flows in networks, the reserve of capacity of arcs, time of delay flows, problems of combinatorial optimization

1 Introduction

The article is an addition to the work [1], in which the Problem of Choosing the Capacity of Arcs (PCCA) for communication network from a given set of discrete integer values with constraint on flows delay time was considered. Delays of flows t_{kl} on arcs are defined as $t_{kl} = f_{kl} / (w_{kl} - f_{kl})$, $\forall kl \in E$, and the constraint on the delay time of flows t_{av} in a network has the following form $t_{av} = 1/U_{\Sigma} \sum_{kl \in E} f_{kl} / (w_{kl} - f_{kl}) \le T_{\max}$. Here $f_{kl} \in Z^+$ — fixed arc flow value for $kl \in E$, E — set of arcs of network, $w_{kl} \in Z^+$ — bandwidth capacity of arc $kl \in E$, $T_{\max}$ — the maximum of flows delay time in network, $U_{\Sigma} = \sum_{ij \in S} u_{ij}$ — total flow in network, $u_{ij} \in Z^+$ — value of the flow from a node i to a node j, S — set of pairs of indexes corresponding nodes in the network. When approaching the magnitude of the flow on the arcs to their carrying capacity, the delay increases and, therefore, network congestion can occur.

The essence of the problem is for fixed flows it is necessary to choose the throughput capacities of arcs from a given set of integers so that the constraint on the delay time of flows is fulfilled and the minimum of some objective function is achieved. For the design of data transmission networks, PCCA was studied in detail in [2-5]. This problem also arises in transport networks when distributing flows according to the criterion of the minimum cost of the network and a given restriction on the delay time of flows [6].

A Problem of Managing the Reserve of Capacity of Arcs (PMRCA) is to parametrically solve a PCCA problem, in which as a variable parameter are selected values $T_{\max}$ with the given sampling step. By controlling the parameter $T_{\max}$ for maximum delay, the data network administrator or the transport network manager can provide the required reserve for the bandwidth capacity of the communication channels or the carrying capacity of vehicles at predicted fluctuations of values of flows on a given time of intervals. A decrease in the parameter $T_{\max}$ (increase in the reserve) leads to a rise in the cost of the network, but reduces the probability of redistribution of flows and technical re-equipment of communication channels or fleet of vehicles at increasing flows and a threat of emergence overloads in the network. An increase in the parameter $T_{\max}$ makes it possible to reduce the capacity of communication channels or the carrying capacity of vehicles and the cost of the network, but increases the risk of redistributing flows and upgrading the network. As a quantitative measure of reserve capacity can be taken as the average load factor of the network arcs.

Topical issues are the affiliation of PCCA and PMRCA to the class of NP-hard problems, and the development of approximate time-polynomial algorithms. The article provides an example of a parametric PCCA solution on a network containing 1000 nodes and 4000 arcs, which clearly demonstrates the methodological approach to solve PMRCA and to the practical choice of required reserve capacity of arcs for the communication network.

2 The formulation and algorithm of solving the problem

We consider a direct connected network $G(N, E)$ with a set of nodes N, $n = |N|$ and a set of arcs E, $e = |E|$. In network for each direct arc kl, ($k < l$) exist back arc lk, ($l > k$). An arc represents a switched communication line in a data network or a vehicle route, the final nodes of which coincide with the initial and final node of the arc. The network may contain loops and parallel arcs, since cyclic and repeating communication lines and communication lines with the same final nodes are allowed. An integer flow matrix is given on the network $U = \| u_{ij} \|_{n\times n}$. Let w_{kl}, $kl \in E$ — sought-for a bandwidth capacity of arcs of the network in transport blocks, $w_{kl} \in \{w_1, w_2, ..., w_\alpha\}$, w_i, $i = \overline{1, \alpha}$ — ascending positive integers; $d_{kl} \in R^+$, $kl \in E$ — arcs lengths; $C_{kl}(w_{kl}, d_{kl}) \in R^+$, $kl \in E$ — discrete values cost of arcs, such that $C_{kl}(w_i, d_{kl}) \le C_{kl}(w_{i+1}, d_{kl})$, $i = \overline{1, \alpha - 1}$; $f_{kl} = \sum_{ij \in S} u_{ij}^{kl}$, $kl \in E$ — fixed total flows in transport blocks, a flowing along the arcs of the network, where u_{ij}^{kl} — is the flow of transport blocks from i to j, which passes along arc kl.

It is required to find the minimum value of the network cost function

$$\min_{w_{kl}} \sum_{kl \in E} C_{kl}(w_{kl}, d_{kl}), \quad w_{kl} \in \{w_1, w_2, ..., w_\alpha\} \tag{1}$$

s.t.

$$\frac{1}{U_\Sigma} \sum_{kl \in E} \frac{f_{kl}}{w_{kl} - f_{kl}} \le T_{\max}, \quad w_{kl} > f_{kl}, \ \forall kl \in E \tag{2}$$

for the parameter of selected values $T_{\max}$, that vary within the following limits

$$\frac{1}{U_\Sigma} \sum_{kl \in E} \frac{f_{kl}}{w_\alpha - f_{kl}} \le T_{\max} \le \frac{1}{U_\Sigma} \sum_{kl \in E} \frac{f_{kl}}{w_{kl,\min} - f_{kl}}, \tag{3}$$

where $w_{kl,\min} = \min w_i > f_{kl}$, $i = \overline{1, \alpha}$.

To estimate the bandwidth reserve for each solution $\tilde{w}_{kl}(T_{\max}) \in \{w_1, w_2, ..., w_\alpha\}$, $\forall kl \in E$, we will calculate the average load factor of arcs for the network

$$ALF = \frac{1}{e} \sum_{kl \in E} \frac{f_{kl}}{\tilde{w}_{kl}(T_{\max})}. \tag{4}$$

Note that problem (1), (2) can be represented as a knapsack problem with Boolean variables and multi-choice (0-1 Multiple-choice Knapsack Problem, 0-1 MCKP), which, as you know, belongs to the class of NP-hard problems [7]. Let $c_{ij} \in R^+$ — discrete values cost of arcs i with capacity $w_{ij} \in \{w_1, w_2, ..., w_\alpha\} \in Z^+$ and length d_i, $j = \overline{1, \alpha}$, $i = \overline{1, e}$; $t_{ij} = f_i / (w_{ij} - f_i)$, $w_{ij} > f_i$, $j = \overline{1, \alpha}$, $i = \overline{1, e}$ —delays of flows on arcs; f_i — flow on the arc i, $i = \overline{1, e}$. Suppose that $x_{ij} = 1$, if for the arc i the capacity w_{ij} is selected, $j = \overline{1, \alpha}$, $i = \overline{1, e}$, and $x_{ij} = 0$ otherwise. We require to find

$$\min \sum_{i=1}^{e} \sum_{j=1}^{\alpha} c_{ij} x_{ij} \tag{5}$$

s.t.

$$\frac{1}{U_\Sigma} \sum_{i=1}^{e} \sum_{j=1}^{\alpha} t_{ij} x_{ij} \le T_{\max}, \tag{6}$$

$$\sum_{j=1}^{\alpha} x_{ij} = 1, \quad i = \overline{1, e}, \tag{7}$$

$$x_{ij} \in \{0, 1\}. \tag{8}$$

Here, the required throughputs w_{ij} correspond to the optimal solution x_{ij}^* to the problem (5) - (8).

It is easy to see that any individual problem formulated in the form of (1), (2) can be transformed in time $O(e\alpha)$ into the corresponding instance of problem (5) - (8). To do this, it is necessary to construct two matrices of size $e \times \alpha$, whose rows correspond to arcs, the columns — to a set of discrete capacities, and the cost of arcs c_{ij} and delays on arcs t_{ij} are taken as matrix elements. The converse is also true.

For the knapsack problem with multi-choice (5) - (8), there are exact pseudo-polynomial algorithms and Fully Polynomial Time Approximation Scheme (FPTAS) [8, 9]. This means that for them there are algorithms that, polynomial time of the size for the input of the problem and $1/\varepsilon$ make it possible to obtain a $(1+\varepsilon)$ - guaranteed approximate solution, where ε is an arbitrarily small positive number. Therefore, to obtain an accurate or guaranteed ε-approximate solution to problem (5) - (8), it is possible to use the algorithms described in [8-13]. These algorithms can also be used to solve the problem in statement (1) - (2).

Despite the existence of exact pseudo-polynomial algorithms, their application for the parametric solution of the PCCA problem is not justified due to the great time complexity of the algorithms.

So, for example, the time complexity of the FPTAS algorithm for solving the classical 0-1 MCKP problem is $O(e^2\alpha/\varepsilon)$ [9].Therefore, in [1], for solving NP-hard problems (1), (2) and (5) - (8), two approximate algorithms were proposed on the basis of the approximation of discrete cost functions by linear ones, and on the method of sequential analysis of options, which was first proposed and investigated in the works [14-17].

The first algorithm uses the Lagrange multiplier method, which allows one to analytically solve a relaxed problem and obtain an exact continuous solution. The second algorithm enumerates the solutions, narrowing the range of feasible solutions at each iteration, and can be used for any monotonically non-decreasing cost of arcs with an increase in their throughput. It can be applied both to the initial statement of the problem, and to the statement in the form (5) - (8).

We consider problem (1), (2) when it is known, that a given discrete values cost of the arcs $C_{kl}(w_{kl}, d_{kl})$ can be approximated with a sufficient degree of adequacy by continuous linear functions

$$C_{kl}(w_{kl}, d_{kl}) = c_{kl}^0 + c_{kl}^1 \cdot w_{kl}, \quad \forall kl \in E \tag{9}$$

where c_{kl}^0, c_{kl}^1 — we found approximation coefficients. For linear cost functions, the analytical solution w_{kl}^*, $C_{\min}^*$, which is obtained by the method of Lagrange multipliers is known as [2, 3]. We write the Lagrange function

$$L = \sum_{kl \in E} c_{kl}(w_{kl}, d_{kl}) + \frac{\beta}{U_\Sigma} \frac{f_{kl}}{w_{kl} - f_{kl}}$$

where β — is the Lagrange multiplier. Equating the partial derivatives of this function to zero, we obtain $\frac{\partial L}{\partial w_{kl}} = c_{kl}^1 - \frac{\beta f_{kl}}{U_\Sigma (w_{kl} - f_{kl})^2} = 0$ where from $w_{kl} = f_{kl} + \sqrt{\frac{\beta f_{kl}}{U_\Sigma c_{kl}^1}}$. Substituting the values w_{kl} in the original equality constraint, we obtain $T_{\max} = 1/U_{\Sigma} \sum_{kl \in E} f_{kl} / (w_{kl} - f_{kl}) = \sum_{kl \in E} \sqrt{\frac{c_{kl}^1 f_{kl}}{\beta U_{\Sigma}}}$ and $\sqrt{\beta} = 1/T_{\max} \sum_{kl \in E} \sqrt{\frac{c_{kl}^1 f_{kl}}{U_{\Sigma}}}$.

Substituting the value of the multiplier in the expression for w_{kl} , we finally obtain the values w_{kl}^* for the optimal capacity of communication lines

$$w_{kl}^* = f_{kl} + \frac{1}{T_{\max}} \sqrt{\frac{f_{kl}}{U_{\Sigma} c_{kl}^1}} \sum_{rs \in E} \sqrt{\frac{c_{rs}^1 f_{rs}}{U_{\Sigma}}} \ ,$$

or after obvious transformations

$$w_{kl}^* = f_{kl} + \frac{f_{kl}}{U_{\Sigma} T_{\max}} \frac{\sum_{rs \in E} \sqrt{c_{rs}^1 f_{rs}}}{\sqrt{c_{kl}^1 f_{kl}}} \ . \tag{10}$$

The optimal network cost is defined as

$$C_{\min}^* = \sum_{kl \in E} c_{kl}^1 f_{kl} + \frac{1}{U_{\Sigma} T_{\max}} (\sum_{kl \in E} \sqrt{c_{kl}^1 f_{kl}})^2 \ . \tag{11}$$

Note that in practice for data transmission networks and transport networks, capacity, as a rule, should be the same for the forward kl and reverse lk directions. Therefore, at practical solution to the problem, two oriented communication lines kl and lk are replaced with one non-oriented communication line $kl, (k < l)$ and selected $f_{kl} = \max\{ f_{kl}, f_{lk} \}$.

The approximation algorithm allows one to quickly get into the neighborhood of continuous points optimum of w_{kl}^* and find an approximate discrete solution $\tilde{w}_{kl}$. The idea of the algorithm is as follows.

Suppose that for all arcs $kl \in E$ the coefficients c_{kl}^0 , c_{kl}^1 of the linear dependence (9) are known. Such coefficients can be obtained for each arc kl of length d_{kl} by linear approximation (for example, by the least squares method) of discrete values cost of the arcs for a number of standard discrete capacity $w_{kl} \in \{w_1, w_2, ..., w_{\alpha}\}$, the same for all arcs.

Knowing the coefficients $c_{kl}^1 = tg\phi$ (Fig. 1) and the values of f_{kl} , through formula (10) you can find the throughput w_{kl}^* . Next in the neighborhood of continuous optimums w_{kl}^* according to a certain procedure, we select the suitable carrying capacity values from a discrete range.

We present a general scheme of the “greedy” algorithm for the parametric solution of problem (1), (2) for linear cost functions.

AP Algorithm

1. For each arc $kl \in E$ and a given range of throughputs $w_{kl} \in \{w_1, w_2, ..., w_{\alpha}\}$, through the least-squares method, it is needed to approximate a discrete cost $C_{kl}(w_{kl}, d_{kl})$ with linear functions. We determine the coefficients c_{kl}^0 , c_{kl}^1 in (9).

2. Through the formula (3), we determine the boundaries of the interval of varia-

tion of the parameter $T_{\max}$. It is arbitrarily to choose the first parameter value $T_{\max}$ from the interval, for example, starting from its left or right border.

3. We calculate w_{kl}^{*}, $C_{\min}^{*}$ according to (10) and (11).

4. From $w_{kl} \in \{w_1, w_2, ..., w_\alpha\}$ for each arc $kl \in E$ we select the closest to w_{kl}^{*} the permissible values w_{kl}^{j}, $j = \overline{1,\alpha}$, such as $f_{kl} < w_{kl}^{j} \le w_{kl}^{*}$. If $w_{kl}^{j} \le f_{kl}$, then as w_{kl}^{j}, we choose the nearest larger value of $w_{kl}^{j} > f_{kl}$ (it may be, that $w_{kl}^{j} \ge w_{kl}^{*}$). If $f_{kl} = 0$, then it is accepted that arc kl does not exist.

5. In the neighborhood of the point w_{kl}^{*} for each arc, we find the values $c_{kl}^{*} = \Delta c_{kl} / \Delta t_{kl}$, where $\Delta c_{kl} = c_{kl}(w_{kl}^{j+1}) - c_{kl}(w_{kl}^{j})$, $\Delta t_{kl} = f_{kl} / (w_{kl}^{j} - f_{kl}) - f_{kl} / (w_{kl}^{j+1} - f_{kl})$, $j \in \{1, ..., \alpha - 1\}$.

6. We arrange all the arcs $kl \in E$ in ascending order of values c_{kl}^{*} and get a set $E^{*} = \{(k,l)_1, (k,l)_2, ..., (k,l)_e\}$. The reason for such an ordering is for all arcs $c_{kl}(w_{kl}^{j}) \le c_{kl}(w_{kl}^{j+1})$, and $t_{kl}(w_{kl}^{j}) = f_{kl} / (w_{kl}^{j} - f_{kl}) > t_{kl}(w_{kl}^{j+1}) = f_{kl} / (w_{kl}^{j+1} - f_{kl})$. We set the initial value of the arcs counter $i = 0$.

7. Let $i \leftarrow i + 1$. We select an arc $(k,l)_i$ from the set E^{*} and go to step 8.

8. We increase throughput $w_{(k,l)_i}^{j}$ for the arc $(k,l)_i$ to the nearest larger value from the discrete row $\{w_1, w_2, ..., w_\alpha\}$, i.e. choose such $w_{(k,l)_i}^{j+1}$, that $w_{(k,l)_i}^{j+1} > w_{kl}^{*} \ge w_{(k,l)_i}^{j}$, $j \in \{1, ..., \alpha - 1\}$. We recalculate the value t_{av} taking into account an increase in throughput of the arc $(k,l)_i$. If $t_{av} \le T_{\max}$, then go to step 9. Otherwise, go to step 7 to increase the counter of arcs. The value of the counter of arcs i cannot exceed the value number of arcs e, since the condition $t_{av} \le T_{\max}$ will be guaranteed to be satisfied in the cycle for i due to the fact that for all arcs may turn out to be $w_{(k,l)_i}^{j+1} \ge w_{kl}^{*}$.

9. The found values $w_{(k,l)_i}^{j}$, $j \in \{1, ..., \alpha\}$, $i \in \{1, ..., e\}$ or $\tilde{w}_{kl}(T_{\max}) \in \{w_1, w_2, ..., w_\alpha\}$, $\forall kl \in E$, are an approximate solution to the problem for the current parameter value $T_{\max}$. Through the formula (4), we calculate the average load factor of the network arcs ALF and the cost of the network $AS = \sum_{kl \in E} C_{kl}(\tilde{w}_{kl}, d_{kl})$.

10. If the choice of the current parameter value $T_{\max}$ is completed, there is the end of the algorithm. Otherwise, we change the parameter value $T_{\max}$ to the selected magnitude and go to step 3.

The time complexity of the AP algorithm is $O(e\alpha^2 + e\alpha + Me\log e + K_1 e)$ and is mainly determined by the time complexity of the algorithm for approximating the cost of arcs with linear functions and of the sorting algorithm, where M — is the number of values $T_{\max}$ with a given sampling step, K_1 — is some constant.

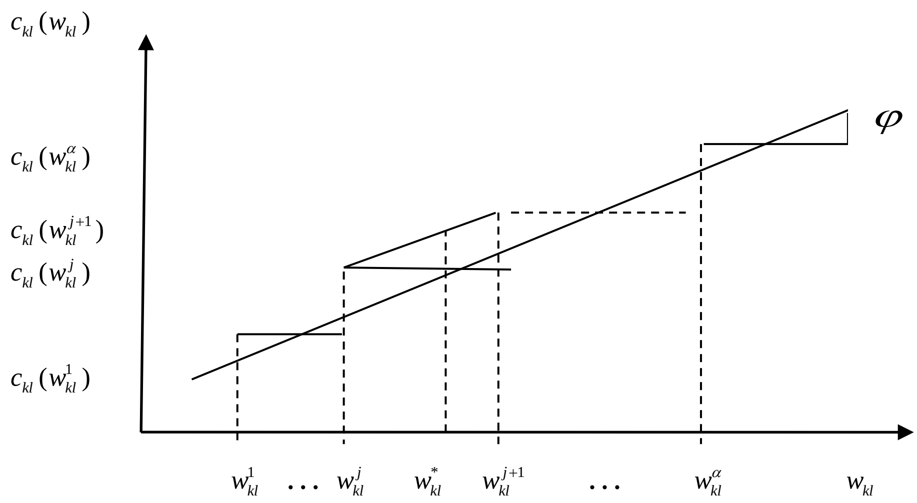


Fig. 1. The cost function of arcs

3 Results of the experimental solution of the problem

The problem was solved by using an example of a network with n = 1000 nodes and e = 4000 arcs, generated by a pseudo random number sensor. The lengths of arcs d_{kl}, $kl \in E$ ranged from 20 to 50 km, and values of flows u_{ij}, $i, j = \overline{1,n}$ from 1 to 2 transport blocks. The f_{kl}, $kl \in E$ values were obtained by distributing all flows along the shortest paths using the two-criteria lexicographic algorithm [18]. The throughput of arcs was selected from the set {5, 10, 15, 20, ...} with a sampling step of 5 units. The cost of arcs for a given set of throughputs of arcs was calculated by the formula $C_{kl}(w_{kl}, d_{kl}) = k_0^j + k_1^j d_{kl}$, $j = 1, 2, ...,$ $kl \in E$. For linear functions, the coefficients $k_0^1 \le k_0^2 \le ...$ and $k_1^1 \le k_1^2 \le ...$ were chosen from the sets {0, 0, 0, 0, 0, ...} and {5, 10, 15, 20, ...}. The number of values in the sets for w_{kl}, k_0^j, k_1^j was determined depending on the maximum flow along the arc $\max f_{kl}$, $kl \in E$, the sampling step of the throughput capacities of the arcs, and the initial value $T_{\max}$. If for the current value of $T_{\max}$, it turned out to be $w_{\alpha} < w_{kl}^{*}$, the set $\{w_1, w_2, ..., w_{\alpha}\}$ can be automatically expanded to the value $w_{\alpha} = \lceil w_{kl}^{*} \rceil^5$, where $\lceil \cdot \rceil^5$ — is the rounding sign to a larger integer multiple of 5. The table 1 shows the results of solving the problem when changing constraint at the delay time from $T_{\max}$ = 0.002 to $T_{\max}$ = 10.0. For all values of the parameter $T_{\max}$, the following are given: Continuous Optimal Solutions, rounded to integers and Approximate Discrete Solutions (AS) in nominal units of cost value; values of the average load factor of arcs for network (ALF); deviations in percent of approximate solution from the continuous optimal solution.

Table 1. Results for an experimental study of the dependence for the network cost and the average load factor of arcs on the value $T_{\max}$ for the AP algorithm

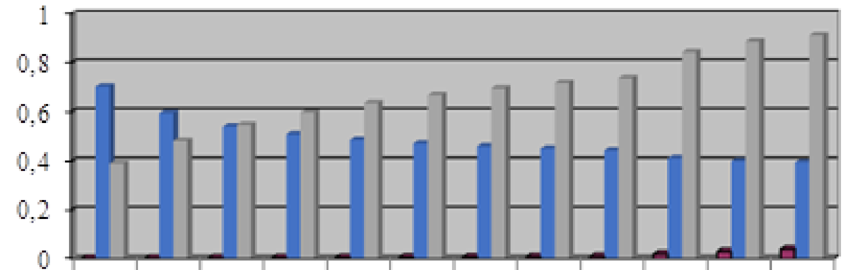
№	$T_{\max}$	Continuous Optimal Solution	Approximate Solution	Average Load Factor of the Arc	Deviation from Continuous Optimal Solution, %
1	0,002	702606912	702629696	0,38942969	0,0032
2	0,003	594371712	594391104	0,48080164	0,0033
3	0,004	540254080	540272512	0,54661781	0,0034
4	0,005	507783520	507800512	0,59667718	0,0033
5	0,006	486136480	486152288	0,63623863	0,0033
6	0,007	470674304	470691072	0,66836989	0,0036
7	0,008	459077664	459092352	0,69505483	0,0032
8	0,009	450058080	450074496	0,71761900	0,0036
9	0,01	442842368	442856736	0,73692870	0,0032
10	0,02	410371808	410387264	0,84252059	0,0038
11	0,03	399548288	399564960	0,88708848	0,0042
12	0,04	394136544	394153120	0,91188490	0,0042
13	0,05	390889472	390907136	0,92762768	0,0045
14	0,06	388724768	388742280	0,93870205	0,0045
15	0,07	387178560	387197728	0,94657397	0,0050
16	0,08	386018880	386038144	0,95283246	0,0050
17	0,09	385116928	385137024	0,95748824	0,0052
18	0,1	384395360	384416224	0,96161634	0,0054
19	0,2	381148320	381177888	0,9798876	0,0078
20	0,3	380065952	380102912	0,98587686	0,0097
21	0,4	379524768	379568832	0,98896205	0,0116
22	0,5	379200064	379250464	0,99074137	0,0133
23	0,6	378983616	379040928	0,99179864	0,0151
24	0,7	378828992	378893472	0,99245179	0,0170
25	0,8	378713024	378784000	0,99292845	0,0187
26	0,9	378622816	378700672	0,99332023	0,0206
27	1,0	378550656	378635168	0,99358821	0,0223
28	2,0	378225952	378363360	0,99453980	0,0363
29	3,0	378117728	378303680	0,99463964	0,0492
30	4,0	378063616	378296352	0,99464238	0,0616
31	5,0	378031136	378295712	0,99464256	0,0700
32	6,0	378009472	378295264	0,99464262	0,0756
33	7,0	377994016	378295008	0,99464267	0,0796
34	8,0	377982432	378295008	0,99464267	0,0827
35	9,0	377973408	378295008	0,99464267	0,0851
36	10,0	377966176	378295008	0,99464267	0,0870

As it is visible in table 1, continuous optimal and discrete approximate solutions are differing slightly with a value from 0.0032% to 0.0796%. In a variant of solution 33 at $T_{\max}$ = 7.0, the lower boundary of the network cost 378295008 is reached, which cannot be improved at the further increasing the value of $T_{\max}$. It follows that the deviations of discrete optimal and approximate solutions will be even smaller.

The most interesting variants for analyzing and deciding the choice of reserve for the capacity of arcs, are the solutions with numbers 1-12, for which the network cost is significantly reduced (by 308476576 units) and load factor of arcs is increased from 0.39 to 0.91. These variants solutions are clearly shown in Fig. 2.

The same results as in table 1 were obtained, when solving the problem with an approximate algorithm on the basis of the method of sequential analysis for variants, which is given in [1]. However, the time complexity of this algorithm is several orders of magnitude greater, and to calculate each solution for the given values $T_{\max}$, it took from 5 to 12 seconds on a PC with a clock frequency of 2.66 GHz. The AP algorithm coped with such tasks in a split second.

The conducted experimental studies showed the sufficient accuracy and speed of the AP algorithm, which in the most cases allows it to be used for engineering calculations on networks containing more than 1000 nodes and 4000 arcs. The PMRCA solution can be useful in solving the practical problems of flow distribution and designing reliable communication networks with discrete parameters and a constraint on the time delay of flows or on the average load factor of arcs of network [19, 20].



	1	2	3	4	5	6	7	8	9	10	11	12
Tmax	0,002	0,003	0,004	0,005	0,006	0,007	0,008	0,009	0,01	0,02	0,03	0,04
AS	0,703	0,594	0,54	0,508	0,486	0,471	0,459	0,45	0,443	0,41	0,4	0,394
ALF	0,389	0,481	0,547	0,597	0,636	0,668	0,695	0,718	0,737	0,843	0,887	0,912

Fig. 2. Change in network cost and load factor of the arcs from Tmax. The values in the AS line is multiplied by 10^9

The experimental results were obtained on a dual-core PC with a clock frequency of 2.66 GHz and 2 GB RAM under Windows XP. All programs are written in software environment Microsoft Developer Visual Studio.

4 Conclusion

The article formulates the problem of managing the reserve of capacity arcs in a communication network with discrete parameters when changing the constrain on the delay time of flows. An approximate polynomial algorithm for solving the problem for the case of linear of arcs cost's functions is proposed and the results of its experimental study are presented. The experimental results allow us to state the practical applicability of the algorithm for solving the problem on large-dimensional networks containing more than 1000 nodes and 4000 arcs.

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