Insightful yet inferential creativity: transduction as a derivation of abduction

Sandra Visokolskis

National University of Cordoba, Cordoba, Argentina
sandraviso@gmail.com

Abstract. This paper deals with Charles Sanders Peirce's conception of "uberty" of an inference to a plausible hypothesis, i.e. its potential to lead to an original idea [30]. Still with other designations that imply the introduction of novelties, Peirce included this trait as the characteristic of abduction, the only type of inference to present it. But in his last writings, he distinguished two tasks that coexisted in abduction: (i) the formation of hypotheses and (ii) the selection or adoption of them, where the first task, the source of new ideas, was only admitted to be by Peirce, the product of instinct; while the second task was specifically inferential. This leads to the dilemma of trying to reconcile reasoning with instinctive insights. However, in contrast with this author, we will assume that the creative formation of hypotheses is much more sophisticated than mere instinct, subsuming all this complex task to what we have called "transduction", a variant of Peirce's abduction, dominated by a cluster of expert-agential cognitive mechanisms and non-deductive activities based on similarity. Transduction, a modelling generation reasoning process, is achieved by means of three mechanisms stemming partly from abduction, partly from induction: (1) evocation of the idea by similarity, (2) analogical transfer, a mix of both processes, following here Aristotle's notion of *paradeigma*, and (3) reasoning from effect to cause, what we would properly call the abductive explanation. The utility of our method is twofold. On the one hand, it extends the source of hypothesis generation to situations not only instinctive but also allows its combination with inferences. On the other hand, our method provides a finer analysis of the cognitive mechanisms that it is supposed to be present in all creative processes and that the abductive Peircean mode of description seems to contemplate although not explicit. The method is tested on several examples.

Keywords: Creativity, Transduction, Expertise

1 Introduction

In the mid-1970s, a body of reports and research on the issue of human reasoning began to emerge in the field of psychology, which established the foundations of what later became known as the *Dual Process Theory* (hereinafter *DPT*)[1]. Research from

[1] Cf. [8] for an overview of the history of *DPT*.

the fields of epistemology and philosophy of the science and mind were added to this type of psychological reasoning, forming a set of ideas that, among other things, favors the role of reason, pushing the notion of associativity into the background.

These theories, extrapolating broadly, postulate a priority of the complex human capacities of reasoning, as opposed to associative mechanisms considered much simpler, thus constituting two different systems of reasoning: system S_1, linked to the associative factors of thought, is generally characterized as an implicit, automatic, fast, innate, and evolutionarily older system, shared with other animal species. On the other hand, system S_2, linked to reflexive reasoning, is characterized as a slow system, which allows hypothetical thinking, and it is the product of more recent evolutionary developments. These two systems have been linked to the dichotomy between a conscious, deliberate, and controlled reflection (S_2), on the one hand, and an associative intuition, also considered by some authors as non-conscious (S_1), on the other.

Notwithstanding the profound implications of this classification, which gave rise to harsh questioning, among which this article constitutes a case to be added to such a list ([14, 23, 27], among others), here we focus specifically on the role of resemblances -a type of association- in scientific reasoning. Is it so that human beings make decisions only based on consciously controlled reasoning? If this is not the case -as we will present it-, to what extent do associative processes impact decision making? What roles do these associations play? And, within the different types of associations [43, 45, 47], what influence do resemblances have? In order to answer these questions, we will recur to the research carried out much earlier (19th and early 20th century) by Charles Sanders Peirce, on the notion of "abduction", and a variant of it, which we propose here and that we have called "transduction" [42, 44]. As we will see in section three, transduction refers to a set of inferential processes and cognitive mechanisms based on the notion of association by similarity. Just as the notion of abduction in Peirce is, according to this author, the only type of inference that introduces novelty, transduction, as a variant of it, deals with the creative emergence of hypotheses, emergence that begins, according to our position, from the combination of the detection and construction of an original resemblance, which will occupy the nucleus of the entire generative procedure. The key, in this matter, is trying to explain how the resulting resemblance is a combination of a discovery and a construction on such inquiry.

Compared to Peircean abduction, transduction only ensures the first inferential, epistemic, and cognitive instances in scientific inquiry, instances of hypothesis generation, while abduction also includes the selection of hypotheses. In the various approaches that Peirce carried out throughout his life regarding the concept of abduction, which was not only changing its name (hypothesis, hypothetical inference, retroduction, presumption, among others) but of referential content, we highlight the skillful Peircean decision to combine both instinct and inferential processes, when describing how to produce novelty in creative acts of various types. But as Paul Thagard [38] points out, Peirce in some sense did not specify how ideas were generated. In this regard and more precisely, we emphasize that Peirce ventured no more than to signal instinct as a trigger for the formation of ideas, which led Peirce to close the issue of how it was possible to produce such changes. If everything reduces to instinct, there is nothing left to explain when it comes to the conceptual and theoretical genesis, but to select among the options that emerge once this has already happened. In this regard, Thagard states:

> Peirce claimed that abduction could generate new ideas, but he did not specify how this could occur. If abduction is analyzed as a logical schema, then *it is utterly mysterious* how any new ideas could arise. The schema might be something like: "q is puzzling, p explains q, so maybe p". But this schema already includes the proposition p, so nothing new is generated. Hence *logic-based approaches to abduction seem impotent to address what Peirce took to be a major feature of this kind of inference*[2]. [38: 453]

In the same spirit of Thagard's critique -which this author solves by means of a neurological subpersonal response-, in this article we opt for a non-instinctive explanation of these genetic processes, and although surely some neurocognitive aspect can be recognized here, we will limit ourselves to developing the notion of transduction, which is managed on a personal level, both conscious and non-conscious.

Such transductive activity is characterized, as a variety of one of Peirce's conceptions of abduction, in terms of a blend between intuitive acts and abductive inferences. Only, unlike this author, (1) instead of instinct mastering intuition, we speak of *expert knowledge* and, (2) we distinguish two stages during abduction: a first that makes the *formation of hypotheses*, and a second, relative to the selection and adoption of one of such hypotheses. While Peirce considers that the formation of hypotheses (stage one) is merely instinctive, we will assume that this task -which we will call "transductive" as a particular and primitive case of the abductive process- is much more sophisticated and complex, and is dominated by a cluster of skills, activities and non-deductive abilities such as: iconic visual inferences, analogies, metaphors, diagrams, among others, all contributing to the construction of one or more hypotheses that explain the emergence of some creative insight, in response to a problem that motivates and drives the creative process. Thagard refers to a type of instinct which Peirce highlights, the instinct to guess, which, according to the Canadian philosopher, is not convincing currently, due to recent neuropsychological research:

> Peirce's suggestion that abduction requires a special instinct for guessing right is not well supported by current neuropsychological findings (…) I prefer the suggestion of Quartz and Sejnowski that what the brain is adapted for is adaptability, through powerful learning mechanisms that humans can apply in many contexts. One of these learning mechanisms is abductive inference, which leads people to respond to surprising observations with a search for hypotheses that can explain them. Like all cognitive processes, this search must be constrained by *contextual factors*[3] such as triggering conditions that cut down the number of new conceptual combinations that are performed. [38: 458]

Although Thagard acknowledges that such instinctive activity does not seem to reveal the prevailing mechanisms in the processes of creative change, and attributes this task only to subpersonal elements, the author considers as "merely contextual" that the intuitive quick decisions of non-instinctive type usually have to do with the

---

[2] The italics are ours.

[3] The italics are ours.

deep knowledge and familiarization that the innovative agent has in relation to her problem solving. In our proposal, the creative agent's expertise which allows for the shortening of paths and making creative shortcuts, is not something accidental but it is the key element in cases where a thorough and deductive reasoner -such as a computer programmed to solve specific problems- would lack the competence to act, at least, to date, not yet in a completely generalized manner, with isolated cases being successful.

The presence of expert knowledge in scientific problem solving will allow, in successful circumstances, to solve complex situations in an accelerated manner, without requiring that the obtained solution constitutes a complete and linguistically formulated representation, but rather a plausible explanation of the reason for such problematization. That is why, in this paper, we will focus on explanations based on similarities, in processes of scientific problem solving. This kind of explanation will turn into patterns of transductive argumentation, the focus of our discussion.

2 Explanations Versus Representations

A common theme in current philosophy of science is the analysis of the relationship between constituted theories and the models used to obtain adequate representations of such theories. This model-theoretical link, widely discussed, has followers who accept the notion of similarity as the bridge established to settle the conceptual distance between these two extremes, theories, and models. We can especially mention Ronald Giere [12, 13], as well as the works of Aronson, Harré and Way [2], Peirce [30] and Teller, among others. There is also a significant number of detractors not only in philosophy but in linguistics and art [7, 15, 16, 17, 18, 19, 37, 53]. It is worth noting that the proposal to establish connections by similarity between theories and models, as a more tenuous variant than that of providing isomorphisms between them, recovers certain characteristics of such linkage. What does similarity transfer between theories and models consist of? How is such a transfer delivered? Are these transfers present only at the model-theoretical level? Here, we are focusing our interest on a characterization of transfers by similarity, not between theories and models but between general objects and signs that represent it. More specifically, we seek to elucidate the relationship by similarity between objects that describe problematic situations and iconic signs that aspire to solve such problematic situations. That is why this article deals with the relationship between the hypothetical postulations of plausible theories and their means of representation. We refer to the processes prior to the systematization of scientific theories in various fields, both formal and natural or social, in which, in order to achieve plausible descriptions of these processes, we do not necessarily appeal to linguistic resources but also to semiotic vehicles that expect to have a first glimpse of them.

Such representational vehicles are not only limited to models [3, 24, 26, 54], the main source of theoretical characterization, but also diagrams, schemes and/or graphs of various kinds [6, 11], as well as metaphors, analogies and similes of different kinds [5, 20, 21, 33].

Taking into account that we will not necessarily adhere to already systematized theories but to hypothetical applications seeking consolidation, this article is oriented toward discovery processes and non-standard representational vehicles, as in the case of icons (images, diagrams and metaphors) that Charles Sanders Peirce characterized in his semiotic writings. Within this thematic framework, we will concentrate on certain patterns of argumentation that we will describe under the label of "transductions". We attempt to put forth a characterization of such transductive arguments having the proposition of an explanatory perspective of the role of the means of theoretical characterization as our main goal, relegating representative functions to the background.

The notion of resemblance (in art) or similarity (philosophy and linguistics) has played a central role in philosophical discussions on the representative potential of a theory or a model. Just as there were important supporters of the acceptance of similarity as a basic characteristic of various representational theories of science, there have also been, and still are, a large number of detractors. In this paper, we posit that the dominant critique of the notion of similarity in the characterization of scientific theories or models is based on the emphasis placed on the notion of representation. We consider that, if, instead, the accent is placed on the notion of explanation, the risk of applying similarities decreases and thus avoiding the elimination of a source that we consider to be constitutive of the processes of formation of ideas, concepts, arguments and/or scientific theories.

Nevertheless, it is also necessary to explain to what notion of explanation we refer. In this sense, in the text *Lessons on pragmatism* [30], Peirce characterized abduction in terms of an explanation of a hypothesis:

The surprising fact, *C*, is observed.

But if *A* were true, *C* would be a matter of course.

Hence, there is reason to suspect that *A* is true. [30: CP 5.189][4]

Here, hypothesis A would be explained from its unintended relationship with the fact *C*: "it is the idea of putting together what we had never before dreamed of putting together which flashes the new suggestion before our contemplation" [30: CP 5.181]. Such hypothesis A is the conclusion that explains facts or evidence (*C*) mentioned in the premises, where its assumption explains the "surprise" [30: CP 5.189] or the "curious circumstance" [30: CP 2.624]: "Hypothesis is where we find some very curious circumstance, which would be explained by the supposition that it was a case of a certain general rule, and thereupon adopt that supposition" [30: CP 2.624].

Since abduction is used as an explanation of the causes that lead to certain effects, it should be clarified that this results in a regressive progression or a retrojection, while, instead, deduction according to Peirce, is a progressive projection, which leads to represent a prediction of causes to effects. The latter seems to contradict the standard inherited conception of explanation in the philosophy of science, which is usually understood as deductive: given a theory *T*, if we indicate its causes as *C* and its effects as the set of statements *E*, the latter requires of an explanation from $T \cup L$, with *L* the logical consequences. But as Wesley Salmon [36] points out, deduction is neither necessary nor sufficient for an explanation. This was already foreseen by

[4] We are using the standard practice of referring to Peirce's *Collected Papers* by 'CP', followed by the corresponding volume and paragraph number.

Peirce, who took, in his last writings, deduction as the second step in scientific inquiry, after the application of abduction, before final inductive testing. Just as Peirce distinguishes abduction -as the first step in the scientific process- from induction -as its last step-, transduction also differentiates them. Indeed, although both types of reasoning are ampliative, they are so for different reasons. Induction consists in the generalization of a particular case (problem) *A*, to a given class Ω, of members with solutions on the set C_{Ω}. Thus, *individuals* belonging to a class already known are amplified. In contrast, in transduction, a new property is conjecturally created for a particular case (problem) *A*, creatively inferred from the capture of a similarity of *A* with another problem *B*. Therefore, the *properties* predicable by *A* are extended, as for example a solution C_A to problem *A*, from an analogy with the known solution C_B of *B*. Furthermore, in the case of transduction, it could eventually also be concluded that a class Ω is created such that $A, B \varepsilon \Omega$; and also that C_{Ω} is created, a kind of problem solving methodology or heuristic of Ω members, such that $C_A, C_B \varepsilon C_{\Omega}$. Thus, induction generalizes, while transduction conjectures.

Given the mentioned separation between deduction and abduction, it can be clarified, as Thagard says "what constitutes the causal relation between what is explained and what gets explained" [38: 449], but, in contrast with his neuronal strategy, we posit as constituents of such an explanatory relationship, to a resemblance, obtained due to latent expert knowledge in the detector agent thereof.

Therefore, the proposal of the notion of transductive argumentation developed herein, seeks to question the importance given to the representational role of the models, contributing with the notion of transductive icons as non-standard representational vehicles oriented toward presenting explanations of scientific practices, instead of constituting types of modelistic idealizations and/or abstractions, which often lack an effective description of the processes of creative change.

Semantic approaches, in general, propose to account for the relationship between theory and model in terms of isomorphisms, thus connecting, on the one hand, a mathematical space of representation, with a model of physical system, on the other. One of its supporters, Ronald Giere, proposes, on the other hand, to privilege the notion of similarity in detriment of the concept of isomorphism as the type of link between theory and model. This "weak" strategy [12: 81] fulfills the function of better describing scientific practices while models remain similar with certain respects that are relevant to the theory, as well as the degree of similarity with which idealizations occur. Although this notion of similarity as a substitute for isomorphism was highly criticized, it gained value in relation to the not-so-considered analysis of contextual epistemic factors, aspects that are also taken into account in our transductive proposal.

However, instead of talking about a "weak" relationship between theory and model through theoretical hypotheses obtained by similarity as linguistic devices that are not necessarily isomorphic, we focus on an also weak (or more comprehensive and general) relationship between a concrete and particular problem, and the finding of a specific and particular "transductive icon", which is linked to the elements of the original problem in a way that is not necessarily linguistic (although it is semiotic, in the Peircean sense), through certain processes of similarity, which do not strictly require a connection of isomorphism, but a weaker one of analogy. Although it should be noted that analogies are usually understood in linguistic terms (not always though), this does not imply falling back into the same difficulties of the correspondence rules applicable in a structuralist style typical of the received view. Also, the not necessarily linguistic mode of expression should not obscure the understanding and explanation of the obtained plausible solution. Let us then make explicit the notion of transduction.

3 What is Transduction?

3.1. Current Motivation for Transduction

In a 2014 article, considering the work of Peirce, Ahti-Veikko Pietarinen and Francesco Bellucci maintain a dichotomous distinction between: (a) instinctive processes, "by which we pass directly from a proposition to another without any mediation", and (b) reasoning, "a process by which we pass from a proposition to another mediately, that is, upon some reason". But what makes the dichotomy much stricter consists of the following division: while "in reasoning there must be some voluntary and controlled act that can evaluate the conclusion with respect to a range of other alternative acts", other kinds of acts, such as instinct, are involuntary ones. From there they conclude that retroduction (abduction) "is subject to logic insofar as is performed according to *some*[5] principles governing voluntary and controlled acts"[6].

Therefore, the reason versus instinct dichotomy, according to these authors, (1) lies at the base of the Peircean notion of abduction, (2) is characterized by attributing voluntary and controlled acts to reasoning while it is not possible to do so when referring to instinctive processes, (3) makes retroduction be a matter of logic as it operates as a reasoning. Consequently, the instinct that Peirce attributes to abduction to generate hypotheses, according to these authors, is an unmediated, involuntary, and uncontrolled task, and thus it is not a matter of logic.

In this way, logic would deal only with "reasoned" aspects of abduction, which would result in the generation of hypotheses (a purely instinctive task) being excluded from its area of influence. In this train of thought, abduction would only intervene in the choice and adoption of hypotheses but not in their formation, which, according to this, is an instinctive task.

From the above, we may ask: (1) are we sure that hypothesis formation can be relegated to instinctive only processes, "instinct for guessing correctly" [28, *MS*[7] 690-692]? If the answer to this question were negative, that is, if there was something else between instinct and reasoning, (2) what will that be? (3) what mechanisms would operate to produce new hypotheses in addition to and apart from Peircean's instinct? If such mechanisms exist, (4) could they be inferential? (5) Should they necessarily be "voluntary and controlled acts" (as Peirce, Pietarinen and Bellucci

---

5 The italics belong to the authors.

6 Cf. [31: 360].

7 All references to Peirce's Manuscripts, numbered in accordance with the R. Robin's *Annotated Catalogue of the Papers of Charles S. Peirce* [28], are abbreviated by '*MS*', followed by the manuscript number.

characterize all reasoning)? (6) Or could there be inferential mechanisms that are not strictly voluntary and deliberate? The answer to these six questions can be obtained if we rely on a distinction that Kapitan [22] and Frankfurt [9] establish around a general notion of Peircean abduction: it is possible to distinguish two tasks that Peirce attributes to abduction: (i) hypothesis formation, and (ii) hypothesis adoption. Again, Peirce attributes task (i) to instinct, while task (ii) is inferential. If, on the other hand, taking distance from Peirce in this matter, we take option (i) as a task that is not purely instinctive but acknowledging also that inferential elements occur too although not necessarily voluntary, deliberate and consciously controlled, then this new type of inference that we have named "transduction" arises [42, 44], and it provides a sequence of logical operations of a particular kind to elucidate, as well as cognitive mechanisms that involve emotional and intentional aspects that contribute to the formation of hypotheses.

3.2. Creative Introduction of Resemblances

When it comes to solving a problem, and the difficulties to find possible solutions persist, the appeal to other similar problems allows, on occasions, to pose like answers as a hypothetical framework, thus opening a new perspective that has been blocked up to this moment. Clearly, the hypothesis formulated acquires a plausible character. In principle, its inconclusiveness is waiting for a later verification. The cognitive opening introduced can be more or less creative depending on the scope of such resemblance. It is known that similarity reasoning is controversial, but, in positive cases, it allows us to extend the analogy to multiple situations, where the benefit of the risk comes from accepting momentarily, tentatively and provisionally this type of argument. Therefore, taking these provisions into account, it is possible to immerse oneself in the world of resemblance, taking it as a primitive, and bet on a creative openness, beyond achieving the justifications of the case. It is possible to observe that this type of reasoning -which goes from the assumption of a similarity and its analogical projection to a greater set of properties in common between the two domains in relation of comparison-, is sustained under the belief of certain principles of permanence (or invariance) and continuity, which apply the similarity in a different domain and extends its incidence to more properties. Another notable characteristic is that it consists of a type of reasoning between particulars, thus escaping generalizations in other contexts. Therefore, it does not seek to cover more cases, as occurs with induction, nor does it pretend to seek generality of the type of a deductive argument. In this sense, we can distinguish at least three types of argumentative inferences that configure work methodologies: top-down (deductive reasoning that goes from generic to generic, or applies generic to particular), bottom-up (reasoning which infers from particular to generic), and what we called "transductive" reasoning (which infers from particular to particular). This last term was coined by Gammerman, Vovk, and Vapnik [10] and applies to processes that try to match a current particular case with a familiar similar one, in order to transfer properties from one to the other, the known and familiar case to the unknown and problematic case that is sought to be solved.

By adopting a transductive inference, an analogy is constructed, described through a function from a domain *A* (the problem to be solved) to an image *B* (another problem already solved previously). This analogy allows us to explore how the problem *A* might be by comparing it with problem *B*, and how it might work if it were like the analogical problem *B*. Since we know a solution of *B*, all we have to do is transfer it to *A*. Therefore, the analogy must reflect the invariant structure that *A* has in terms of some invariance present in problem *B* and similar to that of *A*.

Thusly, we can portray the characteristics of the starting set *A* in terms of some already formed and known set *B* with which we are familiar. In doing so, imagining some aspect or property of the domain *A* in terms of something else, we are able to think about the original problem from their transductive model. This allows solving the initial problem in terms of a solution that is already known in the analogous previously solved problem. The evocation of the familiar and known in advance problem *B*, leads to solving problem *A*, given the similarity between *A* and *B*. In this way, there is a connection –once unthinkable and surprising- between *A* and *B*, in making the terms from *A* to the analogical model *B*, and vice versa as the analogical terms fit back to problem *A*.

The advantage that problem *A* acquires when interacting with problem *B* consists in the creation of a new cognitive scheme to characterize *A* in terms of *B*, embedding *A* into *B* and redirecting *A* to the solution of *B*, which is based on its latent invariants, to finish capturing the invariants in *A* that allow its solution.

Transduction is a kind of reasoning whose function is to introduce original, novel and significant elements in an unfamiliar and problematic context *A*, resorting to other familiar elements belonging to a domain *B* of prior knowledge, of which we could glimpse a plausible solution of a problem within it, which was evoked when trying to solve the problematic issues in *A*. We will say that *A* is the goal or target domain, and *B* is the source or vehicle domain.

We use the transductive framework to explain how ideas arise, and what cognitive mechanisms are involved in inferential processes that lead to the formation of associations between structures of two intervening domains in the resolution of a problem: the source domain, from which the problem and the information on this domain stem from, and the objective domain or destination, which cooperates in the resolution. From the transductive referential framework, for the purpose of solving the problem present in the target domain, not only logical reasoning intervenes in the passage from one domain to the other, but also other epistemic aspects, such as emotions and intentional attitudes.

A transduction is not just a simple passage or transfer (or transport) of certain elements from the source *B* to the goal domain *A*, which, by the way, does occur; but also a process of empathic fusion between elements of both domains, creating something new in such a process, asymmetrically directed from source *B* to target *A*. It should be noted that the inverse fusion of *A* to *B*, if it occurs, would cause another transductive process, which is not necessarily the same as the previous one: comparing 'Achilles' to 'lion' is not the same (as Aristotle [1] did in *Rhetoric*, Book III, chapter 4, 1406b, which recovers, for example, the Achilles property of being bold) than resembling a lion to Achilles (which, for example, evokes an anthropomorphic behavior of the non-human lion animal, to indicate, for example, that lions have frontal eyes like men and, therefore, that they are predatory hunters, unlike other living beings with eyes at their sides)[8].

The asymmetric fusional passage from *B* to *A* creates something new in *A*, from the experience of making it analogous to *B*. For example, the trait of audacity in Achilles is highlighted, which, perhaps otherwise would not have been noticed, but when comparing Achilles to a lion, it emerges evocatively immediately, even if it could have gone unnoticed before. Eventually, new ideas emerge from the experiential formation of the association of *B* with *A*, on the basis of the familiar and prior knowledge that is possessed of *B* and that, therefore, is evoked almost immediately, in order to transfer it transductively to *A*.

The transduction between *A* and *B* does not consist of a single application between *A* and *B* that makes the transfer of elements from *B* to *A* -as it occurs in any analogical transposition-, but in a series of transfer procedures and articulated steps that make up the way of solving the problem that domain *A* departs from. This set of steps is characterized by the initial search (and eventual finding) of a *resemblance M*, which, in subsequent successive stages is creatively elucidated and, in the best-case scenario, it is taking shape to form a plausible solution of the problematic situation (of or in) *A*, from familiar and known elements (of or in) *B*.

The main reason for the importance of similarities *M* is that they have a "beginning of the ball of yarn effect", i.e. they allow us to kick things off when we had nothing but A at hand, or we believed so at least. Similarities offer new elements when there seemed to be no possibilities for future action, when we appeared to be facing aporia, i.e. dead ends. And this is something that, for example, Greek mathematics used when conceiving the notion of "analysis" as a discovery, which consisted in assuming an existing solution to a problem to be solved, even when we do not know if there is one, and then proceed to operate with this supposedly existing putative entity. This situation of "analysis" also occurred -not with that name- in the cultures of Ancient Egypt and Paleo-Babylonian mathematics, when they operated by the method we call today "false position" [48]. The fictitious or putative existence of entities or procedures where there were none (or it was not known of their existence), allowed, in these historical cases, to access a plausible solution path although not necessarily conclusive in a positive sense. That is why transduction, that is, a first stage of abduction not contemplated by Peirce, stems from the "beginning of the ball of yarn" strategy, the "analytical" process in the ancient Greek way of supposing and having something that in principle we do not have to accept as plausible (or reject it), based on the use of some similarity, even if the latter is not yet elaborated.

3.3. Elements and Relations involved in Transduction

By adopting a transductive inference, an iconic sign is evoked, an analogy and an abduction are constructed, described through a function from the domain *A* (the

[8] Aristotle says: "A simile is also a metaphor; for there is little difference: when the poet says [of Achilles], 'He rushed as a lion', it is a simile, but 'The lion rushed' [with 'lion' referring to a man] would be a metaphor; for since both are brave, he used a metaphor [i.e., a simile] and spoke of Achilles as a lion" [1: 205].

problem to be solved) to the icon *B* (another problem already solved previously). The analogy allows us to explore how the problem *A* might be by comparing it with problem *B*, and how it might work if it were like the analogical problem *B*. Since we know a solution to *B*, even though we cannot explain or make it explicit from the beginning, all we have to do is to transfer it to *A*. Therefore, the analogy must reflect the functional invariant structure that *A* has in relation to what caused the problem there, in terms of some invariance present in problem *B* and similar to that of *A*. To capture the analogy between the two domains is to put in evidence, to explain a certain transplant function that connects them and then to backproject it from *B* to *A* and to analyze what happens when "implanting" the found and highlighted properties of *B* now in *A* [42]. If such a transferential implant "catches on" (following the medical metaphor), if it adapts to the new medium *A*, if it fits, if it works in this other context, then we can already try a demarcation in *A* of the properties and relationships that primarily stood out in its initial configuration, and visualize *A* now from the new perspective of the recent set of properties found, thus addressing a new direction of analysis that becomes independent of the iconic relationship and the analogical inference from which they arise, which will converge in a new characterization of the original context *A*, now supported by an hypothesis C_A that solves the problem.

Our account deals with linguistic representations, the tentative and plausible hypothesis. However, that does not always happen in the first steps of the acquisition of icon *B*, being *B*, by its nature, not necessarily linguistic. Icons, as semiotic objects, thus constitute the starting point of the creative process of solving the problematic situation *A*. From there on, there is a whole process in the search and refinement of solutions to *A*, more in line with systematic linguistic styles, typical of scientific research. But the relation of similarity obtained, in itself, does not require, in a first stage, linguistic specifications. This makes the proposed perspective to exhibit subjective nuances.

Figure 1 shows how a complete transductive process develops with all the associative steps involved, which manifests a seemingly linear sequence, i.e. a sequence which although linear in form, is not necessarily linear in act.

Before going on to describe such a sequence of associative phases, we find it necessary to comment the following: that a whole transductive argument is posed sequentially does not mean that every creative subject passes through all instances, consciously or in a non-conscious manner. One can skip stages, start directly with a metaphor (and not with an image or a diagram), or with an analogy (not with a more rudimentary icon), or even with the solution already fully delineated in the mind; or it may even happen that what one finds is all failed, and none of the steps taken pay off; or also, one can be stuck going back and forth repeatedly through previous instances. One can alter the order of the sequence depending on how it raises awareness and realize how to build such a process.

The linearity of a creative model was a characteristic sign of the model proposed by Graham Wallas [52], and one of the reasons why it could be taken as a failed attempt to implement it, as is the case of Pólya's problem solving model [32], which also has a sequence of steps to achieve the desired goal. The key is to think that individuals come and go with ideas, move forward and backward, live a life not only dedicated to solving this specific problem, they think they are right but sometimes they are wrong: the steps of the model are there only as indicators of the inferential elements and the cognitive mechanisms of which the vast and rich creative process is composed, and that, in idealized cases of full awareness, one can capture and/or carry them out.

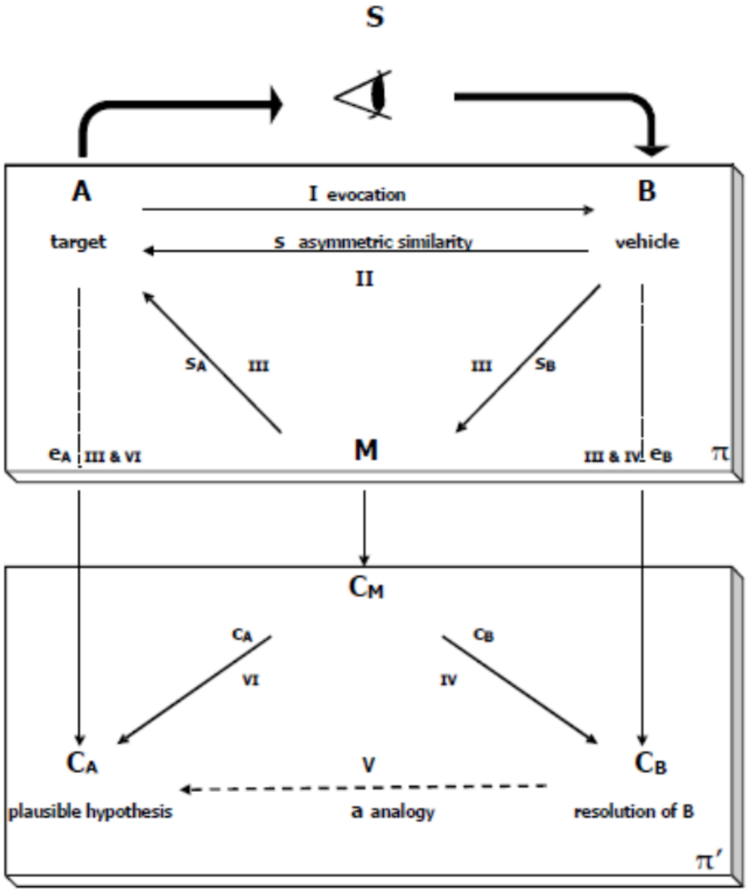

Fig. 1. The transductive process.

Figure 1 shows the five elements at play in a transductive process: the interpretant *S*, the target object *A*, the iconic source sign *B*, the ground *M*, and the consequences C_A, C_B, and C_M from *A*, *B*, and *M* respectively. Let C_A be the proposed hypothesis, which eventually solves the problematic situation (in or of) *A*. In addition, Figure 1 presents six moments that make up the stages of the transductive process. These are:

[i] Evocation of *B*: given *A* and the problem to solve within it, icon *B* emerges.

[ii] Emergence of an asymmetric similarity from *B* to *A*.

[iii] Emergence of an aspect *M* in *B* that is also owned by *A*, and that operates in *A* as it operates in *B*, based on the proposal of a continuous proportion: $\exists M /\ M{:}B{::}B{:}A$.

[iv] Familiar and expertly known solution C_B to *B*.

[v] Extractive analogy of plausible solution C_A for *A*, from known solution C_B for *B*.

[vi] Plausible explanation of such an analogy: the relationship of *M* with *A*; how *M* would operate in *A*.

The formation of the hypothesis C_A is carried out by means of a transductive process of three applications, three types of associations that follow one after the other, in an orderly manner forming a commutative diagram, described in the following three sections.

3.4. Association by Resemblance or Extraction of Similar Features

As previously mentioned, from a gestalt flash, the evocative insight of icon *B* is produced. A subsequent immediate process consists in the search for explanation of such phenomenon, which is assumed as an inferential process of association by similarity. Why is *B* associated with *A*? What kind of association is this? In what sense is similarity discussed here? What elements make up *B* that cause them to emerge during the elucidation of problem (of or in) *A*? What causes the emergence of *B*?

Similarity is a relation (between a target situation *A* and a vehicle *B*) that may be used to explain the problem within *A*, in virtue of the fact that an agent S interprets - creatively and tentatively- that the vehicle *B* is similar to the target *A* in relevant aspects and to a certain degree of similarity. Then we can use *B* to explain *A*. Michael Poznic states:

> Similarity is more fundamental than the condition of inferential capacity. Similarity can be used in order to explain why representational models could have an inferential capacity in the first place. In a nutshell, *only because there is a similarity*[9] between vehicle and target, it is justified to derive certain surrogative relation, the inferences from models to target system can hardly be justified. [34:346]

The explanation process allows to regard *B* as if it conforms a specific fictional scenario which will be the vehicle of our future reasoning [39, 40, 41]. From this scenario in which we have converted *B*, the problem (of or in) *A* acquires other meanings, thus expanding the knowledge of *A*. Such similarity is established based on the iconic interpretation made of *B* in this first associative step: only *B*'s imaginistic aspect is considered here. It may be pointed out that *B* could also be a diagram or a metaphor, not just an image. As an image, *B* resembles *A* in a certain aspect or simple quality. An image achieves similarity by partaking of some of the simple qualities of its object. Therefore, similarity $s{:}B\rightarrow A$ requires a next step that will allow to infer knowledge from this imaginistic configuration: the insight detection of a salient property *M*, -what Peirce calls the ground-, that stands out among others in *B*. Arises $\exists M/\ M{:}B{::}B{:}A$, the middle term that shows the relationship between the extreme terms *B* and *A*, explaining how A resembles or behaves in the way it behaves *B*. Here, we find a phenomenon that we have called "dual cognitive mechanism" [46], where the

[9] The italics are ours.

aspect M fulfills two different roles: on the one hand it indicates a facet of B that wants to be highlighted. On the other hand, it is expected to describe another facet of A. Both the interpretation of B in terms of M first, and then the interpretation of A in terms of M, are what eventually lead to the solution of problem (in or of) A from the known solution of B. Consequently, the plausibility of the emergence of M can be observed. This situation is expressed through the following typically abductive reasoning:

$$\frac{\begin{array}{c} B \underset{\rightarrow}{s} A \\ M \rightarrow (B \rightarrow A) \end{array}}{plausible\, M}$$

The detection process does not trigger properties at random, but it selects a salient one with a priority given to the relation with the target situation A, based on some criteria that dominates agent S, based on its expert knowledge. The resulting selection is a shortcut for the computational explosion, of a totality of properties that B has.

On the other hand, and almost simultaneously, it is sought to understand how the appearance of the middle term M that connects B with A allows to put forward a plausible solution C_A for A. The answer to this question is to establish an analogical relationship with the objective of, in the course of making visible a solution C_B of the familiar and known vicarious problem B, making it possible to similarly extract a solution C_A for A. This reasoning leads to the second type of association.

3.5. Association by Contiguity or Analogy

Next step is to connect the icon B to the target system A and translate claims from the behaviors or functions of the icon B into claims about the target. Once the asymmetric similarity from B to A is pointed out, the agent begins to interpret the target A as something like the vehicle B. This means that whereas previously she took A to have no connection with B, she now understands A in a different way, not merely as a similarity that makes B stand in for A, but the projection of all the inferences from that similarity, i.e. an analogy. If M is an aspect that emerges from the similarity of B with A, then when explaining the relationship of M with B, the key emerges and will explain the relationship of M with A, which could tentatively give the expected solution for A.

Thus, the analogy between B and A generates activation of ideas C_A related to A on the basis of (i) an activation of existing and constructed connections M between A and B, and (ii) a familiar solution C_B of the problem on B, similar to the situation problem on A. This analogy identifies possibly new connections between A and B, and, above all, unexpected effects C_A in A. There are similar patterns of co-occurrence with some plausible effects or consequences (C_B and C_A), co-occurrence across some (not necessarily all) effects from B and A.

In this second associative stage, a specific iconic diagram emerges, which seeks to be closed or commutative so as to indicate an ampliative reasoning. A commutative diagram is a diagram such that all directed path in the diagram with the same start and endpoints lead to the same result. In this case, the directed paths under such conditions are: $e_A \circ s$: $B \rightarrow A \rightarrow C_A$ and $a \circ e_B$: $B \rightarrow c_B \rightarrow c_A$ where s is the relation of similarity, a is the analogical relation between B and A, e_B is the relation of explanation and extraction of consequences from B, and e_A is the relation of explanation from A. This will lead us to state that the existence of C_A is plausible, such that $e_A \circ s = a \circ e_B$ (cf. Figure 1). If B is similar to A, then the effects or resolutive consequences of problem (in or of) B might be similar to those corresponding (in or of) A. Or, also said in terms of decisions: "like cases should be decided alike" (the legal principle of *stare decisis*, the doctrine of precedent, cf. [4: 246].

Here we will talk about a contiguity-based interpretation of analogy, by claiming that A and C_A, on the one hand, and B and C_B, on the other, are contiguous elements in their respective domains. An association by contiguity "supposes that similar ideas have been conjoined in experience until they have become associated" [30, CP 7.499]. It constitutes a type of analogical process, in which, from observed similarities, we can infer a further similarity, when a phenomenon A calls to mind another phenomenon B because A and B have been contiguous, i.e. have been experienced together.

Echoing Bartha here [4:321-322], to express an analogical argument, three steps are needed: (1) the construction of a hypothetical model (or, instead, our iconic sign B), (2) a derivation of some dependence result in that model (our C_B), and (3) an analogical inference (our function a) back to the phenomenon (our A). Thus, the general form of an analogical argument[10] is:

B is similar to A in certain (known) respect M.

B has some further feature C_B.

Therefore, A has some feature C_A similar to C_B.

Bartha proposes to describe analogical arguments from what he calls "prior associations", a vertical relationship within the source domain (our iconic sign B) that one hope to extend to the target (our A): "any acceptable analogical argument must be based upon an explicit prior association in the source domain, and there must be potential for generalizing that association to the target domain" [4: 227].

Moreover, according to Bartha, there are at least four types of analogical arguments: predictive, explanatory, functional and correlative. In our case, it is an explanatory one, where we transfer C_A. In an explanatory analogy, C_B explains B, and this implies that it is plausible that something similar to C_B is C_A, that explains the similar phenomenon A. Here the association is asymmetric, with B being prior to C_B. The prior association is the causal vertical relationship $B \rightarrow c_B$, where C_B explains B, and the direction of the prior association is reversed, from effects to causes: "the analogical argument is meant to provide support for the idea that similar features (…) in the target domain [A] are explained by a similar hypothesis [C_A]" [4: 25]. This abductive (explanatory) analogy involves a transfer of plausible causal knowledge, which has the following scheme:

$$\frac{\underset{\rightarrow}{C_B\, a\, C_A} \qquad C_M \rightarrow (C_B \rightarrow C_A)}{plausible\, C_M}$$

Thus, C_M is the *raison d'être* of the formation of C_A from C_B, all thanks to the detection or construction of the aspect M that causes the link between B and A.

[10] Cf. [4:13].

3.6. Association by Causality or Genesis of Abduction

In the previous associative step, a proportional type analogical transfer was carried out, which not only described the asymmetric similarity of *B* towards *A*, but allowed us to advance in some set of consequences C_A of such a relationship of similarity, by comparison with the inferred consequences C_B that had been obtained for *B*: something will be transferred from *B* to *A* transforming the initial problem *A* into a new situation, overshadowing the previous properties that it possessed, and that previously failed to lead it to its resolution. The salient property *M* allows now to reach to some eventually relevant information to solve the problem at hand.

The third and final associative stage is characterized in metaphorical terms, where a plausible answer is finally obtained for the original problem A: having made (i) an association *s* by similarity, which abducts feature *M* as a cause of similarity, and (ii) an association *a* by analogy, which abducts the effects C_M as their cause, now it would be possible to abduct C_A from *A*, as the analogy plausibly produces C_A. Indeed, identifying these new connections can lead to this novel hypothesis concerning a potential solution for *A*, in the format of a retroductive inference e_A^{-1}, an association of C_A effects to causes *A* (cf. Figure 1). Such inference $e_A^{-1}: C_A \longrightarrow A$ constitutes a metaphorical process, which associates two different elements C_A and C_B, which, however, have some point in contact and connection[11]. As many as their differences may seem, the metaphorical similarity relationship between them shortens this distance, creating an unexpected cognitive shortcut, a surprise. Of the three associations presented to describe a complex transductive argument, the latter e_A^{-1} is where abduction is most explicitly presented as a retroduction: "abduction is the inference of a cause from its effects" [29, *W1*:180[12]]: "I have on reflection decided to give this kind of reasoning the name of retroduction to imply that it turns back and leads from the consequent of an admitted consequence, to its antecedent" [28, *MS* 857: 5, n.d.].

$$\frac{\begin{array}{c} A \\ C_A \xrightarrow{e_A^{-1}} A \end{array}}{plausible\, C_A}$$

Once the hypothesis is put forth, in order to account for some surprising phenomenon, a commutative diagram is obtained. When we have completed this cycle of the commutative diagram, which begins in the problematic situation A and ends in its eventual hypothetical solution C_A, just then we are before what Peirce called a "reasoning from surprise to inquiry (…) a reasoning from consequent [our C_A] to antecedent [our *A*]" [28][13].

Thusly, the cycle presented in Figure 1 is the hypothesis generating process, which is implicit in Peirce or replaced by the commitment to the presence of a conjecture-forming instinct. We seek to replace such instinctive reduction by a whole process that triggers various associative mechanisms: first an association by similarity,

[11] Let's keep in mind that metaphor involves seeing something in terms of something else. Cf. [25].

[12] Citations of the *Writings of Charles Sanders Peirce. The Chronological Edition* are by year, volume, and page number.

[13] Unpublished draft, letter from Peirce to Victoria Welby, 16 July 1905, MS L 463.

secondly an association by contiguity and third, and finally, an association by cause and effect, which will allow the plausible hypothesis to be shaped, which would be the first *abductive* step of scientific inquiry according to Peirce. This would show a whole process hidden by an apparent instinct that would not require, according to Peirce, elucidation.

The above allows characterizing transductions as processes of origin of solutions to a problem, which are also carriers of the causes of such genesis. Therefore, the posed three stages of associative inferences lead to the formation of a hypothesis C_A. Now we are ready to introduce Peircean's abductive reasoning, bypassing its instinctive perspective, since, as Frankfurt says, "it is by such reasoning that we are led to adopt hypotheses for investigation (…) Hypotheses enter *serious thinking*[14] through abduction" [9: 595]. It would seem that Frankfurt values abduction to the extent that, as he presumably understands it, in Peirce, it has a different linguistic format than what it might mean to represent the instinctive stage. On the other hand, in our approach, not everything is linguistically representable in the transductive stage, but when it is, it is the result of the indications that could emerge to consciousness from non-conscious reasoning. Why not also regard as "serious thinking" the previous transductive process that introduces new ideas? "Seriousness" does not mean to reduce everything to linguistic format, as Frankfurt seems to believe. Seriousness should mean making explicit the tacit but also the relevant to inquiry, recognizing that mental life can be elucidated sometimes and in fragments, and that every effort to do so is commendable.

The above transductive method is tested on several examples: Plato's *Meno*'s first mathematical problem [46, 50], Paleo-Babylonian mathematics [48, 50], Hippocrates of Chios's mathematical heuristics [44, 51], and Jean-Baptiste Joseph Fourier's representations of functions [49], among others.

4 Conclusion

The expert-agential similarity approach fulfills the following requirements: [i] not only does it happen that an icon *B* similar to the target problem *A* is activated by a primarily non-conscious incubation process, of which the solution is known by means of C_B, suggesting an analogous tentative alternative of solution C_A of *A*; [ii] not only is it emphasized that the interpretation of an agent *S* is involved, the one using imagination, creating a fictional scenario icon *B* which is the vehicle of further reasoning, so that it makes features of the icon *B* visible, that are usually overlooked, and act imagining situations according to certain implicit rules within the icon *B*; [iii] not only does agent *S* formulate plausible hypotheses to specify similarities between icon *B* and the target system *A*; [iv] not only does agent *S* change the interpretation of *A* while specifying the icon similarity to *A* via the dual cognitive mechanism; [v] not only is similarity the starting primitive point, the 'beginning of the ball of yarn', that indicates that there is no *ex nihilo* creation, even if it is distant in linguistic terms but not in semiotic ones; but, in addition, the process departing from the evocative choice

[14] The italics are ours.

of an icon *B* by similarity with target problem *A* captures what is happening when the agent *S* uses this icon to explain the problem in *A* because of the expertise that *S* possesses to creatively choose *B*, and thus initiating the transductive process (evocation of *B*, asymmetric similarity from *B* to *A*, proportional choice of aspects *M* shared between *B* and *A*, analogy that extracts a plausible solution C_A for *A*, from a known solution C_B for *B*), which eventually offers a plausible solution C_A of *A*.

The consequences of what has been proposed here are that, in order to give a reliable characterization of a creative process that eventually allows us to present a conjecture or plausible hypothesis, it is necessary to accept that not everything can be stated objectively and that, in any case, when such a process is made explicit, it involves the presence of associations governed primarily by similarity, as debatable as this may be, given its high sensitivity to context. Consequently, everything is nuanced by subjective aspects, cognitive mechanisms that partially emerge to the consciousness of the resolver and that, by means of their expertise, acquire a degree of reliability despite their fallibility.

Despite having studied an important number of examples where the transductive methodology is properly applied, more cases can be analysed to gain in-depth understanding of other creative insights. This will lead us to include in future works, among other things, the following central question: how to reconstruct from a formal point of view the logical argument described in the commutative diagram, which schematically and iconically represents the three types of associations that make up a transduction. Carrying out this task implies, among other things, describing an analogical transfer or *paradeigma* based on the assumption of resemblance as a primitive, in terms of proportion-based reasoning [35].

References

[1] Aristotle. *On Rhetoric. A Theory of Civic Discourse. Second Edition*, ed. George A. Kennedy. Oxford University Press, New York, and Oxford, 2007.

[2] Aronson, Jerrold L., Rom Harré, and Eileen Cornell Way. *Realism Rescued*. Duckworth, London, 1993.

[3] Bailer-Jones, Daniela M. When Scientific Models Represent. *International Studies in the Philosophy of Science* 17(1): 59-74, 2003.

[4] Bartha, Paul F.A. *By Parallel Reasoning. The Construction and Evaluation of Analogical Arguments*. Oxford University Press, Oxford, 2010.

[5] Black, Max. Metaphor. *Proceedings of the Aristotelian Society*, New Series 55: 273-294, 1954.

[6] De Toffoli, Silvia, and Valeria Giardino. Forms and Roles of Diagrams in Knot Theory. *Erkenntnis* 79(4): 829-842, 2014.

[7] Eco, Umberto. *Theory of Semiotics*. Indiana University Press, Bloomington and London, 1976.

[8] Evans, Jonathan St. B. T., and Keith E. Stanovich. Dual-Process Theories of Higher Cognition: Advancing the Debate. *Perspectives on Psychological Science* 8(3): 223-241, 2013.

[9] Frankfurt, Harry G. Peirce's Notion of Abduction. *The Journal of Philosophy* 55(14): 593-597, 1958.
[10] Gammerman, Alex, Vladimir Vovk, and Vladimir Vapnik. Learning by Transduction. In *Uncertainty in Artificial Intelligence. Proceedings of the Fourteenth Conference*, eds. G.F. Cooper, and S. Moral, 148-155. Morgan Kaufmann, San Francisco, California, 1998.
[11] Giaquinto, Marcus. *Visual Thinking in Mathematics*. Oxford University Press, Oxford, 2007.
[12] Giere, Ronald. *Explaining Science*. University of Chicago Press, Chicago, 1988.
[13] Giere, Ronald. Using Models to Represent Reality. In *Model-Based Reasoning in Scientific Discovery*, eds. Magnani, Lorenzo and Nancy Nersessian, 41-57. Plenum Publishers, New York, 1999.
[14] Gigerenzer, Gerd, and Terry Regier. How do we tell an Association from a Rule? Comment on Sloman. *Psychological Bulletin* 119(1): 23-26, 1996.
[15] Gombrich, Ernst Hans Josef. *Art and Illusion. A Study in the Psychology of Pictorial Representation.* Princeton University Press, Princeton, 1960.
[16] Goodman, Nelson. *Languages of Art*: An Approach to a Theory of Symbols. Bobbs-Merrill, Indianapolis, 1968.
[17] Goodman, Nelson. Seven Strictures on Similarity. In *Problems and Projects*, Nelson Goodman, 437-446. Bobs-Merril, Indianapolis, 1972.
[18] Goodman, Nelson. *Languages of Art*: An Approach to a Theory of Symbols. Second Revised Edition. Hackett, Indianapolis, 1976.
[19] Gregory, Richard. *The Psychology of Seeing*. Weidenfeld and Nicolson, London, 1966.
[20] Hesse, Mary Brenda. *Models and Analogies in Science.* Sheed and Ward, New York, 1963.
[21] Hesse, Mary Brenda. *Models and Analogies in Science.* University of Indiana Press, Notre Dame, 1967.
[22] Kapitan, Tomis. Peirce and the Structure of Abductive Inference. In *Studies in the Logic of Charles Sanders Peirce*, eds. Nathan Houser, Don R. Roberts, and James Van Evra, 477-496. Indiana University Press, Bloomington and Indianapolis, 1997.
[23] Keren, Gideon, and Yaacov Schul. Two is not Always Better than One: A Critical Evaluation of Two-System Theories. *Perspectives on Psychological Science* 4(6): 533-550, 2009.
[24] Knuuttila, Tarja. Models, Representation, and Mediation. *Philosophy of Science* 72: 1260-1271, 2005.
[25] Lakoff, George, and Mark Johnson. *Metaphors we live by*. University of Chicago Press, Chicago, 1980.
[26] Morgan, Mary S. and Margaret Morrison, eds. *Models as Mediators. Perspectives on Natural and Social Science*. Cambridge University Press, Cambridge, 1999.
[27] Osman, Magda. An Evaluation of Dual-Process Theories of Reasoning. *Psychonomic Bulletin and Review* 11(6): 988-1010, 2004.
[28] Peirce, Charles Sanders. *Manuscripts in the Houghton Library of Harvard University as identified by Richard Robin*. University of Massachusetts Press., Amherst, 1967.

[29] Peirce, Charles Sanders. *Peirce Edition Project: Writings of Charles S. Peirce: A Chronological Edition. Volume 1: 1857-1866.* Indiana University Press, Bloomington, 1982.
[30] Peirce, Charles Sanders. *The Collected Papers of Charles Sanders Peirce*, vol. 1-6, eds. Charles Harshorne & Peter Weiss. The Bleknap Press of Harvard University Press, Cambridge. Vol. 7-8, eds. A. Burks. The Bleknap Press, Cambridge, 1931/1958.
[31] Pietarinen, Ahti-Veikko, and Francesco Bellucci. New Light on Peirce's Conceptions of Retroduction, Deduction, and Scientific Reasoning. *International Studies in the Philosophy of Science* 28(4): 353-373, 2014.
[32] Pólya, George. *How to Solve It.* Princeton University Press, Princeton, 1945.
[33] Pólya, George. *Mathematics and Plausible Reasoning: Induction and Analogy in Mathematics.* Princeton University Press, Princeton, 1954.
[34] Poznic, Michael. Representation and Similarity: Suárez on Necessary and Sufficient Conditions of Scientific Representation. *Journal for General Philosophy of Science/Zeitschrift für Allgemeine Wissenschaftstheorie* 47(2):331-347, 2016.
[35] Prade, Henri, and Gilles Richard. Analogical Proportions: From Equality to Inequality. *International Journal of Approximate Reasoning* 101: 234-254, 2018.
[36] Salmon, Wesley. Four Decades of Scientific Explanation. *Minnesota Studies in the Philosophy of Science* 13: 3-219, 1989.
[37] Suárez, Mauricio. Kinds of Models. In *Model Validation in Hydrological Science*, eds. P. Bates and M. Anderson, 11-21. John Wiley, 2001.
[38] Thagard, Paul. How Brains Make Mental Models. In *Model-Based Reasoning in Science and Technology: Abduction, Logic, and Computational Discovery*, eds. Lorenzo Magnani, Walter A. Carnielli, and Claudio Pizzi, 447-461. Springer-Verlag, Berlin, 2010.
[39] Toon, Adam. Models as Make-Believe. In *Beyond Mimesis and Convention: Representation in Art and Science*, ed. Roman Frigg and Matthew C. Hunter, 71-96. Springer, New York, 2010.
[40] Toon, Adam. *Models as Make-Believe: Imagination, Fiction and Scientific Representation*. Palgrave Macmillan, Basingstoke, 2012a.
[41] Toon, Adam. Similarity and Scientific Representation. *International Studies in the Philosophy of Science* 26(3): 241-257, 2012b.
[42] Visokolskis, Sandra. El fenómeno de la transducción en la matemática. Metáforas, analogías y cognición. In *La metáfora en la educación. Descripción e implicaciones*, eds. Marcel Pochulu, Raquel Abrate, and Sandra Visokolskis, 37-53. Eduvim, Villa María, 2009.
[43] Visokolskis, Sandra, and Gonzalo Carrión. Asociación de ideas en Peirce: de la psicología a la lógica. In *Actas de IV Jornadas "Peirce en Argentina"*, ed. Catalina Hynes, Buenos Aires, 2010. URL: http://www.unav.es/gep/IVPeirceArgentinaVisokolskisCarrion.html
[44] Visokolskis, Sandra. *La noción de análisis como descubrimiento en la historia de la matemática. Propuesta de un modelo de descubrimiento creativo*. PhD Doctoral Dissertation. National University of Córdoba, Córdoba, Argentina, 2016.

[45] Visokolskis, Sandra, and Gonzalo Carrión. Saltos cognitivos: asociaciones y abducción en Peirce. In *Actas de VII Jornadas "Peirce en Argentina"*, ed. Catalina Hynes, Buenos Aires, 2017. URL: http://www.unav.es/gep/JornadasPeirceArgentina.html
[46] Visokolskis, Sandra, and Gonzalo Carrión. Creative Insights: Dual Cognitive Processes in Perspicuous Diagrams. In *SetVR 2018, Set Visualization and Reasoning. Proceedings of International Workshop on Set Visualization and Reasoning*, Volume 2116, eds. Yuri Sato, and Zohreh Shams, 28-43. CEUR Workshop Proceedings, Edinburgh, United Kingdom, 2018.
[47] Visokolskis, Sandra, and Gonzalo Carrión. La noción de semejanza en Peirce desde un punto de vista inferencial. In *Actas de VIII Jornadas "Peirce en Argentina"*, ed. Catalina Hynes, Buenos Aires, 2019.URL: http://www.unav.es/gep/JornadasPeirceArgentina.html
[48] Visokolskis, Sandra. Estimación de una fuente mesopotámica plausible del método geométrico de análisis en la matemática griega antigua: posibles aportes. In *Estudios Interdisciplinarios de Historia Antigua, Volumen V*, eds. Cecilia Ames, M. Sagristani, and Agustín Moreno. Universidad Nacional de Córdoba, Córdoba., 2019a.
[49] Visokolskis, Sandra. Transduction: Peirce's Creative Aspects on Theorem Proving. In *Handbook of Abstracts. Creativity 2019*, eds. Jean-Yves Beziau, and Arthur Buchsbaum, 69-70. Paper presented at Workshop Nº 5, "Creativity and Logic in/of Charles Sanders Peirce", organized by Cassiano Terra Rodrigues, Sociedade Brasileira de Logica. Universidade Federal do Rio de Janeiro, Rio de Janeiro, Brazil, 2019b.
[50] Visokolskis, Sandra. Transductive Mechanisms of Creative Research in Mathematics: Three Case Studies. In *Handbook of Abstracts. Creativity 2019*, eds. Jean-Yves Beziau, and Arthur Buchsbaum, 127-128. Paper presented at Workshop Nº 11, "Mathematical Creativity", organized by Irina Starikova, Sociedade Brasileira de Logica. Universidade Federal do Rio de Janeiro, Rio de Janeiro, Brazil, 2019c.
[51] Visokolskis, Sandra, Vargas, Evelyn, and Gonzalo Carrión. Transductive Reconstruction of Hippocrates' Dynamical Geometrical Diagrams". In *Diagrams 2020, LNAI 12169, Diagrammatic Representation and Inference*, eds. Ahti-Veikko Pietarinen, Peter Chapman, Leonie Bosveld-de Smet, Valeria Giardino, James Corter, and Sven Linker, 1-16. Springer, Switzerland, 2020.
[52] Wallas, Graham. *The Art of Thought*. Harcourt, Brace and Company, New York, 1926.
[53] Walton, Kendall R. *Mimesis as Make-believe: On the Foundations of the Representational Arts.* Harvard University Press, Cambridge, Massachusetts, 1990.
[54] Weisberg, Michael. Who is a Modeler? *British Journal for Philosophy of Science* 58: 207-233, 2007.