Representation of Knowledge Using Different Structures of Concepts

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Abstract. The paper is devoted to the problem of knowledge representation using concepts of different levels of generality. Model-theoretical methods are being developed for translating data and knowledge presented in the language of low-level concepts into knowledge presented using concepts of a high level of generality. The formalization of knowledge is carried out in the terms of FCA, which allows us to move from low-level concepts to more general concepts and, as a result, generate new, more general knowledge about the domain.

Keywords: Knowledge Representation, FCA, Precedent Model, Boolean-valued Model, Semantic Domain Model

1 Introduction

Knowledge representation is a central part of the development of intelligent systems. Today, there are several different methodologies for representing knowledge: frames, semantic networks, production systems, etc. [1].

One of the most developed and popular methodologies are the logical knowledge representation. Logical systems are highly expressive. Representation of the data array in the form of an algebraic system makes it possible to work with two levels of information consideration: the level of the initial data on which the analysis is performed, and the level of knowledge – generalized laws formulated in the form of sentences of first-order predicate logic. Consideration of a set of different situations (precedents) of the given domain as a class of algebraic systems allows us to work with statistical knowledge about the domain [2].

On the other hand, FCA is a powerful tool for presenting and processing knowledge [3-5]. FCA methods are widely used for ontology engineering [6], machine learning [7], semantic web [8] and so on. The connection between FCA and the theory of axiomatizable classes of algebraic systems was shown in [9]. There was described a method for constructing a formal context for the case model (using the notion of a Boolean-valued model) and a method of transitioning to an object-clarified context [10, 11].

Recently, much attention has been paid to approaches to concept mining simplification. For example, in [12], the concept indices were studied and their applications for evaluating interestingness measures of formal concepts.

In this paper we consider various levels of knowledge representation, which are presented using concepts of varying degrees of generality. For example, knowledge of a lower level of generality is:

– symptoms of diseases, test results;

– specific numerical data on exchange rates, oil prices;

Knowledge of the high level of generality is, for example:

– patients' diseases, syndromes, complications of diseases;

– economic forecasts, expectations of stability or instability, currency crises, etc.

Knowledge containing statements of different levels of generality is formalized in the form of algebraic systems of different signatures. For these systems, formal contexts are constructed that describe them. The properties of lattices of constructed formal contexts are studied.

2 Semantic Domain Model

We start the formalization with a finite set $\mathbb{E} = \{E_1, \dots, E_n\}$ of domain precedents. We have a set (signature) σ of low-level concepts of this domain. Each domain precedent $E_i \in \mathbb{E}$ is formalized as a model $E = \langle A, \sigma \rangle$ [13, 14].

Next, we will bring together knowledge about of all precedents from the class $\mathbb{E}$. For this we need to enrich the signature σ with a set of constants $\boldsymbol{C_A} = \{c_a \mid a \in A\}$, i.e. to take $\sigma_A = \sigma \cup \boldsymbol{C_A}$. Now we may consider the set $S_a(\sigma_A)$ of all atomic sentences of the signature σ_A as a formalization of low-level concepts of the object domain and the set $S(\sigma_A)$ of all sentences of the signature σ_A as a formalization of all possible concepts of the domain.

Definition 1 [15]. *Ordered triple* $\mathfrak{A}_{\mathbb{E}} \leftrightharpoons \langle A, \sigma_A, \tau \rangle$ *is called* ***Precedent Model*** *generated by the set of the precedents* $\mathbb{E}$*, if for any sentences* $\varphi(c_{a_1}, \dots, c_{a_n}) \in S(\sigma_c)$ *we have*

$$\tau\left(\varphi(c_{a_1}, \dots, c_{a_n})\right) = \{E \in \mathbb{E} \mid E \vDash \varphi(a_1, \dots, a_n)\}.$$

The definition of ***Boolean-valued Model*** one can find in [15].

It was shown in [15] that for any Precedent Model it is possible to construct a Boolean-valued Model isomorphic to it.

When there is a formalization of individual precedents of the given domain, it is often necessary to describe low-level concepts. So, for example, in the subject domain of computer security [16] we formalized the knowledge obtained from texts in natural language. Of these texts, it was often possible to single out very small (in volume) concepts. For example, in some precedent it was said about an attack using the *Randex virus*, in another precedent the *CMJ virus* was used, and in the third - *MrKlunky*. For each of these viruses it will be difficult to identify any regularities, since the ratio of the number of attack precedents, where there was a mention of this particular virus to the total number of precedents, is very small. Thus the evalution μ on the predicates formalizing these concepts will be very close to zero.

On the other hand, all these viruses have many common characteristics, so it's reasonable to combine them into one, more general concept "*TSR Viruses*" and to study the properties of this new concept. Note that each such concept is expressible in the signature σ through some formula.

So, we select the set of formulas $F \subseteq F(\sigma)$ and enrich the signatureσ with the set of new predicts $\boldsymbol{P}_F = \{P_\varphi \mid \varphi \in F\}$. Let $\sigma_F = \sigma_A \cup \boldsymbol{P}_F$. Next, we extend the Boolean-valued model $\mathfrak{A}_{\mathbb{B}}$ to the signature σ_F, i.e. put

$$\mathfrak{A}'_{\mathbb{B}} = \mathfrak{A}_{\mathbb{B}} \downharpoonleft \sigma_F,$$

where the estimation $\tau' : S(\sigma_F) \to \mathbb{B}$ is redefined as follows. For each $P_\varphi(x_1, \dots, x_n) \in \boldsymbol{P_F}$ and for any elements$a_1, \dots, a_n \in A$ we have

$$\tau'\left(P_\varphi(c_{a_1}, \dots, c_{a_n})\right) = \tau\left(\varphi(c_{a_1}, \dots, c_{a_n})\right).$$

Now we can remove low-level concepts from the signature σ_F leaving only concepts of a higher level, i.e. put $\sigma^* = \boldsymbol{C_A} \cup \boldsymbol{P_F}$. The model $\mathfrak{A}''_{\mathbb{B}} = \mathfrak{A}'_{\mathbb{B}} \upharpoonright \sigma^*$ is a formalization of the subject domain at a higher level.

Let $\mathfrak{A}_{\mathbb{B}}$ be an atomic Boolean-valued model. Denote

$$At(\mathbb{B}) = \{a \in \mathbb{B} \mid a \text{ is an atom}\}.$$

Consider the formal context

$$\boldsymbol{K}(\mathfrak{A}_{\mathbb{B}}) = \left(At(\mathbb{B}),\ S_a(\sigma_{\mathrm{A}}),\ I_\tau\right),$$

where

$$a\ I_\tau\ \varphi \Leftrightarrow\ a \le \tau(\varphi).$$

We say that the formal context $\boldsymbol{K}(\mathfrak{A}_{\mathbb{B}})$ describing the Boolean-valued model $\mathfrak{A}_{\mathbb{B}}$ [17].

In the next section, we will consider various generation algorithms of the model $\mathfrak{A}''_{\mathbb{B}}$ from the model $\mathfrak{A}_{\mathbb{B}}$ and show how the formal contexts describing them will change.

3 Formal contexts representing higher-level concepts

When forming a set of $\boldsymbol{P}_F$ of higher-level concepts, first of all, we consider the possibility of obtaining these concepts directly from the formal context itself that describes this Boolean-valued model.

Definition 2. *Consider the formal context* $\boldsymbol{K} = (G, M, I)$. *Denote by* $\widetilde{M}$ *the set of contents of all concepts of the context* $\boldsymbol{K}$, *i.e.*

$$\widetilde{M} = \{B \subseteq M \mid B^{\downarrow\uparrow} = B\}.$$

Consider the formal context $\widetilde{\boldsymbol{K}} = (G, \widetilde{M}, \widetilde{I})$, *where for any object* $g \in G$ *and for any* $B \in \widetilde{M}$ *we have*

$$g \widetilde{I} B \ \Leftrightarrow g \in B^{\downarrow} \text{ in the context } \boldsymbol{K}.$$

Proposition 1. *The concept lattices generated by the formal contexts* $\boldsymbol{K} = (G, M, I)$ and $\widetilde{\boldsymbol{K}} = (G, \widetilde{M}, \widetilde{I})$ are isomorphic, i.e. $\underline{\mathfrak{B}}(\boldsymbol{K}) \cong \underline{\mathfrak{B}}(\widetilde{\boldsymbol{K}})$.

Thus, this Proposition shows that in order to move to the meta level it is not enough to have only "internal" information contained in a formal context. So, for example, in [16] it was proposed to obtain "external" information through cauterization of the set of objects' properties and, when moving to the meta-level, consider generalized properties that characterize different clusters.

We describe this approach in the terms of FCA [18].

Let an equivalence relation $\sim$ be defined on the set M. We will denote by $[m]_\sim$ the equivalence class generated by the element m and denote by $M/_\sim$ the factor-set.

Definition 3. Consider the formal context $\boldsymbol{K} = (G, M, I)$ *and the equivalence relation* $\sim$ *defined on the set* M.

1. *By* $\boldsymbol{K}_\wedge = (G, M/_\sim, I_\wedge)$ *we denote the formal context in which*

$$g\, I_\wedge\, [m]_\sim \iff \forall n \in [m]_\sim\ gIn.$$

2. *By* $\boldsymbol{K}_\vee = (G, M/_\sim, I_\vee)$ *we denote the formal context in which*

$$g\, I_\vee\, [m]_\sim \iff \exists n \in [m]_\sim\ gIn.$$

Proposition 2. *Consider the formal contexts* $\boldsymbol{K} = (G, M, I)$ *and* $\boldsymbol{K}_\wedge = (G, M/_\sim, I_\wedge)$. *Then the lattice* $\underline{\mathfrak{B}}(\boldsymbol{K}_\wedge)$ *is a sublattice of the lattice* $\underline{\mathfrak{B}}(\boldsymbol{K})$.

Theorem 1. *Consider the formal contexts* $\boldsymbol{K} = (G, M, I)$ *and* $\boldsymbol{K}_\vee = (G, M/_\sim, I_\vee)$. *We define a map* $h: \underline{\mathfrak{B}}(\boldsymbol{K}) \to \underline{\mathfrak{B}}(\boldsymbol{K}_\vee)$ *as follows:* $h\big((A, B)\big) = \big(A^{\uparrow\downarrow}, A^{\uparrow}\big)$. *Then for any* $(A_1, B_1), (A_2, B_2) \in \underline{\mathfrak{B}}(\boldsymbol{K})$ *we have:*

1. $(A_1, B_1) \le (A_2, B_2) \Rightarrow h\big((A_1, B_1)\big) \le h\big((A_2, B_2)\big)$;
2. $h\big((A_1, B_1)\big) \cap h\big((A_2, B_2)\big) = h\big((A_1, B_1) \cap (A_2, B_2)\big)$;
3. $h\big((A_1, B_1)\big) \cup h\big((A_2, B_2)\big) \le h\big((A_1, B_1) \cap (A_2, B_2)\big)$.

Thus, it follows from the Theorem 1 that the map $h: \underline{\mathfrak{B}}(\boldsymbol{K}) \to \underline{\mathfrak{B}}(\boldsymbol{K}_\vee)$ is a homomorphism of the lower semilattices.

Corollary 1. A map $h: \underline{\mathfrak{B}}(\boldsymbol{K}) \to \underline{\mathfrak{B}}(\boldsymbol{K}_\vee)$ is homomorphism of the lattices if and only if for any $A_1, A_2 \in G$ follow condition is satisfied:

$$(A_1 = A_1'' \text{ and } A_2 = A_2'') \Rightarrow A_1 \cup A_2 = (A_1 \cup A_2)''.$$

Note that in the general case, the homomorphism h may not possess the properties of injectivity and surjectivity. This means that when moving from the context $\boldsymbol{K}$ to the context $\boldsymbol{K}_\vee$, on the one hand, some concepts will be combined into more general concepts, and on the other hand, new concepts will appear. On the other hand, when moving from the context $\boldsymbol{K}$ to the context $\boldsymbol{K}_\wedge$, some concepts will be "forgotten" only.

The results obtained in the paper may be applied to the development of ontological [19-22] and semantic [23-28] technologies.

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