Density-Based Risk Assessments within Soft Safety Domains

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Abstract. This work is aimed at the study of ways to assess risk in complex intelligent systems, which do not involve a human and focused mainly on issues related to the joint activity of unmanned systems in a dynamic environment, where risk appears due to their interaction. A simple and fast qualitative risk assessment method that can provide assessments reliably, efficiently and timely is proposed. The proposed method combines a matrix-based approach and a geometric approach to process numerical data at the input. A soft set is proposed as a suitable tool to avoid uncertainties related to incomplete and inaccurate information. The model of a dynamic system under risk based on n-dimensional stateful interaction space and corresponding topological structures is defined. It is proposed to estimate numerically only inexpensive parameters and then provide sophisticated quantification of them to avoid extensive calculations. The risk levels are assessed with respect to the certain cells of the discretized n-dimensional interaction space. The levels of risk assessed in different cells of interaction space allow assessing risk distributed over the interaction space to prioritize risks properly. Due to uncertainty and lack of sufficient data, a density-based metric, which represents a relative density of interacting objects, is proposed to use instead of traditional probabilities or frequencies to estimate the chance of the object being exposed to the undesired event. The algorithm of the proposed risk assessment method is presented. The proposed method provides the acceptable performance of risk assessment enough to the real-time.

Keywords: Qualitative Risk Assessment, Interaction space, Density-Based metric, Soft Set, Topological space, Risk Level, Unmanned Systems

1 Introduction

The world around us is a source of any risk. Most of the people's usual activities are risky. Often, people are forced to cope with risk in order to make a proper and comprehensive decision in various ordinary activities such as driving a car, buying expensive goods, lending from a bank, etc. In any case, to be responsible for the decisions, people must be aware of their consequences, that is why many people use risk assessment techniques explicitly or implicitly [1]. All of the above even more applies to institutions such as banks, insurance, logistic and trade companies, etc. Responsible persons of such institutions use various management methods based on a risk assessment. Thus, most people aim to investigate, correctly assess, eliminate and minimize risk both in their life and business because this allows them to maintain the property, vital and business activity at the appropriate level, to save their health and even life in many dangerous cases, etc. Although the risk assessment and analysis are mainly subjective and uncertain, they are considered to be very important and difficult tasks [2]. This paper substantially addresses the real-world risk assessment problem. However, the paper is not about people making decisions under risk.

Even though a person remains an important element of many complex systems since she manages them, makes important decisions, and, therefore, is responsible for their results, the modern level of technology development brings us new challenges. Unmanned systems are slowly but firmly and ruthlessly displacing people and taking full responsibility. Today, we are surrounded by lots of complex intelligent systems forced to make decisions on their own.

In human-based (ergatic) systems, the decision-maker mitigates the effect of subjectivity and uncertainty taking responsibility for the final result, but nobody can take control and responsibility in an autonomous unmanned system, so such a system must assess risk by itself. Thanks to intensive improvements, unmanned systems have become more and more intelligent and cheaper. This ensures the start of the massive use of unmanned systems that work together. Performing certain functions simultaneously, they interact with each other in a complex way entailing risk, which must be assessed reliably and timely.

It is not surprising, but even the most perfect system based on artificial intelligence cannot be compared with a human in terms of the considered issues. Therefore, serious challenges related to the limited resources of such systems, their limitations in the computational performance, great sensitivity to incompleteness and inaccuracy of information [3] is an addition to other challenges related to the risk assessment by a human in human-based systems. An even greater challenge is the non-linearity of the methods of artificial intelligence since they are implemented by exhaustive algorithms, which can not guarantee a final computation time during the risk assessment [4]. Obviously, methods of risk assessment, which can be efficiently used in complex intelligent systems, significantly differ from methods used in human-based systems, but such methods are not good enough studied for today.

Thus, the study of ways to assess risk in complex intelligent systems, which do not involve a human, is a topic of our interest. In this paper, we focused mainly on issues related to the joint activity of unmanned systems in a dynamic environment, where risk appears due to their interaction. To assess risk in such conditions, we need a simple and fast method that can provide assessments reliably, efficiently, and timely.

2 Related Works

Understanding of risk is domain-dependent, so its definitions differ essentially [5]. In general, the risk is about unpleasant consequences caused by exposure to hazard and a chance of being exposed to the hazard. In most cases, risk can be defined as a product of the likelihood and the magnitude of harms or losses caused by hazard [6]. The consideration of harms and losses depends on the purpose and often can not be reduced to a money equivalent (e.g., the consequences are about the life and health of people).

We consider dynamic systems that include a multitude of dynamic objects interacting in an uncertain and unpredictable environment. Since risk arises directly from the interaction of dynamic objects, the following assumptions can be introduced:

- there is no risk if there is no dynamic and, consequently, there is no interaction;
- there is a risk as soon as objects interact;
- since the system is dynamic, the risk is also dynamic (i.e., it is not constant);
- the risk could be applied to all participants in the interaction but varying degrees.

Thus, emerging risk should not be ignored and must be assessed on time to provide a corresponding influence on decision-making at each interacting object. Since the risk is almost always present, it is necessary to assess its acceptability in a certain context.

Risk assessment is an important tool to minimize risks, and there are qualitative and quantitative approaches to assess risk [7].

A quantitative approach has been proposed to estimate risk as a numerical value based on the exposure, frequency (or probability), and consequence (i.e., the severity of potential loss) [8]. The simplest way to determine the value of risk is to multiplicate severity and probability measures. The reliability of risk assessment depends on the availability of data, so this approach can be successfully used wherever it is possible to rely on sufficient data; insurance companies and banks are a prime example of its scope. However, if reliable data are unavailable or their retrieval is too expensive, the quantitative approach is weakly appropriate.

A qualitative approach is most relied on the subjective judgment of the competent person to determine an overall risk and can be mainly applied whenever the quantitative risk assessment cannot be used for ethical reasons because the consequences are harmful to people or death [9]. According to this approach, decision-makers often evaluate risk in comparison to another risk. In this case, an expert or decision-maker must compare risk and choose a decision (or, at least, define functions that allow such comparison). For example, subjective estimations are often modeled by fuzzy values [10] and their membership functions need to be determined by experts. Thus, the main disadvantage of the qualitative approach is its subjectivity: a human is always needed to assess the risk. Such subjectivity leads to unclear results of risk assessment.

Let us make a small comparison. The qualitative approach is subjective but relatively simple. It less depends on exactly measured data than quantitative one because experts' judgments supplement and clarify unavailable data.

The quantitative approach is very complex but more credible and more objective because they rely on meaningful statistics as well as on mathematical methods.

It is known that unmanned systems usually receive information about themselves and the world around them using a variety of sensors. Some sensors provide measurements, while others provide images, which require processing and analyzing. In any case, all types of sensors can not provide complete information about the world because they have limited accuracy [11]. As a result, the vast majority of information to cope with risks is incomplete, imprecise, inconsistent, and sometimes vague or fuzzy. Overall, the most common features of the considered dynamic system, which include a multitude of interacting dynamic objects, are the following:

- dynamics of the interaction.
- lack of information;
- lack of communication;
- lack of “last chance” decision-maker, i.e. person who could correct errors;
- the unpredictability of the environment and behavior of dynamic objects.

Taking into consideration above-mentioned features, we conclude that both pure qualitative and pure quantitative risk assessment approach is inappropriate for such kind of systems. Hence, we need to combine these approaches to develop a suitable risk assessment method.

There are several well-known and widespread risk assessment methods, which can be used for this. Statistical methods [12] are based on descriptive statistics; they are quite simple, fast, and easy to apply. However, they cannot deal with incomplete, imprecise, and inconsistent information. If some numerical data are unavailable, they rely entirely on the expert’s opinions.

Another well-known method is the matrix method [13], which has such advantages as simplicity, adjustability, and validity for the prioritization of hazards. The main drawbacks are required contingency and subjectivity in prioritizing risks.

There is a wide range of geometric methods based on the multidimensional representation of the state space of a certain system and the ranging of hazards using well-defined space and time limits (minima) [14, 15], which are relatively simple and based on quantitative calculations. However, most of them rely on a weak assumption of static limits and constant state changes.

Decision trees are used widely to assess risk [16] and have such advantages as well-applicability to combine the likelihood and impact of risks as well as flexibility in determining mitigation strategies. Their main disadvantages are a need for sufficient historical data and an inability for risk criticality assessment.

The Monte Carlo Simulation is another widespread risk assessment method [17] applicable to contingency modeling and generation of probability distributions but it requires a huge amount of data and it cannot establish an accurate estimate of the risks.

Artificial neural networks are widely used to quantify risks [18] and have some advantages since their inputs and outputs can be defined subjectively and do not need statistical distributions. Their disadvantages are mainly related to an inability to apply risk mitigation strategies.

Sensitivity analysis [19] is another method to assess risks that enables the definition of risk dimensions and ranking of hazards in their order of significance. However, it relies on a weak assumption that hazards arise one-at-a-time.

Fault tree analysis [20] provides experts a possibility to use linguistic terms rather than numerals to assess the probability of occurrence of hazards. Despite some advantages, this method depends only on experts' opinions to analyze the hazards.

Fuzzy logic [21] is a method capable to deal with the vagueness and imprecision relevant to the qualitative risk assessment process but it also relies on the subjective opinions of experts in the definition of fuzzy membership functions.

Considering the advantages and disadvantages of the above-mentioned methods with respect to the objective of the research at hand, all methods that rely on statistics or the subjective opinion of experts are unacceptable since unmanned systems have neither statistics nor experts. Since the risk assessment in the context of our consideration is not concerned with obtaining precise outcomes, a qualitative assessment is quite acceptable at the output, but the main requirement is to prioritize risks in the order of their mitigation. At the same time, most of the available information is numerical but has limited accuracy and maybe blurred.

Thus, the use of the matrix method could be a good idea. It is also reasonable to combine the matrix method with the geometric one for processing numerical information at the input. Furthermore, a proper tool must supplement such a combination to process incomplete and inaccurate information. Since complicated problems cannot be solved using classical methods, there are several well-known tools to describe various kinds of uncertainty, such as fuzzy sets [10], rough sets [22], vague sets [23], and others, but all of them have difficulties [24] in a deal with uncertainties due to their essential nonlinearity and computational complexity, which prevents their efficient use in the conditions of strong time constraints. Soft sets that have been introduced in [25] to overcome such difficulties seem to be quite suitable.

3 Model of a Dynamic System under Risk

Consider an abstract dynamic system Ω within a certain n-dimensional stateful space Ξ. Suppose the system Ω includes a set of m dynamic objects $\{\omega_1,...\omega_m\}$, each of which has an explicitly described state $X_m \in \Xi$ that can change in time. Suppose such objects $\omega_i \in \Omega$ can interact in a certain way within a given state space Ξ.

Suppose the n-dimensional space Ξ is Euclidean, linear, and uniform.

Let Y be a set of certain elements $\{y_1,...y_z\}$. Let T be an infinite set of time points $\left[t_0,...t_y\right)$ strictly ordered by $<_T$, t_0 be an initial count, and Δt is a time slice. Thus, $\left(T,t_0,\Delta t,<_T\right)$ is a timescale defined over T.

Suppose ξ_T is a metric defined on T such that $\left\|t_i - t_j\right\|_T \to \tau$ endowed with the following properties:

1. $\xi_T\left(t_i,t_j\right) = 0 \Leftrightarrow t_i = t_j$;

2. $\xi_T\left(t_i,t_j\right)=\xi_T\left(t_j,t_i\right)$;
3. $\xi_T\left(t_i,t_k\right)\leq \xi_T\left(t_i,t_j\right)+\xi_T\left(t_j,t_k\right)$,

$\forall t_i,t_j,t_k \in T$.

Suppose a norm $\|y\|_{\Xi}=\min\limits_{t\in[0,T)}\left(y(t)\right)$ within space Ξ, where $y\in Y$, $t\in T$. Let us define a corresponding metric $\xi_{\Xi}\left(y_1,y_2\right)=\|y_1-y_2\|$ endowed with the properties:

1. $\xi_{\Xi}\left(y_1,y_2\right)=\|y_1-y_2\|=0 \Leftrightarrow y_1=y_2$;
2. $\xi_{\Xi}\left(y_1,y_2\right)=\|y_1-y_2\|=\|y_2-y_1\|=\xi_{\Xi}\left(y_2,y_1\right)$;
3. $\xi_{\Xi}\left(y_1,y_2\right)=\xi_{\Xi}\left(y_1+a,y_2+a\right)$;
4. $\xi_{\Xi}\left(\lambda y_1,\lambda y_2\right)=\lambda\cdot\xi_{\Xi}\left(y_1,y_2\right)$ where $y_1,y_2,a\in Y$

Suppose a certain basis $e_1,...,e_n$ holds uniformity of the metric ξ_{Ξ} within n-dimensional space Ξ. Thus, a certain state X_i within space Ξ of the dynamic object $\omega_i\in\Omega$ can be described as $X_i=\left(x_{1i},...,x_{ni}\right)$ whereas $x_{1i},...,x_{ni}$ are the state parameters that correspond to the given basis $e_1,...,e_n$.

Let us discretize space Ξ by a metric grid D using certain lines spaced with size δ. Suppose a linear mapping $f:\Xi\rightarrow D$ transforms the given space Ξ into a grid of n-dimensional cells of size δ in each n dimension.

As the result, we obtain the grid $D=\left\{d_{x,...z}\right\}$ of isometric n-dimensional cells $d_{x,...z}$, where $x,...z$ correspond to the cell state parameters $x_1,...,x_n$. In this way, each n-dimensional cell $d_{x,...z}\in D$ is the smallest (discrete) subspace of the space Ξ. Therefore, the discrete state of each dynamic object $\omega_i\in\Omega$ is referenced to the corresponding cell within space Ξ. The size δ cell can usually be determined by the technical capabilities of sensors and the computing capabilities of onboard computers.

Suppose the proposed discretized model of space is consistent with the information captured by sensors. Let us build the corresponding topology.

Let D be a non-empty n-dimensional set of cells, $\mathrm{R}^{\geq 0}$ be a set of non-negative real numbers, and ξ_D be a function $D\times D\rightarrow \mathrm{R}^{\geq 0}$. Obviously, ξ_D can be a suitable distance function (metric), if its values satisfy the conditions:

1. $\xi_D\left(d_1,d_1\right)=0$ if and only if $d_1=d_2$;
2. $\xi_D\left(d_1,d_2\right)=\xi_D\left(d_2,d_1\right)$;
3. $\xi_D\left(d_1,d_2\right)+\xi_D\left(d_2,d_3\right)\geq\xi_D\left(d_1,d_3\right)$

for each $d_1,d_2,d_3\in D$. In this case, the function $\xi_D\left(d_1,d_2\right)=\|d_1-d_2\|$ provides a certain distance from a cell d_1 to a cell d_2 within the grid D, and a couple $\left(D,\xi_D\right)$ is a metric space.

Let $\Re_D \subseteq D \times D$ be a reflexive, symmetric, and transitive relation defined on the n-dimensional set of cells D. It represents an indiscernibility relation $\Re_D(d_1, d_2)$ between two cells d_1 and d_2 such that $d_1, d_2 \in D$ in terms of certain value $y \in Y$, if $(\forall d_1, d_2 \in D)(\forall y \in Y)\left[y(d_1) = y(d_2)\right]$. In this case, the cells d_1 and d_2 are y-indiscernible. Since the indiscernibility relation $\Re_D(d_1, d_2)$ is definitely an equivalence relation, it helps us to determine equivalence classes of D with respect to $\Re_D$.

Suppose $D / \Re_D$ is a factor set that consists of all equivalence classes of D with respect to $\Re_D$. Let D be a universal set and $apr_D = (D, \Re_D)$ be an approximation space defined by a composite set that is a finite union of elementary sets, each of which corresponds to an empty set or an element of the factor set. Thus, the equivalence class $\Re_D(d)$ containing a specified cell $d \in D$ clearly determines a family of all composite sets $Def(apr_D)$ that, in turn, uniquely determines a topological space $\mathcal{T} = (D, Def(apr_D))$ based on the approximation space $apr_D = (D, \Re_D)$.

In this case, $Def(apr_D)$ is a topology on D and $\mathcal{T} = (D, Def(apr_D))$ is a corresponding topological space, if all subsets of the set $Def(apr_D)$ satisfy the conditions:

1. $\varnothing \in Def(apr_D),\ D \in Def(apr_D)$;
2. $A, B \in Def(apr_D) \Rightarrow A \cap B \in Def(apr_D)$;
3. $A, B \in Def(apr_D) \Rightarrow A \cup B \in Def(apr_D)$.

Certainly, each cell $d \in D$ is an element of the topological space $\mathcal{T}$.

Suppose each dynamic object $\omega_i \in \Omega$ performs a certain function changing its state $X_i(t) \in \Xi$ over time. The state of the system Ω is defined by a set of states of all dynamic objects $\{\omega_1, \ldots \omega_m\} \in \Omega$ that constitute this system, so that $X_\Omega(t) = \{X_i(t)\}_{i=1}^{n}$.

Since the state $X_i(t)$ of each dynamic object $\omega_i \in \Omega$ is uniquely determined by a correspondent cell $d_{ix,\ldots iz}$ of the grid D, as well as the overall state $X_\Omega(t)$ of the system Ω is determined by a set of cells $\{d_{ix,\ldots iz}\}_{i=1}^{n}$, the topological space $\mathcal{T}$ based on D can be a relevant tool for risk assessment within space Ξ.

4 Proposed Method of Risk Assessment

In the human-based system, a responsible person has to decide under risk rationally taking into account moral, ethics, and other reasons. In contrast, unmanned systems are based solely on feasibility expressed through the prism of given criteria. If there are many such criteria set simultaneously, then decision making will be multi-criteria.

Thus, the risk assessment is considered to be a very important task of a complex intelligent unmanned system. As we found out above, “no risk” situations are not mostly observed in the considered class of dynamical systems. Indeed, such a system is dynamic because its objects interact generating various threats and risks. Hence, for such a system, achieving the "near zero risk" is either impossible or too expensive. Thus, a much more important and achievable task for each of the dynamic objects is establishing an acceptable level of risk and use risk assessment methods to minimize risk during decision-making.

It should be noted that the dynamic object does not need to calculate the degree of risk quantitatively, but it needs to evaluate risk qualitatively in comparison to another risk to prioritize them. Therefore, even if there are ways to calculate risk parameters accurately, the use of complex time-consuming algorithms is not necessary.

In this paper, it is proposed to use a simple and fast qualitative method based on a risk matrix. The risk matrix should correspond to the available data and the exposure, frequency (probability, possibility), and consequences (severity of potential loss). Quantitative risk assessment requires numerical estimations of such parameters.

We propose to estimate numerically only inexpensive parameters and then provide sophisticated quantification of them to avoid extensive calculations. Such quantified values can then be used as inputs to assess a risk level at the output based on the risk matrix. In this way, we assess the risk level with respect to the certain cells $d_{ix,\dots iz}$ of the discretized interaction space D. Ultimately, knowing the levels of risk in different cells of space, we can assess risk distributed over this space to prioritize risks properly. To mitigate the effect of quantification, we use soft sets, which allow us to construct a distribution of risk levels over the space D.

4.1 The Algorithm of the Risk Assessment

The risk assessment is a continuous process that includes (Fig. 1):

- determination of the risk context;
- identifications of threats;
- risk estimation:

  1. estimation of exposure;
  2. estimation of possibility;
  3. estimation of consequences;

- risk evaluation;
- risk prioritization.

Determination of the risk context is aimed at analyzing possible interactions between dynamic objects and evaluating all three components necessary for the risk assessment.

At the next stage, some of these interactions, which can pose risk, are identified as threats. Since threats can cause injury, damage, or loss, the purpose is to identify as many threats as possible.

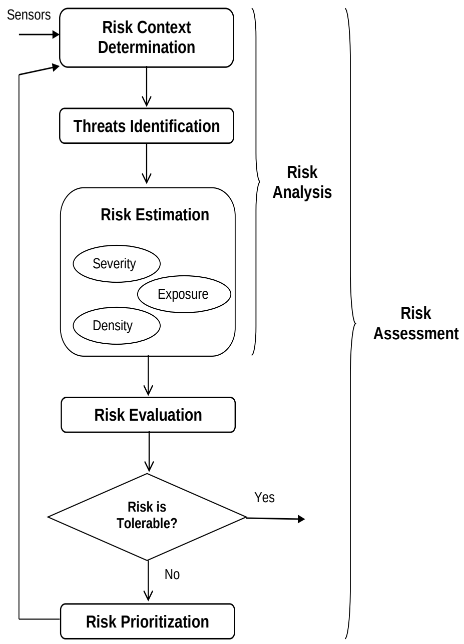

Fig. 1. The Algorithm of the Risk Assessment

At the next stage, the qualitative assessment is provided to determine the level of risk associated with each specific threat identified at the previous stage. Usually, this stage is based on the estimations of probability, exposure, and severity of loss as the consequences of an undesirable event. In our case, the exposure and severity can be estimated using the interaction analysis, but the probability can not be estimated due to a lack of sufficient data, so we propose to use density estimations instead of probability estimations.

Finally, risks must be evaluated and prioritized to reduce, mitigate, or eliminate them in the context of all components. Since such components of risk (e.g., density, exposure, and severity) usually cannot be identified unequivocally, we use quantified levels that can be estimated using ordered scales of limits. Thus, in our case, the risk assessment can be reduced to a matrix based on levels of all three components.

4.2 Determination of the Risk Context

Suppose dynamic objects $\omega_i, \omega_j \in \Omega$ interact within space D during their joint activity. The trajectory $Tr(\omega_i)$ of each object ω_i can be represented as a continuous sequence of its states $\left[X_i(t_j),...X_i(t_k)\right]$ on a time interval $\left[t_j,...t_k\right] \in T$, while the state $X_i(t)$ of the ω_i at the moment $t \in T$ corresponds to the certain cell $d_{ix,...iz} \in D$ determined by its parameters within D.

The objects, which states are within the interaction space D do not necessarily interact. Their trajectories at different time points can approach, intersect, or diverge from each other. Basically, their interaction can be defined based on the local proximity D^* of the dynamic objects within the interaction space D, which can be estimated based on the given metric ξ_D (Fig. 2).

Thus, D^* is a subspace of the interaction space D, $D^* \subseteq D$, usually represented by a closed n-dimensional figure having the maximum surface distance $\mathcal{R}$ from a certain base point d_0 such that $d \in D^* \Leftrightarrow \xi_D(d_0, d) \leq \mathcal{R}$.

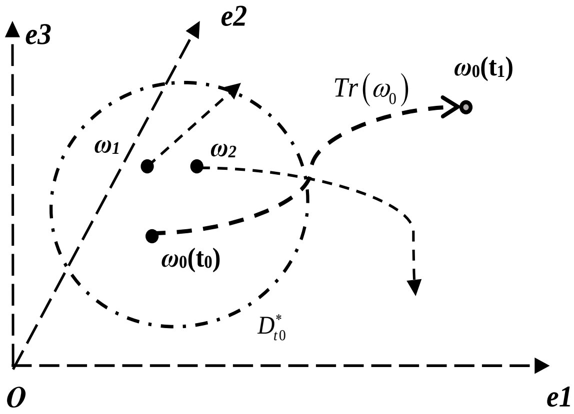

Fig. 2. The interaction space

Let $c(\Omega,\Omega)$ be a relation between two dynamic objects $\omega_i, \omega_j \in \Omega$ that restricts a subset of interacting objects Ω^c, i.e. $\{\omega_i, \omega_j\} \in \Omega^c \leftrightarrow c(\omega_i, \omega_j)$. Due to the dynamics of the interaction, the composition Ω^c also changes dynamically.

Let $r_i(t) = \{r_{i0}(t), \ldots r_{il}(t)\}$ be a set of n-dimensional margins evaluated for a certain object ω_i at the time t based on the above-mentioned metric ξ_D such that $r_{ij}(t) = [r_{ij1}(t), \ldots r_{ijn}(t)]$. Since the trajectory $Tr(\omega_i)$ of each object ω_i is characterized by the different dynamic (e.g., speed and acceleration) and functional parameters (e.g., prudence or persistence), each of the interacting objects will simultaneously have its own set of margins that can differ from the set of margins for other objects.

Suppose a function $X(\omega_i, t)$ returns the state $X_i(t)$ of the object ω_i at the time t as well as the function $X(\omega_k, t)$ returns the state $X_k(t)$ of the object ω_k at this time. Thus, a certain distance $\Delta(\omega_i, \omega_k)$ between their trajectories can be evaluated by $\|X(\omega_i, t) - X(\omega_k, t)\|$ for each couple (ω_i, ω_k) of objects with respect to the object ω_i. Clearly, the distance $\Delta(\omega_k, \omega_i)$ with respect to the object ω_k can differ from $\Delta(\omega_i, \omega_k)$.

Assume that unpleasant consequences are concerned with an excessive rapprochement or intersection of the trajectories of interacting objects ω_i and ω_k.

Thus, we define interaction function $\chi_i : D \to r_{ij}$, which describes a subset of objects $\Omega^{\chi_i} \subseteq \Omega$ interacting with ω_i, as a surjective anisometric mapping based on the evaluated sets of n-dimensional margins $r_i(t)$ and $r_k(t)$ (Fig. 3).

Suppose $\tau_{ik}(t) = \{\tau_0(t), ... \tau_q(t)\}$ is a set of time limits given with respect to the interaction of ω_i and ω_k based on the metric ξ_T over T such that $\|t_i - t_k\|_T \to \tau$.

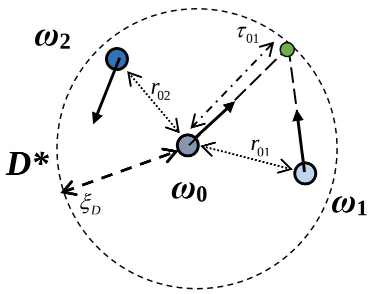

Fig. 3. Interaction Metrics

Let $\succ_r$ be a partial order $r_{i0}(t) \succ_r r_{i1}(t) \succ_r ... \succ_r r_{iz}(t)$ given on the set $r_i(t)$ that implies the corresponding ordinal scale $R_i(t) = \{r_{i0}(t), ... r_{iz}(t)\}$. Suppose μ_i is a function that uniquely takes each distance $\Delta(\omega_i, \omega_k)$ onto a certain level of the scale $R_i(t)$, such that $\mu_i : \|X(\omega_i, t) - X(\omega_k, t)\| \to r_{ij}(t) \in R_i(t)$. Thus, the function μ_i can be used for quantification of the states' distances onto the scale $R_i(t)$.

The dynamic object ω_i *interacts* with ω_j (in other words, there is a relation $\chi_i(\omega_i, \omega_j)$), if and only if $\|X(\omega_i, t) - X(\omega_k, t)\| \geq r_{i1}(t)$. Consequently, $\forall \omega_i, \omega_k \; \chi_i > r_{i1} \Leftrightarrow \omega_i, \omega_k \in A^{\chi_i}$.

The dynamic object ω_i interacts with ω_j *dangerously*, if $\|X(\omega_i, t) - X(\omega_k, t)\| \geq r_{i2}(t)$, and interacts with ω_j *critically*, if $\|X(\omega_i, t) - X(\omega_k, t)\| \geq r_{i3}(t)$.

Thus, the *interaction set* Ω^{χ_i} around ω_i includes all objects $\omega_j \in \Omega, \forall j \neq i$, which have established the interaction with ω_i based on χ_i.

Clearly, the margin limits outline n-dimensional areas within subspace D^*, which represent exposure levels needed for the risk assessment. Definitely, all objects that interact with the object ω_i (i.e., their distances are equal or greater than $r_{i1}(t)$) pose risk to ω_i, so the object ω_i is exposed to risk.

Let us use margin limits as it is shown in Table 1.

Table 1. Margin limits and corresponding exposure levels

	Level	Exposure due to the interaction
0	r_{i0}	«no interaction»
1	r_{i1}	«safe interaction»
2	r_{i2}	«dangerous interaction»
3	r_{i3}	«critical interaction»

At the same time, severity levels can be evaluated based on the inertial properties of the objects within the interaction space D. Indeed, to avoid the intersection of trajectories objects can speed up, slow down, or deviate, while the environment can both increase the inertial force or weaken it. Thus, the time-to-intersect $\tau_{ik}(t)$ can be a relevant measure, which makes it possible to assess how great is the influence of the dynamics of the objects' ω_i and ω_k interaction on its possible consequences.

Let $\prec_t$ be a partial order $\tau_{ik0}(t) \prec_t \tau_{ik1}(t) \prec_t \ldots \prec_t \tau_{ikq}(t)$ given on the set $\tau_{ik}(t)$ that implies the corresponding ordinal scale $U_{ik}(t) = \{\tau_{ik0}(t), \ldots \tau_{ikq}(t)\}$. Suppose η_{ik} is a function that uniquely takes time-to-intersect $\tau_{ik}(t)$ onto a certain level of the scale $U_{ik}(t)$, such that $\eta_{ik} : \tau_{ik}(t) \to \tau_{ikj}(t) \in U_{ik}(t)$. Thus, the function η_{ik} can be used for quantification of the remaining time during the interaction onto the scale $U_{ik}(t)$.

Based on the time-to-intersect $\tau_{ik}(t)$ and the scale $U_{ik}(t)$, the interaction space D can be divided into the subspaces of easy activity D_{ik}^0, restricted activity D_{ik}^1, desired deviation D_{ik}^2, and obligatory deviation D_{ik}^3, which can be outlined around the state $X(\omega_i, t)$ of the object $\omega_i \in \Omega^c$ at the time t, i.e., around the cell that reflect the current state of the object ω_i within D.

Naturally, the less time it takes to prevent an undesirable event, the less is the possibility of preventing such event and the more serious are its consequences. Hence, we can define the severity levels based on the time-to-intersect limits.

Let us use time limits as it is shown in Table 2.

Table 2. Time limits and corresponding severity levels

	Level	Severity	Activity
3	τ_{ik3}	Critical	«obligatory deviant»
2	τ_{ik2}	Major	«recommended deviant»
1	τ_{ik1}	Minor	«restricted»
0	τ_{ik0}	Minimal	«mutually free»

The next and last component of risk assessment is a chance to the object of being exposed to the unpleasant event and its consequences. Usually, this chance can be accurately estimated using the probability of an event or the frequency of occurrence of an unpleasant event. However, as shown above, it is not always possible to obtain such estimates. Moreover, in the considered class of dynamic systems, the statistics of the occurrence of undesirable events are unavailable due to the non-representativeness of the sample. Therefore, we propose to use density-based metrics to estimate the chance of the object being exposed to an unpleasant event. The relative density of interacting objects is the simplest density metric that can be easily applied to the risk assessment. Consider the *interaction set* Ω^{χ_i} for the object ω_i. This set includes all objects that interact with ω_i based on the relation χ_i. These objects are dispersed over the subspace D^* of the interaction space defined as the closed n-dimensional area described by the maximum surface distance $\mathcal{R}$ starting from the point d_0, which can be spatially aligned with the cell d_i that represents the state $X_i(t)$ of the object ω_i within the interaction space D.

Obviously, the higher is the density of objects within the subspace D^*, the higher is the likelihood of an unpleasant event. Therefore, the relative density of interacting objects $s_i(t)$ with respect to the object ω_i can be estimated as the ratio of the number of interacting objects N that constitute the interaction set Ω^{χ_i} to the relative volume of the subspace D^*, which can be obtained as the number of cells within the grid D that are inscribed in the subspace D^*. Consequently, $s_i(t) = \left|\Omega^{\chi_i}\right| / \left|D^*\right|$, where $|\cdot|$ is the cardinality of the corresponding set.

Let $\prec_s$ be a partial order $\sigma_{i0}(t) \prec_s \sigma_{i1}(t) \prec_s \ldots \prec_s \sigma_{iu}(t)$ given on the set $s_i(t)$ that implies the corresponding ordinal scale $\mathsf{S}_i(t) = \{\sigma_{i0}(t), \ldots \sigma_{iq}(t)\}$. Suppose γ_i is a function that uniquely takes the relative density $s_i(t)$ onto a certain level of the scale $\mathsf{S}_i(t)$, such that $\gamma_i : s_i(t) \to \sigma_{ij}(t) \in \mathsf{S}_i(t)$. Thus, the function γ_i can be used for quantification of the relative density of interacting objects within the interaction space D.

We can use a limited number of density levels to speed up the risk assessment as is shown in Table 3.

Table 3. Density levels

	Level	Density
0	σ_{i0}	Sparse
1	σ_{i1}	Low
2	σ_{i2}	Medium
3	σ_{i3}	High

Thus, we have three components of risk represented qualitatively by certain levels and we are ready to assess the risk level.

4.3 Risk Assessment

The goal of risk assessment is to determine risk acceptability, often by comparison to similar risks. Thus, risk assessment is relative.

Table 4 shows the risk categories and correspondent risk levels ensured by the proposed method.

Table 4. Risk categories (levels)

		Level	**Category**
5	y_5	X - Extreme	Fatal
4	y_4	H - High	Unacceptable
3	y_3	M - Medium	Unacceptable
2	y_2	L - Low	Tolerable
1	y_1	I - Minor	Acceptable
0	y_0	Z - Near Zero	Negligible

Estimated levels of all three components of risk intersected by rows and columns create a Risk Assessment Matrix. Risk Assessment Matrix is a generally accepted definition for risk levels given in Table 5.

Table 5. Risk Assessment Matrix

Density	**Exposure**	**Severity**			
		Minimal	**Minor**	**Major**	**Critical**
0	0	Z	Z	M	H
	1	I	I	M	H
	2	I	I	M	H
	3	I	L	H	H
1	0	Z	I	M	H
	1	I	L	M	H
	2	I	L	H	H
	3	I	L	H	X
2	0	I	L	M	H
	1	I	L	H	X
	2	L	M	H	X
	3	L	M	H	X
3	0	I	L	H	X
	1	L	M	H	X
	2	L	M	X	X
	3	L	M	X	X

4.4 Soft Risk Assessment

Soft sets enable to approximately represent risk levels distributed over the interaction space using the topologies defined in Section 3.

Let y_i be an i-th risk level and $Y = \{y_i\}_{i=0}^{5}$ be an ordered set of risk levels. Suppose D is a universe and Υ is a mapping that takes Y onto a set of all subsets of the set D, i.e. $\Upsilon : y_i \to 2^D$. Thus, a pair (Υ, Y) represents a soft set of cells [24]. In other words, the pair (Υ, Y) is a family of subsets of the set of cells D, all of which are parameterized by the set Y. Each value $y_i \in Y$ defines a certain set of y_i-approximated elements of the soft set (called y_i-elements of the soft set [25]). Let us denote such y_i-elements by Υ_i.

Soft set (Υ, Y) divides the universe D into the set of y_i-elements such that $\Upsilon = \cup\{\Upsilon_i\}_{i=1}^{k}$. A dynamic y_i-indiscernibility relation $\Re_D^{y_i}(t)$ can be defined on the universe D as $(\forall y_i \in Y)\ \Re_D^{y_i}(t) = \{(d_m, d_n) \in D \times D \mid y_i(d_m, t) = y_i(d_n, t)\}$. Using this relation, each y_i-element of the soft set Υ_i represents a certain equivalence class at the moment t. Thus, the parameterized family of subsets of the universe D constitutes the y_i-element of the set Υ_i, which uniquely determines a factor-set $D / \Re_D^{y_i}(t)$. The factor-set consists of all equivalence classes of D induced by the relation $\Re_D^{y_i}(t)$. Therefore, a pair $apr_D = (D, \Re_D^{y_i}(t))$ defines the dynamic approximation space. Definitely, $Def(apr_D)$ is a family of all compound sets and $\mathcal{T}_D^{\Re_D^{y_i}}(t) = (D, Def(apr_D))$ is a dynamic soft topological space, which uniquely corresponds to the dynamic approximation space.

Each y_i-element of the soft set Υ_i enumerates cells, which corresponds to the certain i-th risk level. Thus, different y_i-elements of the soft set have distinctive risk levels. The cells, which belong to the y_i-element of the soft set Υ_i, constitute the topologic structure represented by the corresponding equivalence class $D / \Re_D^{y_i}(t)$. Since the assessments of risk usually change over time, the soft set (Υ, Y) of cells is dynamic as well as risk assessments. The representation of the dynamic soft set of risk assessments distributed over the interaction space is shown in Fig. 4.

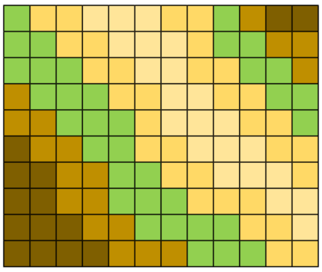

Fig. 4. Representation of the dynamic soft set of risk distributed over the interaction space

5 Results

The proposed method has been tested in the unmanned vehicle's onboard system Breeze [26] based on embedded microcontroller STM32F429 (180 MHz Cortex M4, 2Mb Flash/256Kb RAM internal, QSPI Flash N25Q512). The proposed algorithm of the risk assessment has been implemented using the C++ programming language as well as the ToPo and SoFTo library, which offer a set of operations for topologies including their addition and subtraction, determining unions, intersections, closures, and interiors, and a wide range of operations with soft sets.

The efficiency of the proposed method has been examined based on its comparison with traditional geometric (quantitative) and decision-tree based (qualitative) risk assessment methods. The total time of the risk assessment has been evaluated with respect to the number of interacting dynamic objects varied from 10 to 100. The results are shown in Fig. 5, they show that the proposed method provides acceptable performance, which allows it to be used in real-time unmanned systems.

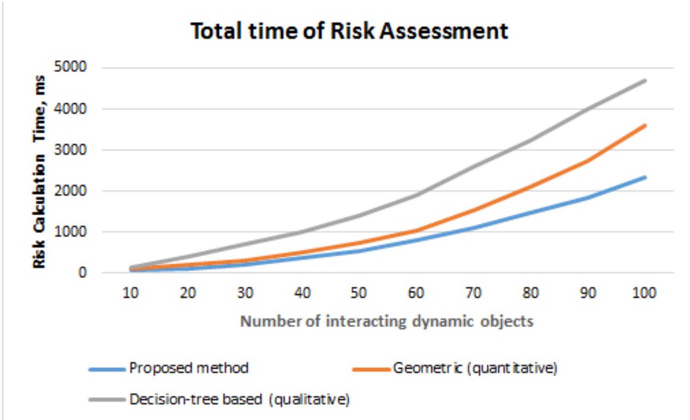

Fig. 5. Simulation results

6 Conclusions

The problem of risk assessment in complex intelligent systems, which do not involve a human is addressed in the paper. The paper is focused on issues related to the joint activity of unmanned systems in a dynamic environment, where risk appears due to their interaction. The authors propose the method that combines a matrix-based approach and a geometric approach to process numerical data at the input and uses soft sets as a suitable tool to avoid uncertainties related to incomplete and inaccurate information. The proposed method is based on the model of a dynamic system under risk based on n-dimensional stateful interaction space and corresponding topological structures. Authors propose to estimate numerically only inexpensive parameters and then provide their quantification to avoid extensive calculations. The risk levels are assessed with respect to the certain cells of the discretized n-dimensional interaction space. The levels of risk assessed in different cells of interaction space allow assessing risk distributed over the interaction space to prioritize risks properly. Due to uncertainty and lack of data, a density-based metric, which represents a relative density of interacting objects, is proposed to use instead of traditional probabilities or frequencies to estimate the chance of the object being exposed to the undesired event.

The proposed qualitative method is simple and fast, it provides the acceptable performance of risk assessment enough to the real-time unmanned systems.

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