Analytic Hierarchy Process Sustainability at the Significant Number of Alternatives Ranking

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Abstract. This paper focuses on the need to evaluate the sustainability of Analytic hierarchy process at the Ranking of more than 10 alternatives. The proposed method is based on simulation modeling of the process of improving expert pair-wise comparison judgments. The represented method provides a step-wise improvement of the pair-wise comparison matrix transitivity. The average discrepancy and coincidence of ranks in multiple modeling are proposed as estimates of the rating stability. The application of the developed method was studied on a statistical sample formed according to the final tables of the England, Germany and Spain football championships. The method for determining probability of some alternatives ranks is developed. It is possible to modify the method for predicting the results of sports competitions and for the case of ranking with partially missing expert ratings.

Keywords: Analytic Hierarchy Process, Ranking, Sustainability, Consistency, Simulation Modeling

1 Introduction

Analytic hierarchy process (AHP) [1-3] is applied in many areas, such as economy, industry, social sphere, ecology, politics, military science while solving such problems as: choice and evaluation of decision alternatives and decision factors, resource allocation, analysis of benefits-costs-opportunities-risks, forecasting, analytical planning, construction and evaluation of development scenarios and other [4].

AHP solves the ranking alternatives problem on the basis of expert filled pair-wise comparison matrix (PCM).The result of the ranking is subjective.

The property of the ranking stability lies in the different susceptibility to changes in the results with insignificant variations in the assessments of the expert.

The question is how the results of ranking the same alternatives may differ from another expert with the same knowledge and experience remains open. It is difficult to assess how much the order of alternatives can change with minor or small changes in the expert's opinion when comparing several pairs of alternatives. Another problem with AHP is that removing one or more alternatives from the list of ranked ones can change the remaining alternative ranks relative to each other.

The stability problem is considered when solving two problems [5]: the selection of the best alternative, and the ranking of all alternatives.

2 Research Goals

Let PCM $A=[a_{ij}]$ with $i,j=1,\dots,n$ be a reciprocal positive matrix corresponding to the paired comparisons on a Saaty scale [1-3] of the n elements or alternatives A_i, $i=1,\dots,n$ with respect to a single criterion.

Objectives of the study (in relation to PCM of significant dimension):

- offer indicators of ranking stability for AHP;
- establish patterns of change in these indicators with a slight variability in expert assessments;
- propose a method for assessing the stability of ranking in solving specific problems;
- assess the stability of the ranking with some expert incompetence, with the possibility of evaluations such as “hard to say” or “I don't know”.

3 Related Work

This paper investigates the ranking stability of a single-level AHP to change part of the expert’s ratings with an improvement in the degree of the PCM transitivity and consistency. The results of the research are also applicable to AHP modifications with ranking alternatives for incomplete data.

The ranking for incomplete data [6] extends to situations in which the expert is allowed to answer “I don’t know” or “not sure” to some of the questions. The Harker approach is based on the definition of quasi-inverse symmetric matrices. A similar approach to identifying priorities for an incomplete inverse symmetric matrix was proposed in [7]. Other shortcut PCM formation procedures and methods for supplementing the missing assessments [8-10] can be distinguished.

Harker [11] on PCM with the number of alternatives 6–9 filled randomly investigated the possibilities of 5% underfilling.

In most cases, the stability of the ranking is determined by the “what happens if” principle, interactively changing grades and tracking the ranking results. The authors of [12] integrate the AHP with the stochastic multicriteria acceptability analysis, an inverse-preference method, to make pair-wise comparisons uncertain. A simulation experiment is used to determine how the accuracy of the decisions and the ability of the model to find the best option deteriorates as the uncertainty increases.

The next area of research [4, 5, 13, 14] of the AHP ranking stability is aimed at establishing possible (small) variations in expert estimates providing an unchanged result.

Aguarón and Moreno-Jiménez [5] proposed a method of determine of local stability intervals. This method, based on an inverse sensitivity analysis the final ranking of the alternatives, deals with the relationship between changes in the judgments and the rank reversal of the alternatives. The local stability intervals are determined for each judgment, for each alternative or element, and for the PCM associated with the criterion (reciprocal matrix of pair-wise comparisons) which ensures that the best alternative and ranking are maintained, respectively.

The method was further developed in [4]. As an example it was used in solving a multiple-criteria decision-making problem of evaluation of renewable energy technologies for an eco-house, using the AHP. The stability intervals allow finding so-called critical elements of a decision-making problem. Critical expert pair-wise comparison judgments can be found that are sensitive to changes of a local ranking of decision alternatives. Also critical hierarchy elements, i.e. decision criteria, decision goals etc. can be determined – elements that are characterized by the least changes of their weights necessary for a global ranking changes of decision alternatives. Later [15] the consistency stability interval associated with each judgment, in which expert judgmentscan oscillate without exceeding a value of the consistency measure fixed in advance were obtained.

To calculate these intervals, the row geometric mean method as the prioritization procedure, the geometric consistency index as the consistency measure and a local situation with one criterion were considered.

There are several other interesting studies to improve the consistency of PCMs. An optimization algorithm for minimizing the consistency ratio (CR) has been proposed, assuming that the expert statements have an accuracy of 10% [17]. The places of greatest inconsistency in [18-21] are calculated by the matrix model of the induced bias of the Hadamard product [19], by the M Outflow method [21]. The expert is encouraged to change his opinion on his own or on the basis of a determined recommendation. In [22], the number of paired comparisons is reduced to ensure that the ranking is not precise but approximate. A variety of other options were considered in [23] to solve the problems of inconsistency of expert judgments and incompleteness of PCM.

The research [4] solves such problems as evaluating sensitivity of a decision alternatives local ranking to changes in expert pair-wise comparison judgments (elements of a PCM); evaluating sensitivity of a global ranking of decision alternatives to changes in hierarchy elements weights; search critical and stable expert pair-wise comparison judgments; search critical and stable elements.

The stability of AHP ranking at significant PCM dimensions was not researched in the reviewed papers.

4 The Method of Improve Consistency Expert Comparison Judgments

The method of improvement consistency expert comparison judgments (ICECJ) is based on simulation modeling of the expert's work to improve the consistency of his assessments.

Consider transformations of PCM ***A***. Part of the expert's ratings, randomly selected, is reset to zero. Then the zero values are replaced by averaged estimates according to the transitivity property of the remaining estimates, according to the sequence of calculations:

$$a_{ij} = \frac{1}{n_1}\sum_{k=1}^{n} a_{ik} \cdot a_{kj}, \qquad a_{ji} = \frac{1}{n_1}\sum_{k=1}^{n} a_{jk} \cdot a_{ki}, \ ,.$$

then

$$a_{ij} = (a_{ij} + \frac{1}{a_{ji}})/2,$$

and then

$$a_{ij} = \begin{cases} [a_{ij}], \text{ if } a_{ij} \geq 1 \\ [1/a_{ij}] \text{ otherwise} \end{cases}, \ a_{ij} = \begin{cases} 9, \text{ if } a_{ij} > 9 \\ a_{ij} \text{ otherwise} \end{cases}, \ a_{ji} = 1/a_{ij} \qquad (1)$$

where $n_1 = \sum_{i=1}^{n} \delta(a_{ik} \cdot a_{kj}, 0)$, $\delta(x, y) = \begin{cases} 1, \text{ if } x = y \\ 0, \text{ otherwise} \end{cases}$, $[x]$ –the integer part of number x.

Thus, PCM is modified, AHP ranking is performed. Repeatedly executing this procedure allows you to get a set of solutions for ranking alternatives. The analysis of many solutions allows us to determine some estimates of the solutions accuracy:

- the probability of an alternative appearing at each ranking level;
- the average divergence of ranking in the solutions set;
- the average coincidence of ranking in the solutions set;
- the average divergence of ranking place in the initial solution and on the set of modified PCMs for all alternatives.

5 Experimental Research

5.1 Statistics Data

Due to the subjectivity of the AHP process, the ranking results have some error (instability). To estimate this error, methods of probability theory and mathematical statistics should be applied.

To do this, you must have a sufficient amount of statistical information. Due to the fact that in most cases AHP is used in strategic planning, there are few numbers of its constant and repeated use, especially with a large number of alternatives (more than 10), and the authors are not aware of such information.

To assess the sustainability of the ranking results, this paper uses ranking data without an expert. The role of the expert is played by nature. For research, we will use the information about the ranking of teams as a result of the football championships: Germany Bundesliga 53 seasons from 1963/1964 to 2015/2016, Spain La Liga 29 seasons from 1987/1988 to 2015/2016 and England Premier League 24 seasons from 1992/1993 to 2015/2016. A total of 105 final tables of football championships were processed. They were attended by 16 (2 times), 18 (49 times), 20 (49 times) or 22 (5 times) teams.

PCM is also filled here and rankings are performed, but on a different scale than proposed by T. Saaty. Moreover, paired comparisons are performed by nature and depend on many random factors (player disqualifications, injuries, physical and psychological state of the players, etc.). PCM has some natural inconsistency, but the ranking results are fairly objective.

5.2 Convert the Final Tables of the Football Championships to PCM with Saaty Scale

To reseach AHP, there was made a transition from the traditional football championships of Spain, England and Germany to the Saaty scale, in which the superiority of one team over another is estimated by integers from 1 to 9 and, accordingly, concession by the inverse number from 1 to 1 / 9.

The converting was performed as follows:

$$a_{ij} = \begin{cases} 1, \text{ if } m_{ij} = 0 \\ [bm_{ij} + c], \text{ if } [bm_{ij} + c] \le 9 \,\&\, m_{ij} > 0 \\ 9, \text{ if } [bm_{ij} + c] > 9 \,\&\, m_{ij} > 0 \\ 1/[bm_{ij} + c], \text{ if } [bm_{ij} + c] \le 9 \,\&\, m_{ij} < 0 \\ 1/9, \text{ if } [bm_{ij} + c] > 9 \,\&\, m_{ij} < 0 \end{cases}, \qquad (2)$$

where are a_{ij} – the PCM elements, m_{ij} – is the difference between goals scored and conceded in two championship matches between the *i*-th and *j*-th teams, *b* and *c* are some (desired) coefficients, $[x]$ – is rounding to the nearest integer.

To assess the quality of the transition from a traditional football table to PCM on the Saaty scale, the following indicators are proposed:

– the average discrepancy of places in the traditional table and ranking by AHP $S_j^R = \frac{1}{n_j}\sum_{i=1}^{n}(O_{ij}^T - O_{ij}^S)$, where n_j – the number of teams that participated in the j-th championship, O_{ij}^T – the rank of the i-th team in the traditional table of the j-th championship, O_{ij}^S – the team rank after ranking by AHP using (2);

– the average proportion of team that retained their ranks after the conversion $S_j^E = \frac{1}{n_j}\sum_{i=1}^{n}\delta(O_{ij}^T, O_{ij}^S)$.

The optimal values of *b* and *c* are determined for each of the three championships separately. They were determined as a result of two-criteria optimization: minimizing the discrepancy of ranks and maximizing the number of teams that retained their rank after converting the tables of football championships to PCM on the Saaty scale:

$$(b,c) = \arg(\min_{b,c} \frac{1}{N} \sum_{j=1}^{N} \frac{S_j^R(b,c) - \min_j S_j^R(b,c)}{\max_j S_j^R(b,c) - \min_j S_j^R(b,c)} - \frac{S_j^E(b,c) - \min_j S_j^E(b,c)}{\max_j S_j^E(b,c) - \min_j S_j^E(b,c)}) , \quad (3)$$

where N – is the number of championship tables, $\arg$ – defines functions arguments b,c at which a minimum is reached. The corresponding quality indicators and Consistency ratio (CR) are given in table 1.

Table 1. Indicators of the transition from the traditional tables of football championships to PCM of AHP

Championship	b	c	S_j^R		S_j^E		CR
			By (3)	min	By (3)	max	By (3)
Germany	0,2	2,3	1,57	1,54	0,31	0,31	0,21
Spain	0,33	1,2	1,7	1,70	0,29	0,33	0,12
England	0,6	2,5	1,61	1,59	0,34	0,34	0,39

An example of converting the traditional German championship table of the 1963/1964 season to PCM is given in table 2 and 3.

In table football teams marked as C_1 – Braunschweiger TSV Eintracht 1895, C_2 – Eintracht Frankfurt, C_3 – BV Borussia 09 Dortmund, C_4 – SV Werder Bremen, C_5 – Hamburger SV, C_6 – Hertha BSC, C_7 – MSV Duisburg, C_8 – FC Köln, C_9 – FC Kaiserslautern, C_{10} – Karlsruher SC, C_{11} – TSV 1860 München, C_{12} – FC Nürnberg, C_{13} – SC Preußen 06 Münster, C_{14}– FC Saarbrücken, C_{15} – FC Schalke 04, C_{16}– VfB Stuttgart 1893.

To unify the processing, points were awarded according to modern rules: for a victory – 3 points, a draw – 1 and a loss – 0. Therefore, some of the ranking results in the final tables of the championships (including the ones given in table 2) differ from the official ones.

In the above example, the following indicators were obtained: CR = 0.15, $S^R = 1,75$ and $S^E = 0,375$. The results of the conversion to PCM (average discrepancy of about two ranks and one third of the teams that retained their rank) are close to the average for the Germany Bundesliga. You should not expect a better matching because of a significant, non-linear change in the measurement scale

As a result of the transformations, the statistical material PCM (105 matrices) was obtained, which is close to that was formed by nature and sufficient to study the stability of PCM.

5.3 Experimental Results and Discussion

Experiment 1. The goal is to assess the probability of an alternative being placed in appropriate ranks.

Table 2. Summary table German Bundesliga 1963/1964

	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}	C_{11}	C_{12}	C_{13}	C_{14}	C_{15}	C_{16}	rank
C_1	x	0:3	2:0	1:1	2:1	1:1	0:0	1:1	0:1	2:0	0:1	2:0	1:0	3:1	4:3	2:0	10
C_2	3:0	x	2:1	7:0	2:2	4:0	2:2	2:1	1:1	0:3	5:2	2:3	3:0	3:1	4:2	3:2	2
C_3	3:0	3:0	x	4:3	5:2	7:2	0:0	2:3	9:3	3:2	3:3	3:1	0:0	2:1	3:0	7:1	4
C_4	2:3	4:1	3:2	x	4:2	2:2	1:1	1:1	2:0	0:0	4:1	2:1	4:2	0:3	1:0	2:2	11
C_5	2:1	3:0	2:1	1:1	x	5:1	3:3	1:1	7:3	1:1	5:0	2:2	5:0	4:2	3:1	1:1	6
C_6	1:2	1:3	0:0	5:2	1:2	x	5:2	0:3	2:2	2:3	3:1	1:1	2:0	3:2	1:0	0:2	13
C_7	5:1	3:1	3:3	1:0	4:0	1:3	x	2:2	3:0	2:0	3:0	0:0	0:0	3:1	3:0	3:0	3
C_8	4:1	1:1	5:2	4:3	4:1	3:1	3:3	x	5:1	4:0	2:2	5:0	3:0	1:3	2:2	2:1	1
C_9	2:1	1:1	0:1	3:0	3:2	3:0	1:1	3:3	x	1:0	2:1	3:1	0:0	2:4	2:3	1:3	12
C_{10}	3:1	1:2	1:3	1:1	0:4	1:1	1:4	2:2	5:1	x	1:0	1:3	4:2	2:2	1:1	0:3	14
C_{11}	1:1	1:1	6:1	3:2	9:2	1:2	0:0	1:3	3:0	1:0	x	5:0	3:1	7:1	7:1	1:1	7
C_{12}	1:0	1:0	4:0	3:0	3:2	2:3	2:0	2:2	0:5	2:4	2:2	x	2:2	2:0	0:2	0:0	9
C_{13}	0:2	1:3	1:2	1:3	1:1	4:2	4:2	0:2	1:0	0:0	0:0	0:1	x	2:1	2:2	4:2	15
C_{14}	2:2	0:4	2:1	3:2	1:1	3:0	0:2	0:2	2:4	1:3	1:2	3:5	1:1	x	1:1	0:1	16
C_{15}	2:0	1:2	3:1	2:3	1:0	1:0	2:2	2:3	4:0	2:1	2:1	4:1	1:2	4:1	x	2:0	8
C_{16}	5:0	0:0	2:1	2:0	2:2	2:0	1:2	0:1	4:0	4:1	1:1	1:0	0:3	3:1	2:0	x	5

Table 3. PCM formed based on table 1

	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}	C_{11}	C_{12}	C_{13}	C_{14}	C_{15}	C_{16}	λ	rank
C_1	1	1/4	1	1	1	1	1/3	1/2	1	1	1	1	2	1	1	1/2	0,043	15
C_2	4	1	1	3	1/2	4	1	1	1	1	2	1	4	4	2	1	0,091	2
C_3	1	1	1	1	1	4	1	1/3	5	2	1/4	1	1	1	1	4	0,071	5
C_4	1	1/3	1	1	1	1/2	1	1	1	1	1	1	3	1/3	1	1	0,050	10
C_5	1	2	1	1	1	4	1/3	1/2	2	3	1	1	4	1	1	1	0,071	6
C_6	1	1/4	1/4	2	1/4	1	4	1/4	1/2	1	2	1	1	1	1	1/3	0,055	9
C_7	3	1	1	1	3	1/4	1	1	2	4	2	1	1	3	2	3	0,090	3
C_8	2	1	3	1	2	4	1	1	3	3	1	4	4	1	1	1	0,094	1
C_9	1	1	1/5	1	1/2	2	1/2	1/3	1	1/2	1	5	1	1	1/4	1/4	0,046	12
C_{10}	1	1	1/2	1	1/3	1	1/4	1/3	2	1	1	1	1	1	1	1/4	0,040	16
C_{11}	1	1/2	4	1	1	1/2	1/2	1	1	1	1	4	1	5	4	1	0,081	4
C_{12}	1	1	1	1	1	1	1	1/4	1/5	1	1/4	1	1	3	1/4	1	0,045	13
C_{13}	1/2	1/4	1	1/3	1/4	1	1	1/4	1	1	1	1	1	1	1	4	0,046	11
C_{14}	1	1/4	1	3	1	1	1/3	1	1	1	1/5	1/3	1	1	1/2	1/2	0,043	14
C_{15}	1	1/2	1	1	1	1	1/2	1	4	1	1/4	4	1	2	1	1	0,059	8
C_{16}	2	1	1/4	1	1	3	1/3	1	4	4	1	1	1/4	2	1	1	0,067	7

The ICECJ method of increasing PCM transitivity with a small percentage of redefined expert ratings (5% here) and a sufficiently large number of parallel experiments

(1000 here) allows us to establish probabilistic estimates of certain (calculated) ranks for each alternative.

Note that minor changes in expert ratings are aimed to improving their consistency. The variability of the computed ranks of alternatives is associated with this.

The obtained results allow us to conclude that the stability of the occupied place by one or another alternative.

For example, for PCM table 4, the probabilities of occupied ranks are calculated (here, as a percentage). A number of alternatives $(C_4, C_6 \ldots C_8, C_{11}, C_{15}, C_{16})$ fairly steadily occupy their ranks. You can count on the objectivity of this situation. Alternative C_1 is almost the same for the 14th and 15th positions. For this alternative you need to provide additional criteria.

Experiment 2. The goal is to identify patterns of change in the sustainability indicators of ranking results and PCM consistency.

In the experiment, the number of modified estimates varied from 5 to 30%.

Each experiment was repeated 50 times.

Table 4. Ranks probabilities (percent)

	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**
C_1											3	8	17	35	34	3
C_2	25	50	24	1												
C_3				4	42	45	9	1								
C_4								1	11	77	11	1				
C_5				3	52	40	4									
C_6							3	26	61	5	1		1	2	2	1
C_7	10	25	61	3	1											
C_8	65	25	11													
C_9									2	8	39	33	7	3	4	4
C_{10}												1	3	5	19	72
C_{11}			4	89	3	3										
C_{12}									1	3	11	2	31	14	14	5
C_{13}									1	3	28	3	18	9	8	4
C_{14}										1	6	6	23	32	21	11
C_{15}							8	65	22	3	2					
C_{16}					2	12	76	7	3							

To assess the stability of the ranking, the previously introduced indicators and were used S_j^R and S_j^E .

As a result of the experiment, it was possible to quantitatively establish the degree of improvement in PCM consistency when changing expert estimates (and increasing the degree of transitivity) and the corresponding losses estimated by averaged: discrepancies before and after changes in the ranking of alternatives, fractions of alternatives that have retained their place and the difference in weight of alternatives (Fig. 1 and 2).

The studies were conducted separately on PCM, transformed according to the football championships results of the three countries.

The reliability of the results obtained is confirmed by its proximity and the small confidence intervals obtained for both 90 and 95% level of consistency. An example of the dependence S_j^R for PCM formed on the results of the Germany Bundesliga is shown in Fig. 3.

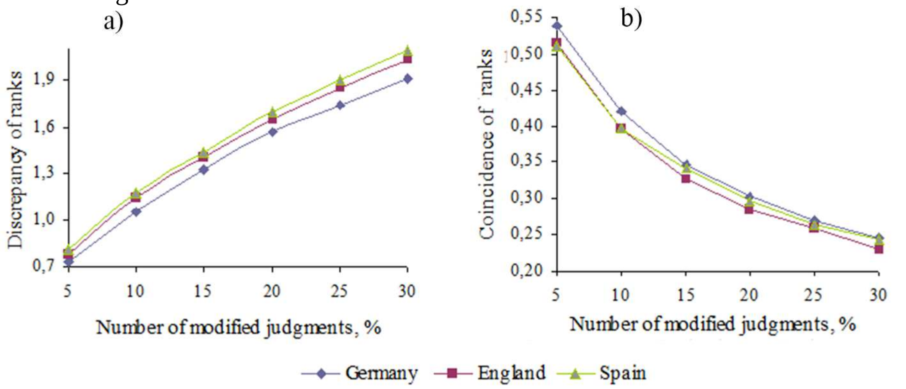

Fig. 1. Average discrepancy (a) and coincidence (b) ranks according to AHP before and after ICECJ

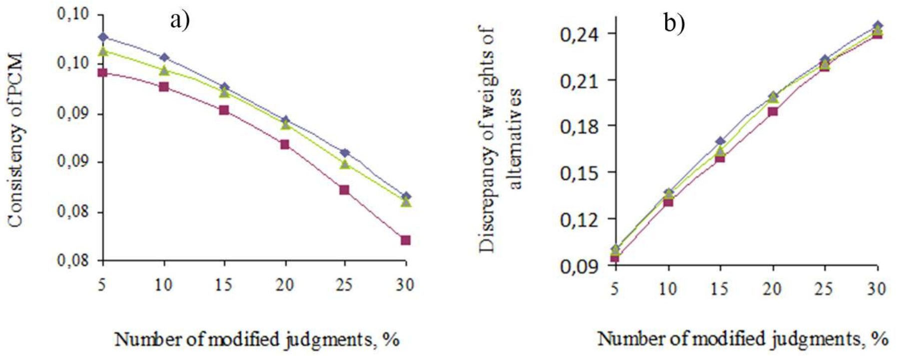

Fig. 2. Average CR and weights of alternatives before and after ICECJ

From 2% (with 5% of the changed estimates) to 20% (at 30%), the consistency of MPS improves, from CR= 0.113 to CR= 0.09. In this case, the average discrepancy in the weights of the alternatives varies by 10-25%, respectively.

It was found that even insignificant (in five percent) changes in the estimates of alternatives (even in the direction of improving consistency) lead to 45% of the divergence of places in the ranking. With a 5% change in expert ratings, about 50% of teams retain their places, and with 30% only one fifth.

We take as a basis the accuracy of the transition from traditional methods of pairwise comparisons in football championships to the assessment according to AHP, as natural. It can be noted that with 20-25% of the expert's assessments being changed, the percentage of teams that have retained their place and the average divergence of places are approaching natural accuracy.

In other words, when additionally determining 20-25% of the missing expert ratings in AHP, the accuracy of the ranking corresponds to the natural accuracy of reproducibility.

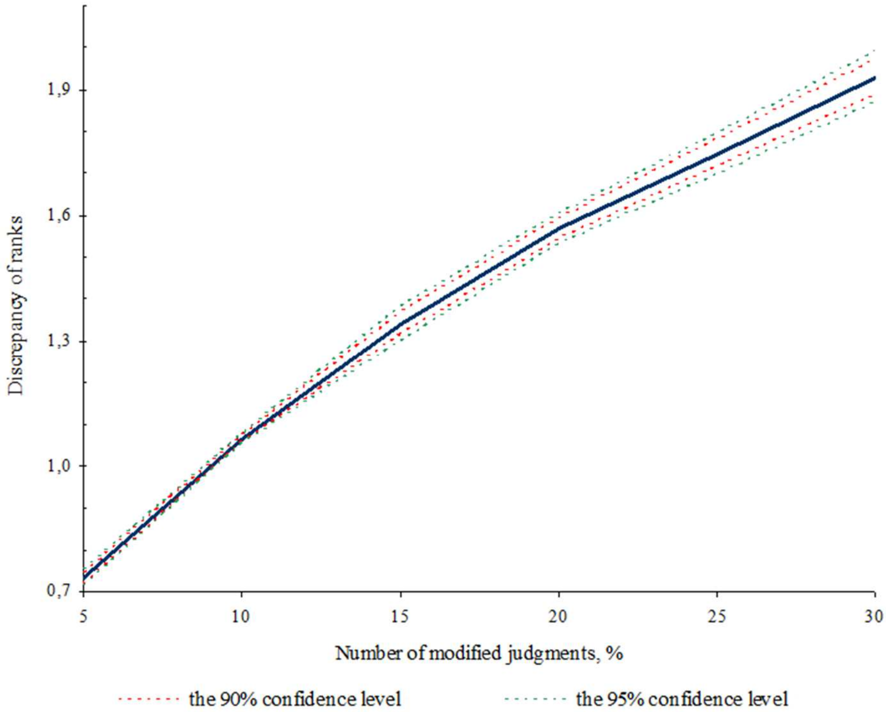

Fig. 3. The discrepancy of ranks on PCM formed at Germany Bundesliga with confidence intervals

6 Conclusions and Future Work

The teams ranking by generally accepted rules of football championships is also in some measure unstable. The place of the team may depend on one or two points (goals) and even the difference between goals scored and goals that were conceded. If at least productive judicial errors are corrected, the ranking results may differ by 1-3, and sometimes even more, positions.

The AHP-based ranking is also in some measure unstable. The expert makes a decision on the basis of information available to him, which is fundamentally incomplete, changes over time, and is also partially fuzzy, contradictory and erroneous.

Since the proposed ICECJ method involves improving the transitivity in expert assessments, the PCM consistency assessment is improved. Consequently, the stability estimates obtained are “optimistic”.

Many users of information systems do not adequately interpret the results of tasks. Sometimes the results are perceived as the ultimate truth, the characteristic statement is “that was counted by computer”. Moreover, this applies to integer results, in our case these are ranks, places of alternatives. The concepts of error, stability among users are often absent.

Therefore, information systems, solving such problems, should inform the user about the errors of decisions and their stability. The results of the study show that they are quite significant. Changing several estimates of pairs of alternatives can often lead to a shift of the alternative in the resulting ranking by 1-5 positions. This also applies to the leading alternative.

In this paper, we studied the stability of ranking alternatives by AHP with one criterion. Further development of the work is connected with multicriteria tasks. At the same time, tasks with a significant number of criteria and alternatives (over 10), a small number of criteria and alternatives, a significant number of criteria, and a small number of alternatives should be considered separately.

It can be assumed that with an increase in the total number of pair-wise comparisons, in view of their certain (albeit small and constant) inconsistencies, ranking errors increase statistically.

In the future, the ICECJ method to predict the results of sports competitions (at the final stage) can be modified. The solution of similar problems is a separate area of research (for example, [14, 15]).

It is also assumed to be promising for methods application in the case of AHP ranking with part of missing expert estimates and a predicted error estimate (solution stability).

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