Digital Transformation of School and the Role of Mathematics and Informatics within It Problems and Paradoxes of Mathematics Education and their Digital Solution[1]

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Abstract. Mathematics is the backbone of digital technology, critical to our entire civilization. The demand for mathematical competence in professional activity as well as in everyday life is raising. Paradoxically, interest for studying math decreasing in general education. The paradox is caused by the huge digital gap dividing pre-digital, even anti-digital school from digitally accelerating society. The paper revises the goals of mathematical and computer science education in the 21st century, and the role of this education in the general school. Using digital instruments for the main activities for students' work in learning and applying mathematics, as well as for communication with teachers and collaborative work on digital platforms for learning, should be the critical factor of improvement and sustainable development. At the same time mathematics as a field of discovery of novel and un-expected can play an important role in the whole agenda of formation of an individual in the VUCA world. The paper considers the case of general education as well as teachers' preparation in Russia in the perspective of the framework described.

Keywords: digital transformation of education, mathematics education, mathematical literacy, digital (computational) competence.

1 Introduction

There is a "paradox" of mathematics education:

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- Mathematics is becoming an increasingly important element of modern civilization: all digital technologies are built on mathematical methods and results.
- The attitude of schoolchildren to basic mathematics in many countries is deteriorating: children lose interest in it and perceive it as meaningless. The level of mathematics education of different categories of high-school graduates is falling.

To deal with this paradox, it is useful to understand how mathematics is actually used in the work and daily life of a person in the 21st century.

The number of professionals working in the field of fundamental mathematics is growing, but remains insignificant. This does not mean that children, who could become future mathematicians, need not be found, motivated and supported, for example, by organizing specialized schools for them. But this is not about them, but about mainstream schools.

In Russia, as in many other countries, there is a shortage of IT professionals in the broadest sense, ranging from chip developers to applied mathematicians, creators of new algorithms and models of reality and behavior. This entails working with mathematics at school on a larger scale than previously. Some successful attempts to introduce "coding" in the kindergarten exemplify such work [1]. The variability of the Technology school curriculum, which will be discussed below, is especially important.

It is widely (and probably reasonably) accepted that the mainstream school should take into account the presence of these two categories of professionals (pure mathematicians and the users of mathematics), give all children a certain minimum level and interest in mathematics, involve most children in its study, etc. Then, there will be more chances to find future researchers. And this is one of the reasons why it is important to stop the decline in interest in mathematics at school. The country that manages to do this will receive a competitive advantage.

There is a significant number of professionals (although it is still a minority) who use sophisticated software in their work. These are, for example, designers, engineers, doctors, lawyers, and financial analysts. For many of them, it is not so mandatory to understand "how it works inside". It is also not mandatory to understand this for an ordinary "user", pulling a mobile phone out of his pocket to make a call or recalculate the tax amount.

We said "not mandatory", but still believe that it is "desirable". For a car mechanic, it is essential to understand how an internal combustion engine works, for a taxi driver, this is advisable, for a car enthusiast, it is desirable, for a taxi passenger not at all. But sometimes it is important for everyone to understand "what is going on" when "the spark is failing". It is also desirable to understand how graphs are arranged in the Esquire or Bloomberg Businessweek magazine, or how video files are archived. This issue becomes vital in the 21st century, and every year (month, day) it is becoming more and more urgent, as the development of digital technologies is accelerating, and people are constantly facing new situations. One has to build models from ready-made mathematical bricks, for example, choosing the shortest route or calculating their expenses, or planning a renovation, etc. At the same time, one usually holds a calculator in their hand. But it is even more important that one is increasingly surrounded by artificial intelligence, which will increasingly be entrusted with modeling reality and decision-making. It becomes critical to understand how artificial intelligence works, and understand what decisions it makes and how these decisions are justified. And at the heart of this understanding (as well as at the heart of AI construction) is mathematics.

Thus, we sometimes need, albeit very rough, models of reality, and mathematics is a part of these models.

But in addition to the external reasons for studying mathematics, there is also an internal motivation, which is more significant for an average student. The math problem may be interesting in itself, and not because its content will be useful "in real life", but more especially if it comes in handy in ten years' time. This means that the task should look new and unexpected, and have the right individual level of difficulty for the student (be in the zone of his proximal development).

Another goal of school mathematics, significant for everyone, is usually spoken of. Ivan Yakovlevich Depman apocryphally ascribed the following motto to Lomonosov: "And mathematics should be taught at least because it puts the mind in order" [2]. This motto corresponds to our dream that our high-school graduates, having learned the mathematical way to reason, give definitions, find a mistake in proof, give a disproving example, etc., would be able to do it independently, and not only in the field of mathematics and its application in real life, but also in a wider context, e. g. a legal one. It is clear that in order to achieve the goal of transferring the methods of reasoning from mathematics to new contexts, it is necessary to provide such new contexts for students, at least in mathematics. Another reason for the importance of the novelty factor in mathematics is the usefulness of the human quality of pre-adaptability — the readiness to face something UNexpected, UNforeseen, and cope effectively with it.

2 The situation in Russian schools

So, here are the goals that are desirable to achieve, and which, hopefully, will be effectively achieved and contribute to an increased interest in mathematics, and the motivation to study it. It is necessary to form the ability of students:

- to reason logically, even outside of mathematics.
- to model reality using ready-made mathematical models and creating new ones.

The undoubted advantage of Russian, in particular Soviet school mathematics, is that it is "problem-based" [3]. This means not learning facts, but applying them to solving problems. Today it is commonly called the "competency-based approach", minus the significant fact that these problems are taken not from real life, but from a problem book. In the school course of algebra, there are few "theorems" and many problems, whereas in the geometry course there are many theorems, but there are also many problems. However:

- The problems of school mathematics are monotonous. Of course, when solving trigonometric equations, there may be some "subtle, unexpected moves", but the average student does not get to the subtleties. It is significant that the introduction of the problems on integer numbers, with some restrictions into the word problems of the Russian Unified State Examination (EGE), such as “how many boxes will be enough”, was perceived almost as revolutionary. Of course, these problems quickly became “standard”, but still expanded the “scope of the standard”.

- From an “applied” point of view, all school equations can be solved by computer algebra systems and it is precisely these systems that a professional uses if necessary. Word problems, as already mentioned, are monotonous and oversimplified. The applied value of school geometry can be boiled down to some facts that occupy only a small portion of the curriculum.
- There is almost no logical reasoning in school algebra. In fact, most students simply learn to follow a given pattern. This blind following causes the logic to disappear. In school geometry, the proofs of theorems are also learned by heart, and not found independently, there is not so much reasoning in geometric problems, and the degree of novelty in these problems is insignificant.
- School mathematics, the EGE, textbooks, and teachers alike, ignore digital tools for mathematics. It is noteworthy that, for example, school “mathematical statistics” and data analysis in mainstream Russian textbooks, do not involve any digital means. Although this section already appeared in Russian textbooks in the digital age, at the beginning of the “era of big data”, schools still remain in the pre-digital world.

3 What can we do?

The EGE is often blamed for the loss of the quality of mathematics education and the loss of interest in it. Indeed, it is hard to both achieve our goals and prepare for the mandatory exam in traditional mathematics, whether it be the EGE or the final exam of a Soviet school. Perhaps we need to reconsider the content of both the school course and the exam.

In short, the most important thing that needs to be done in school mathematics is to give a lot of essentially new and unique problems, taking into account topics that are more directly focused on the modern world, in particular, digital technologies. This is the world of logic, language, combinatorial objects — finite symbolic sequences (chains) and finite multisets (bags).

Such problems can be found in the so-called “recreational mathematics”. Note that the word “recreational”, interestingly, is not only an advertising gimmick in this case. There is a real difference between these problems and the flow of monotonous school problems.

In connection with the discussed issues, it is worth dwelling on the Russian course of computer science, first of all, on its mathematical component. This course was designed by mathematicians (including the author of these lines) who in one way or another came into contact with digital technologies, primarily with programming and teaching it. For us at that time, the introduction of computer science to schools was, among other things, a way to update school mathematics “from the outside”. To some extent, this attempt was successful:

1. The problems on algorithm design and the possibility of their visual execution on the computer screen significantly expand the field of mathematical objects, the essential novelty is achieved much easier than in school algebra.
2. Some “recreational” problems arise naturally, essentially consisting in algorithm design. Problems on the development of “algorithmic thinking” often take on a visual form, e. g. for a robot in a maze.
3. Many techniques in algorithm design, on the one hand, relate precisely to those models that arise in mathematics and can be transferred to other areas (for example, the “divide and conquer” strategy), and on the other hand are also used in serious, adult programming, and in real life.

Of course, the computer science course offers an abstract mathematical description of the functioning of a computer, along with a mathematical model of program execution.

Does this mean that “continuous” mathematics should be completely abandoned? Of course not. The abstraction of a real number is the most important achievement of mathematics, as well as elementary functions — the sine, the exponential function. But the center of gravity should be shifted from “simplifying expressions for taking logarithms”, to independent discovery of the properties of functions, basic formulas and identities, general formulas for solving an equation, as well as solving the simplest equations in an amount sufficient to understand the general principle, but without the obligatory achievement of a high level of faultlessness, and the technical sophistication in symbolic transformations. After that, the student can delegate the solution of the equations to a computer algebra system and, with the help of this system, solve all school and non-school equations.

We can approach the goal of teaching and learning modeling in the Russian course of physics, primarily in middle school. Ideally, there are real experiments: the ball is rolling along the gutter, the rheostat engine is moving, the pendulum is swinging. Note that this ideal can be achieved thanks to digital technologies: digital sensors of position, temperature, pressure, current, etc., and computer algebra systems. On the one hand, this reasonably reduces the time spent on conducting an experiment and a visual presentation of its result. On the other hand, students actually use some real objects — digital sensors equipped with a large number of physical effects and technological principles. Computer algebra systems also allow students and teachers to overcome algebraic difficulties and errors more easily and focus on the physical essence of phenomena and their mathematical models. Another role of a physical experiment, both in the physics classroom and remotely via telecommunications, is to be a source of data for analysis using statistical and machine learning methods. Thus, we are moving towards an important goal of mathematics education — learning the elements of artificial intelligence. The topic of school physics as the right place for mathematical modeling deserves separate consideration.

What about geometry? Its importance at school fell sharply during the couple of years when it was not included in the EGE. Now it is being restored. As evident from the above, we believe that the role of geometry is primarily the solving of various problems that are appropriately difficult for every student. The number of geometry lessons can even be slightly increased. At the same time, the course should be designed for students of different levels to have a sufficient number of problems to solve. It is important that logic is supported by visualization. This property of school geometry is greatly enhanced in dynamic geometry systems, where one can accurately and beautifully create a drawing, then transform it maintaining the configuration (incidence of elements).

What should the "Mathematics, Computer Science, Physics, Technology" school education sector look like? The following vector of development of school mathematics education emerges:

1. Expansion of the range of problems and a significant increase in their novelty, which is even more significant than the inclusion of a particular area of mathematics into the curriculum. Inclusion of the principle of novelty in the EGE framework.
2. The use of a computer as a tool for mathematical activity, in particular, for experiments, visualization, data analysis (statistics), and algebraic calculations.
3. Physics as a natural field for mathematical modeling and data analysis using digital technologies.
4. Algebra — achievement of all the results required by the Federal State Educational Standard and curriculum guidelines, and many other results achieved by students using computer algebra systems.
5. Geometry — formation of a system of goals and a system of tasks (as well as research tasks, projects) of various difficulties. These systems, placed on the digital learning platform, allow to build individual educational routes designed for the mandatory achievement of all set goals. Such a personalized approach to teaching, if compliant with the Federal State Educational Standard, is also applicable to other areas and subjects, but it is especially important for geometry. In addition, dynamic geometry should be used for experiments.
6. Computer science — the use of algorithms as a source of a wide range of new tasks and puzzles, and a computer as a tool for experiments, debugging, finding errors in one's own work. A common system of basic objects should be used for both elementary mathematics and mathematical computer science.

4 What is already being done?

How does the content of modern Russian mainstream school mathematics relate to all this? This content is mostly determined by the EGE, mainstream textbooks, an average teacher, and the way of training new teachers in pedagogical (and other) universities. Unfortunately, the above-mentioned development trends are implemented to a very small extent in all these elements, even if we consider (as formulated in the Federal State Educational Standard) mathematics and computer science combined. Moreover, the above-mentioned decline in interest is accompanied by a "decline in morals". Students don't see any point in solving problems independently and use answer books from the Internet, at best complementing them with an effective Russian computer algebra system UMS [4], and more recently, the Croatian product Photomath [5].

Nevertheless, some changes are taking place. For example, the international Kangaroo contest has been very popular in Russian primary schools for decades, in many ways it is the creation of our compatriot, Professor Mark Bashmakov from St. Petersburg [6]. The problems of this competition represent a wide range of mathematical questions of varying difficulty. “Kangaroo” sets an example for motivating students. The problems of this competition are solved every year by millions of children in Russian primary schools. At the same time, the competition is not supported by the state, and even meets certain resistance from the educational authorities.

The systematic development of the modern content of mathematics + computer science has been going on for three decades. The textbooks are published by Prosveshchenie publishing house [7] and are used in hundreds of Russian schools. They attempt a balanced introduction of modern, combinatorial, logical, and algorithmic content, along with the traditional numerical one.

Since the 1960s there is a system of mathematical schools in Russia that has gained worldwide fame. One of the powerful branches of this system implements the methodology of N. N. Konstantinov. Without trying to describe it as a whole, let us pay attention to just one of its features. Within this framework, students “create” mathematics (not so important what kind of mathematics) themselves. The problems they receive have a high degree of novelty — these are individual lemmas, steps in proving important, understandable, motivating theorems, and not monotonous solutions of equations that do not make sense to the student.

As already mentioned, we largely associate the progress in mathematics education, in particular, the resolution of our initial paradox, with the practice of using digital technologies in school. One of the approaches to this practice is the Framework of the Subject Area Technology at School [8]. This framework implies significant variability in the content of school technology education and the application of technology in various school subjects. Based on these provisions, a team of authors is preparing for publication, a textbook on computer science and ICT. In its digital form there are modules on the use of digital technologies in the process of learning various school subjects. In particular, in the module on mathematics, there will be an introduction to the dynamic geometry system GeoGebra, which is becoming the most popular digital mathematical tool in school [9], and there will also be a tutorial on the Mathematical Constructor [10, 11]. In the physics module, digital sensors and computer algebra will be taught.

Of course, the EGE will apparently remain the main regulator and, to a certain extent, a drag on the modernization of school mathematics education. However, there are also modest victories here. In 2021, for the first time, the computer science exam will be run on computers, and real programming will be found in the corresponding tasks. Hopefully, it will be allowed to use an ordinary fraction as an answer in the EGE in mathematics, and the computer will understand this.

The number of participants in mathematical modeling Olympiads is growing [12, 13].

Of course, training new mathematics teachers is key. For several years, the principles of the above approach have been implemented in the training of primary school teachers and mathematics teachers at Moscow State Pedagogical University [14]:

- the problem component was strengthened, primarily due to the “school” problems,

- computers were used for mathematical activity.

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