# Coverage of the coronavirus pandemic through entropy measures

Vladimir N. Soloviev<sup>*a,b*</sup>, Andrii O. Bielinskyi<sup>*a*</sup> and Natalia A. Kharadzjan<sup>*a*</sup>

<sup>a</sup>Kryvyi Rih State Pedagogical University, 54 Gagarin Ave., Kryvyi Rih, 50086, Ukraine <sup>b</sup>Bohdan Khmelnitsky National University of Cherkasy, 81 Shevchenko Blvd., Cherkasy, 18031, Ukraine

#### Abstract

The rapidly evolving coronavirus pandemic brings a devastating effect on the entire world and its economy as a whole. Further instability related to COVID-19 will negatively affect not only on companies and financial markets, but also on traders and investors that have been interested in saving their investment, minimizing risks, and making decisions such as how to manage their resources, how much to consume and save, when to buy or sell stocks, etc., and these decisions depend on the expectation of when to expect next critical change. Trying to help people in their subsequent decisions, we demonstrate the possibility of constructing indicators of critical and crash phenomena on the example of Bitcoin market crashes for further demonstration of their efficiency on the crash that is related to the coronavirus pandemic. For this purpose, the methods of the theory of complex systems have been used. Since the theory of complex systems has quite an extensive toolkit for exploring the nonlinear complex system, we take a look at the application of the concept of entropy in finance and use this concept to construct 6 effective entropy measures: Shannon entropy, Approximate entropy, Permutation entropy, and 3 Recurrence based entropies. We provide computational results that prove that these indicators could have been used to identify the beginning of the crash and predict the future course of events associated with the current pandemic.

#### Keywords

coronavirus, Bitcoin, cryptocurrency, crash, critical event, measures of complexity, entropy, indicator-precursor

# 1. Introduction

The novel coronavirus outbreak (COVID-19) quickly became a catastrophic challenge for the whole world, and it would be even more of a "black swan" than the global financial crisis and Great recession of 2008-2009 [1]. While the governments are focused on saving lives and preventing the virus from spreading further, at the moment, it continuous to led to a substantial disruption of the world financial system. Furthermore, economic worries are also increasing dramatically. At first, people have worried about potentially become one of the infected. Then,

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<sup>☆</sup> vnsoloviev2016@gmail.com (V.N. Soloviev); krivogame@gmail.com (A.O. Bielinskyi); n.a.kharadzjan@gmail.com (N.A. Kharadzjan)

https://kdpu.edu.ua/personal/vmsoloviov.html (V.N. Soloviev);

https://kdpu.edu.ua/personal/nakharadzhian.html (N.A. Kharadzjan)

D 0000-0002-4945-202X (V.N. Soloviev); 0000-0002-2821-2895 (A.O. Bielinskyi); 0000-0001-9193-755X (N.A. Kharadzjan)

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when the situation escalated, people started to worry about the impact on their economic situation [2, 3, 4]. To prevent exponential growth of the disease incidence bans on traveling the world, visiting public places, and businesses were established that had particularly a destructive impact on financial markets, and the cryptocurrency market, showing that for most systems it was unexpectable. Subsequent days on the market promise to be more volatile and riskier than before. And it seems that Bitcoin is going to fall into a new Great recession along with the most influential stock indexes. As it was noted by Pier Francesco Procacci, Carolyn E. Phelan and Tomaso Aste [5], the crisis-state representative and eventually becomes extremely dominant in March, but a lot of factors have to be included to make an adequate analysis of this problem.

The doctrine of the unity of the scientific method states that for the study of events in socioeconomic systems, the same methods and criteria as those used in the study of natural phenomena are applicable. The increasing mathematical knowledge of complex structure provides us with such methods and tools that are interconnected and can be used to study so relevant and "almost unpredictable" situations as the coronavirus pandemic. By almost unpredictable, we mean that although past experience and data are not so relevant in this case, we can use some mathematical methods (models) that are useful in current situations, including current statistics, relevant parameters, and even the inner complexity of the system. As an example, Alexis Akira Toda [6] estimated the Susceptible-Infected- Recovered (SIR) epidemic model for COVID-19 because of its desire to help individuals in managing their investments. Governments also need help in making an informed decision on imposing travel restrictions, social distancing, closure of schools and businesses, etc., and, mainly, for how long [7]. In the paper of Bohdan M. Pavlyshenko [8] was studied different regression approaches for modeling COVID-19 spread and its impact on the stock market. The logistic curve model was used under Bayesian regression for predictive analytics of the coronavirus spread. The obtained results showed that different crises with different reasons have a different impact on the same stocks. Bayesian inference makes it possible to analyze the uncertainty of crisis. Michele Costola, Matteo Iacopini and Carlo R. M. A. Santagiustina [9] regarding the public concern of 6 countries during the outbreak of COVID-19 find that the Italian index in the current situation is relevant in explaining index returns for other studied countries. Assuming that COVID-19 has a deterministic, exogenous impact on the market, Karina Arias-Calluari, Fernando Alonso-Marroquin, Morteza Nattagh-Najafi and Michael Harré [10] forecast it with a model of the stochastic and systematic risk. Here, such a model is assumed to be q-Gaussian diffusion process which is accompanied by three spatio-temporal regimes. In this case, the results were achieved with 85% accuracy. Presented results are promising not only for markets but also for risk control in other areas such as seismology, communication networks, etc. In the paper [11] the bandwidth and the discount factor are proposed to minimize a criterion consistent with the traditional requirements of the validation of a probability density forecast. They use Kolmogorov-Smirnov statistic and a discrepancy statistic to build a quantitative criterion of the accuracy of pdf which they try to maximize when selecting the bandwidth and the discount factor of their time-varying pdf. Such an approach allows exposing an accurate chronology of the current pandemic. Ayoub Ammy-Driss and Matthieu Garcin [12] explore novel pandemic using two efficiency indicators: the Hurst exponent and the memory parameter of a fractional Lévy-stable motion. Presented results highlight the occurrence of inefficiency at the almost beginning of the crisis for US indices. Asian and Australian indices are seemed to be less affected during this period, i.e., their inefficiency is even questionable. A. Fronzetti Colladon, S. Grassi, F. Ravazzolo and F. Violante [13] present a new textual data index, which assesses the related most usable keywords and semantic network position, for predicting stock market data. They apply it to the Italian press and use it for predicting recent periods of Italian stock, including the COVID-19 crisis. According to the results, it can be observed that the index is characterized by strong predictability. Unfortunately, by the luck of observation and potentially diseased people that are surrounding us at the moment, it is still difficult to predict further dynamic during the pandemic.

As it can be observed, markets have seen significant numbers of investors selling off and rebalancing their portfolios with less risky assets. That has been leading to large losses and high volatilities, typical of crisis periods. The economy key for preventing such activity may lie in cryptocurrency and constructing effective indicators of possible critical states that will help investors and traders fill in safety. Bitcoin, which is associated with the whole crypto market, has such properties as detachment and independence from the standard financial market and the proclaimed properties that should make it serve as the 'digital gold' [14]. As was shown by Ladislav Kristoufek [15], Bitcoin promises to be a safe-haven asset with its low correlation with gold, S&P 500, Dow Jones Industrial Average, and other authoritative stock indices even in the extreme events. But authors please not overestimate the cryptocurrency since according to their calculations and, obviously, the current structure of the system, gold remains more significant. But for ten years, this token has been discussed by many people, it has experienced a lot in such a short period, many people believe in it, and it has managed to form a fairly complex and self-organized system. The integrated actions from real-world merge in such dynamics and relevant information that is encoded in Bitcoin's time series can be extracted [16, 17, 18]. In the context of volatile financial markets, it is important to select such measures of complexity that will be able to notify us of upcoming abnormal events in the form of crises at an early stage.

In this article, we:

- present such measures;
- study critical and crash phenomena that have taken place in the cryptocurrency market;
- try to understand whether a crash caused by the coronavirus pandemic could have been identified and predicted by such informative indicators or not.

According to our goals and actions, the paper is structured as follows. In Section 2, we presented a brief overview of the studies in this field of science. In Section 3, relying on these researches and the experience of other scientists, we present our classification of Bitcoin's crises for the period from 1 January 2013 to 9 April 2020. In Section 4, we describe the applied methods and present some empirical results with their subsequent description. Section 5 concludes.

### 2. Review of the previous studies

For its short history of existence, bitcoin has experienced many events, periodic rises and sudden dips in specific periods, regulatory actions, and discussions about whether it can become a universally mature commodity around the world or not, and therefore its largely unexplored dynamics is still presenting new opportunities and challenges for traders, economists, and researchers from different fields of science to highlight chaos that is hidden in it. Although officially bitcoin is considered to be a commodity rather than a currency, the comprehensive analysis of the applicability of the instruments which have been used for mature financial markets for a long time can be made for cryptocurrencies and, namely, for the Bitcoin market.

A vast amount of different methods, as an example, from the theory of complexity, the purpose of which is to quantify the degree of complexity of systems obtained from various sources of nature, can be applied in our study. Such applications have been studied intensively for an economic behavior system. As an example, Miguel Henry and George Judge [19] used an information theoretic-symbolic logic approach which is based on Shannon's information entropy and called *Permutation entropy* (PEn). The entropy is applied to the Dow Jones Industrial Average to extract information from this complex economic system. The result demonstrates the ability of the PEn method to detect the degree of disorder and uncertainty for the specific time that is explored. In such paper, [20] presented by Higor Sigaki, Matjaž Perc, and Haroldo Valentin Ribeiro, the PEn and statistical complexity over sliding time-window of daily closing price log-returns are used to quantify the dynamic efficiency of more than four hundred cryptocurrencies. Authors address to the efficient market hypothesis when the values of two statistical measures within a time-window cannot be distinguished from those obtained by chance. They find that 37% of the cryptocurrencies in their study stay efficient over 80% of the time, whereas 20% are informationally inefficient in less than 20% of the time. Moreover, the market capitalization is not correlated with their efficiency. Performed analysis of information efficiency over time reveals that different currencies with similar temporal patterns form four clusters, and it is seen that more young currencies tend to follow the trend of the most leading currencies.

Steve M. Pincus and Rudolf E. Kalman [21], considering both empirical data and models, including composite indices, individual stock prices, the random-walk hypothesis, Black- Sholes, and fractional Brownian motion models to demonstrate the benefits of *Approximate entropy* (ApEn), a quantitative measure of sequential irregularity. On the example of the applied models and empirical data, the authors presented that ApEn can be readily applied to the classical econometric modeling apparatus. This research the usefulness of ApEn on the example of three major events of the stock market crash in the US, Japan, and India. During the major crashes, there is significant evidence of a decline of ApEn during and pre-crash periods. Based on the presented results, their research concludes that ApEn can serve as a base for a good trading system. This article [22] gives evidence of the usefulness of Approximate Entropy. The researchers quantify the existence of patterns in evolving data series. In general, scientists observe that the degree of predictability increases in times of crisis. However, the presented results do not demonstrate that the regularity techniques studied in this paper can serve to predict an imminent crash.

Also, there is a paper [23] that is dedicated to Ethereum. Here, the concept of entropy is

applied for characterizing the nonlinear properties of the cryptocurrencies. For their goal, Shannon, Tsallis, Rényi, and Approximate entropies are estimated. From their empirical results, it is obtained that all entropies are positive. Of great interest is the results of ApEn which demonstrates larger value for Ethereum than for Bitcoin. In this case, it concludes that Ethereum has higher volatility. The same result for other measures. Daniel Traian Pele and Miruna Mazurencu [24] investigate the ability of several econometrical models to forecast value at risk for a sample of daily time series of cryptocurrency returns. Using high-frequency data for Bitcoin, they estimate the entropy of the intraday distribution of log-returns through the symbolic time series analysis (STSA), producing low-resolution data from high-resolution data. Their results show that entropy has strong explanatory power for the quantiles of the distribution of the daily returns. They confirm the hypothesis that there is a strong correlation between the daily logarithmic price of Bitcoin and the entropy of intraday returns Based on Christoffersen's tests for Value at Risk (VaR) backtesting, they conclude that the VaR forecast built upon the entropy of intraday returns is the best, compared to the forecasts provided by the classical GARCH models.

During our investigation, we found that many theses and papers that have been studied the dynamics of financial markets rather than cryptocurrencies using the measures from the theory of complexity. Thus, we are interested in contributing to the study of the dynamics of a potentially strong currency that affects the world economy and faces the same problems as even the most quoted financial markets. Thus, the construction of predictive models and measures of unexpectable and critical events in the cryptocurrency market remains relevant.

### 3. Data preparation and classification

The ecosystem of cryptocurrencies has been growing at an increasing pace. More and more of them become tradable and begin to inspire confidence for many people. Such events as the coronavirus threat that are presented to be unpredictable, break this confidence and the subsequent anxiety of traders shape the extent of economic worries and collapse.

Previously, we conducted research on the dynamics of both stock and cryptocurrency markets where were provided results confirming the effectiveness of using quantitative methods of the theory of complexity which can serve as a base for estimation of indicators-precursors of the critical states. This paper presents a comparative analysis of the measures based on the concept of entropy: Shannon, Approximate, Sample, Permutation, and Recurrence based entropies. Here, each entropy is applied to the entire market and separate bubble, crashes, and critical events that have taken place in this market. Thus, we advanced into action and set the tasks:

- Classification of such bubbles, critical events and crashes.
- Construction of such indicators that will predict crashes, critical events in order to give investors and ordinary users the opportunity to trade in this market.

At the moment, there are various research papers on what crises and crashes are and how to classify such interruptions in the market of cryptocurrencies. Taking into account the experience of previous researchers and our own [25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36],

we present our classification of such leaps and falls from the previous articles [28, 29, 30, 33] on the cryptocurrency topic, relying on Bitcoin time series during the period (01.01.2013 – 09.04.2020) of verifiable fixed daily values of the Bitcoin price (BTC) (https://finance.yahoo. com/cryptocurrencies).

For our classification, crashes are short, time-localized drops, with the strong losing of price per each day, which are formed as a result of the bubble. Critical events are those falls that could go on for a long period, and at the same time, they were not caused by a bubble. The bubble is increasing in the price of the cryptocurrency that could be caused by certain speculative moments. Therefore, according to our classification of the event with the number (1-3, 6-8, 12-15, 17) are the crashes, all the rest - critical events. More detailed information about crises, crashes, and their classification following these definitions are given in table 1.

#### Table 1

BTC HISTORICAL CORRECTIONS. LIST OF BITCOIN MAJOR CORRECTIONS  $\ge 20\%$  SINCE APRIL 2013

Nº	Name	Days in correction	Bitcoin High Price, \$	Bitcoin Low Price, \$	Decline, %	Decline, \$
1	08.04.2013-15.04.2013	8	230.00	68.36	70	161.64
2	04.12.2013-18.12.2013	15	1237.66	540.97	56	696.69
3	05.02.2014-25.02.2014	21	904.52	135.77	85	768.75
4	12.11.2014-14.01.2015	64	432.02	164.91	62	267.11
5	26.01.2015-31.01.2015	5	269.18	218.51	20	50.67
6	09.11.2015-11.11.2015	3	380.22	304.70	20	75.52
7	18.06.2016-21.06.2016	4	761.03	590.55	22	170.48
8	04.01.2017-11.01.2017	8	1135.41	785.42	30	349.99
9	03.03.2017-24.03.2017	22	1283.30	939.70	27	343.60
10	10.06.2017-15.07.2017	36	2973.44	1914.08	36	1059.36
11	31.08.2017-13.09.2017	13	4921.85	3243.08	34	1678.77
12	16.12.2017-22.12.2017	7	19345.49	13664.9	29	5680.53
13	13.11.2018-26.11.2018	13	6339.17	3784.59	40	2554.58
14	09.07.2019-16.07.2019	7	12567.02	9423.44	25	3143.58
15	22.09.2019-29.09.2019	7	10036.98	8065.26	20	1971.7
16	27.10.2019-24.11.2019	28	9551.71	7047.92	26	2503.79
17	06.03.2020-16.03.2020	11	9122.55	5014.48	45	4108.07

Accordingly, during this period in the Bitcoin market, many crashes and critical events shook it. Thus, considering them, we emphasize 17 periods on Bitcoin time series, whose falling we predict by our indicators, relying on normalized returns, where usual returns are calculated as:

 $G(t) = \ln x (t + \delta t) - \ln x (t) \approx \left[ x (t + \delta t) - x (t) \right] / x (t)$ 

and normalized returns as:

$$g(t) \cong [G(t) - \langle G \rangle] / \sigma,$$

where  $\sigma$  is a standard deviation of G,  $\delta t$  is a time lag (here  $\delta t = 1$ ), and  $\langle ... \rangle$  denotes the average over the period under study.

Further calculations were carried out within the framework of the algorithm of a moving window. For this purpose, the part of the time series (window), for which there were calculated measures of complexity, was selected, then the window of a length 100 was displaced along with the time series in a one-day increment and the procedure repeated until all the studied series had exhausted. Further, comparing the dynamics of the actual time series and the corresponding measures of complexity, we can judge the characteristic changes in the dynamics of the behavior of complexity with changes in the cryptocurrency. The key idea here is the hypothesis that the complexity of the system before the crashes and the actual periods of crashes must change. This should signal the corresponding degree of complexity if they are able to quantify certain patterns of a complex system. A significant advantage of the introduced measures is their dynamism, that is, the ability to monitor the change in time of the chosen measure and compare it with the corresponding dynamics of the output time series. This allowed us to compare the critical changes in the dynamics of the system, which is described by the time series, with the characteristic changes of concrete measures of complexity. It turned out that quantitative measures of complexity respond to critical changes in the dynamics of a complex system, which allows them to be used in the diagnostic process and prediction of future changes.

Moreover, using the measures already mentioned, we are trying to understand whether certain patterns or information were hidden in the market that could have enabled us to predict the current crisis that was caused by the coronavirus pandemic, or not. Therefore, in the next section, we give a brief description of the applied methods and present the empirical results.

For further analysis, the computations in a rolling window algorithm with a length of 100 and a step of 1 for the entire time series and local crashes we made. From the classification table, we selected 4 crashes with numbers 1, 3, and 7. This choice was due to the fact that the current crisis is essentially a crash according to our classification, and therefore for convenient presentation of complexity measures in further, we should select a few of any crashes, the nature of which in some sense would be similar to the main crash of the presented article. For the entire Bitcoin time series, each crash and the critical event will be marked by the arrow in accordance with the table. For local crashes, the beginning of each crash has to be indicated by our measures in the time of 100.

### 4. Related methods

Nowadays, the most important quantity that allows us to parameterize complexity in deterministic or random processes is entropy. Originally, it was introduced by Rudolf Clausius [37], in the context of classical thermodynamics, where according to his definition, entropy tends to increase within an isolated system, forming the generalized second law of thermodynamics. Then, the definition of entropy was extended by Boltzmann and Gibbs (BG) [38, 39], linking it to molecular disorder and chaos, to make it suitable for statistical mechanics, where they combined the notion of entropy and probability [40].

After the fundamental paper of Claude Elwood Shannon [41] in the context of information theory, its notion was significantly redefined. After this, it has been evolved along with different ways and successful enough used for the research of economic systems [42, 43, 44]. In

what follows, the theoretical background and empirical results for the most popular entropy measures are presented.

#### 4.1. Shannon entropy

*Shannon entropy* (ShEn) was proposed as a measure of uncertainty by its author – Claude Elwood Shannon in his famous paper "A Mathematical Theory of Communication" [41], where, at first, its purpose was to quantify the degree of 'lost information' in phone-line signals. His contribution has proved that this approach can be generalized for any series where probabilities exist. Comparatively to the entropy of Clausius and Boltzmann that were valid only for thermodynamic systems, it was significant progress. Formally, his approach can be briefly defined as the average amount of 'information' and 'uncertainty' encoded in patterns recorded from a signal, or message. Other interpretations refer to entropy as a measure of 'chaos' or disorder in a system. During decades the generalization of Shannon's entropy has been applied to various domains, particularly to the financial sector.

The general approach can be described as follows. Formally, we represent the underlying dynamic state of the system in probability distribution form P and then ShEn *S* with an arbitrary base (i.e. 2, *e*, 10) is defined as:

$$S[P] = -\sum_{i=1}^{N} p_i \log p_i,$$
 (1)

where  $p_i$  represents the probability that price *i* occurs in the sample's distribution of the Bitcoin time series, and *N* is the total amount of data in our system. When dealing with continuous probability distributions with a density function f(x), we can define the entropy as:

$$H(f) = -\int_{-\infty}^{+\infty} f(x) \log f(x).$$
<sup>(2)</sup>

According to the approach, the negative log increases with rarer events due to the information that is encoded in them (i.e. they surprise when they occur). Thus, when all  $p_i$ 's have the same value, i.e. where all values are equally probable, and S[P] reaches its minimum for more structured time series (events that are more certain). Equation 2 is obeyed to the same rules as discrete version of this method. In the figure 1 are the empirical results for ShEn, for the entire time series (1a), and local crashes (1b).



**Figure 1:** ShEn dynamics along with the entire time series of Bitcoin (a). The dynamics of ShEn for the local crashes (b) in accordance with the table 1

#### 4.2. Approximate entropy

To gain more detail analysis of the complex financial systems, it is known other entropy methods have become known, particularly, ApEn developed by Steve M. Pincus [45] for measuring regularity in a time series.

When calculating it, given N data points  $\{x(i) | i = 1, ..., N\}$  are transformed into subvectors  $\vec{X}(i) \in \Re^{d_E}$ , where each of those subvectors has  $[x(i), x(i+1), ..., x(i+d_E-1)]$  for each  $i, 1 \le i \le N-m+1$ . Correspondingly, for further estimations, we would like to calculate a probability of finding such patterns whose Chebyshev distance  $d[\vec{X}(i), \vec{X}(j)]$  does not exceed a positive real number r:

$$C_i^{d_E}(r) = (N - d_E + 1)^{-1} \sum_{j=1}^{N - d_E + 1} \mathcal{H}(r - d[\vec{X}(i), \vec{X}(j)])$$

where  $\mathcal{H}(\cdot)$  is the Heviside function which count the number of instances  $d[\vec{X}(i), \vec{X}(j)] \leq r$ .

Next, we estimate

$$F^{d_E}(r) = (N - d_E + 1)^{-1} \sum_{i=1}^{N - d_E + 1} \ln(C_i^{d_E}(r)),$$

and ApEn of a corresponding time series (for fixed  $d_E$  and r) measures the logarithmic likelihood that patterns that are close for  $d_E$  adjacent observations remain close on the next comparison:

$$ApEn(d_E, r, N) = F^{d_E}(r) - F^{d_E+1}(r).$$
(3)

If equation (3) is small, then we should expect high reqularity in our data (sequences remain close to each other), and extreme value of ApEn indicates independent sequential processes.

The calculation results for the full time series (2a) and the local ApEn values (2b) are presented in figure 2.



**Figure 2:** ApEn dynamics along with the entire time series of Bitcoin (a). The dynamics of ApEn for the local crashes (b) in accordance with the table 1

#### 4.3. Permutation entropy

PEn, according to the previous approach, is a complexity measure that is related to the fundamental Information theory and entropy proposed by Shannon. Such a tool was proposed by C. Bandt and B. Pompe [46], which is characterized by its simplicity, computational speed that does not require some prior knowledge about the system, strongly describes nonlinear chaotic regimes. Also, it is characterized by its robustness to noise [47, 48] and invariance to nonlinear monotonous transformations [49]. The combination of entropy and symbolic dynamics turned out to be fruitful for analyzing the disorder for the time series of any nature without losing their temporal information. According to this method, we need to consider "ordinal patterns" that consider the order among time series and relative amplitude of values instead of individual values. For evaluating PEn, at first, we need to consider a time series {x(i) | i = 1, ..., N} which relevant details can be "revealed" in  $d_E$ -dimensional vector

$$\dot{X}(i) = [x(i), x(i + \tau), ..., x(i + (d_E - 1)\tau)],$$

where  $i = 1, 2, ..., N - (d_E - 1)\tau$ , and  $\tau$  is an embedding delay of our time delayed vector. After it, we map By the ordinal pattern, related to time *t*, we consider the permutation pattern  $\pi_l(t) = (k_0, k_1, ..., k_{d_{E-1}})$  where  $1 \le l \le m!$  if the following condition is satisfied:

$$x(j + k_0\tau) \le x(j + k_1\tau) \le \dots \le x(j + k_{d_F-1}\tau).$$
(4)

Then, regarding the probability distribution P of each ordinal pattern, we finally define the *normalized permutation entropy* as:

$$E_{s}[P] = \frac{-\sum_{l=1}^{d_{E}!} p_{l} \ln p_{l}}{\ln d_{E}!},$$
(5)

where  $p_l$  is the relative frequency of each ordinal pattern.

The same idea has the permutation entropy. With the much lower entropy value, we get a more predictable and regular sequence of the data. Therefore, PEn gives a measure of the departure of the time series from a complete noise and stochastic time series.

There must be predefined appropriate parameters on which PEn relying, namely, the embedding dimension  $d_E$  is a paramount of importance because it determines  $d_E$  possible states for the appropriate probability distribution. With small values such as 1 or 2, parameter  $d_E$ will not work because there are only few distinct states. Furthermore, for obtaining reliable statistics and better detecting the dynamic structure of data,  $d_E$  should be relevant to the length of the time series or less [50]. For our experiments,  $d_E \in \{3, 4\}$  and  $\tau \in \{2, 3\}$  indicate the best results. Hence, in figure 3 we can observe the empirical results for PEn where it serves as indicator-precursor of the possible unusual states.



**Figure 3:** PEn dynamics along with the entire time series of Bitcoin (a). The dynamics of PEn along with the local crashes (b) in accordance with the table 1

#### 4.4. Recurrence based entropies

The corresponding measure of entropy is related to the recurrence properties that may be peculiar for the nonlinear complex system. A method that allows us to visualize and understand the recurrence behavior of the system is the *recurrence plot* (RP). A recurrence plot reflects the binary similarities of pairs of vectors in phase space. In fact, we visually represent the reconstructed  $d_E$ -dimensional phase space of the system according to *Takens' theorem* [51] by a square similarity matrix that can be defined as:

$$R_{i,j} = \mathcal{H}(\epsilon - \|\vec{\mathbf{X}}(i) - \vec{\mathbf{X}}(j)\|),$$

where i, j = 1, ..., N;  $\epsilon$  is a threshold that determines the similarity of two probably neighborhood trajectories, and previously mentioned  $\mathcal{H}$  is a Heaviside function that represents an answer in binary form; || || is the Chebyshev distance, and N is the size of the analyzed data. If we keep the fixed radius condition and use  $L_{\infty}$ -norm, as a result, the binary matrix captures a total  $N^2$  similarity values. Further recurrence plot is the representation of similar vector pairs that are represented by black dots, whereas cells referring to dissimilar pairs of vectors are presented by white dots.

Such representation of the systems can be, certainly, useful. But it has a limitation. As an example, with the increasing size of a plot, it is hard to display it graphically as a whole. It has to be resized, which in turn will lead to new artifacts and distortion of the patterns. These types of plots may cause incorrect interpretation.

However, to enable an objective assessment, the graphical representation of RP allows us to derive qualitative characterizations of the dynamical systems within a recurrence plot. For the quantitative description of the dynamics, the small-scale patterns in the RP can be used, such as diagonal and vertical lines. The histograms of the lengths of these lines are the base of the *recurrence quantification analysis* (RQA) [52, 53, 54].

A large number of different approaches have been developed to obtain as much as possible information about the nature of the studied phase space. An important class of recurrence quantifiers is those that try to capture the level of complexity of a signal. In accordance with this study, the *entropy diagonal line histogram* (DLEn) is of the greatest interest which uses the Shannon entropy of the distribution of diagonal lines P(l) to determine the complexity of the diagonal structures within the recurrence plot. One of the most know quantitative indicators of the recurrence analysis can be defined as:

$$DLEn = -\sum_{l=l_{min}}^{l=l_{max}} p(l) \log p(l)$$

and

$$p(l) = \frac{P(l)}{\sum_{l=l_{min}}^{N} P(l)},$$

where p(l) captures the probability that the diagonal line has the exactly length l, and DLEn reflects the complexity of deterministic structure in the system. Further calculations were provided and presented in figure 4 for both Bitcoin time series 4a and its several local crashes 4b.



**Figure 4:** DLEn dynamics along with the entire time series of Bitcoin (a). The dynamics of DLEn for the local crashes (b) in accordance with the table 1

However, as follows from the analysis of the entropy indicators, the results may differ for different data preparation. Thus, further in the paper, we take into account two types of en-

tropy based on the general Shannon's approach: *recurrence period density entropy* (RPDEn) and *recurrence entropy* (RecEn).

The RPDEn is the quantitative measure of the recurrence analysis that is useful for characterizing the periodicity or absolutely random processes in the time series. It is useful for quantifying the degree of repetitiveness [55, 56]. Considering embedded data points  $\vec{X}(i)$  and suitable threshold  $\epsilon$  in  $d_E$ -dimensional space, we are following forward in time until it has left this ball of radius  $\epsilon$ . Subsequently, the time j at which the trajectory first returns to this ball is recorded, and the time difference T of these two states is recorded. The procedure is repeated for all states of the embedded vector, forming a histogram of recurrence times R(T). The histogram is then normalized to give the *recurrence time probability density*:

$$P(T_i) = \frac{R(T_i)}{\sum_{i=1}^{T_{max}} R(T_i)},$$

where  $T_{max} = \max \{T_i\}$ . The normalized entropy of the obtained density can be defined as:

$$RPDEn = \frac{-\sum_{i=1}^{T_{max}} P(T_i) \ln P(T_i)}{\ln T_{max}}.$$
(6)

In fact, based on the length of the sequences of neighboring points in the phase space: the more points are neighborhoods, the lower the value of the entropy according to equation (6). The comparing of RPDEn and the Bitcoin's critical states can be seen in figure 5.



**Figure 5:** RPDEn dynamics along with the entire time series of Bitcoin (a). The dynamics of RPDEn for the local crashes (b) in accordance with the table 1

However, recent articles [57, 58] present a slightly different technique for calculating recurrent entropy using a novel way to extract information from the recurrence matrix. To properly define it, we need to define the microstates  $F(\epsilon)$  for the RP that are associated with features of the dynamics of the time series. Selecting the appropriate metric and using the Heaviside function, we evaluate the matrices of dimension  $N \times N$  that are sampled from the RP. The total number of microstates for a given N is  $N_{ms} = 2^{N^2}$ . The microstates are populated by  $\overline{N}$  random

samples obtained from the recurrence matrix such that  $\overline{N} = \sum_{i=1}^{N_{ms}} n_i$ , where  $n_i$  is the number of times that a microstate *i* is observed.

The probability of occurrence of the related microstate *i* can be obtained as  $P_i = n_i \cdot (\overline{N})$ . The RecEn of the RP associated with the probability distribution of the corresponding microstates is given by the following equation:

$$RecEn = \sum_{i=1}^{N_{ms}} P_i \ln P_i.$$
<sup>(7)</sup>

In figure 6 we can see the performance of RecEn accordingly to the described above method.



**Figure 6:** RecEn dynamics along with the entire time series of Bitcoin (a). The dynamics of RecEn for the local crashes (b) in accordance with the table 1

# 5. Conclusions

Definitely, the situation with coronavirus is of paramount importance and is of significant danger. From the literature overview, we have understood that the peak and ultimate duration of the outbreak is going to be undetermined for a long time. Yesterday's, today's, and tomorrow's events associated with this pandemic will not disappear without a trace, but will also affect the fate of both individuals and the States in which we live in the long term.

In order to give reliable, powerful, and simple indicators- precursors that are able to minimize further losses as a result of critical changes, we addressed the theory of complexity and the methods of nonlinear dynamics that can identify special trajectories in the complex dynamics and classify them.

The obtained quantitative methods were applied to classified crashes of the Bitcoin market, where it was seen that these indicators can be used to protect yourself from the upcoming critical change. Due to the nature of some crashes and critical events, at any moment there does not exist such a predictive model that could foreshadow them in advance. Thus, the most understandable crashes were selected along with the last critical fall that was caused by COVID-19 due to the desire to understand whether this crash was predictable or not. We

tested 6 measures of complexity on the familiar and past crises, along with the latest ongoing crash, to demonstrate their effectiveness. According to our result, it can be said that general Shannon entropy might be a robust indicator of crashes and critical events not only in the stock market but also in the cryptocurrency market, whereas 5 other measures can serve as efficient harbingers of these events.

Apparently, the impact of the pandemic was reflected in the cryptocurrency market, and therefore, the beginning of the subsequent crisis could be predicted using the appropriate indicators of the theory of complexity. In our further studies, we are going to continue exploring and analyzing other methods from the theory of complexity, and particularly, make research on the fields of artificial intelligence, machine learning, and deep learning [59, 60, 61, 62, 63, 64, 65].

## References

- [1] J. Lonski, Coronavirus may be a black swan like no other, https://www.moodysanalytics.com/-/media/article/2020/ weekly-market-outlook-coronavirus-may-be-black-swan-like-no-other.pdf?source= news\_body\_link, 2020.
- [2] T. Fetzer, L. Hensel, J. Hermle, C. Roth, Coronavirus perceptions and economic anxiety, The Review of Economics and Statistics (2020) 1–36. doi:10.1162/rest\_a\_00946.
- [3] M. Feldkircher, F. Huber, M. Pfarrhofer, Measuring the effectiveness of us monetary policy during the covid-19 recession, 2020. arXiv: 2007.15419.
- [4] R. Cerqueti, V. Ficcadenti, Anxiety for the pandemic and trust in financial markets, 2020. arXiv:2008.01649.
- [5] P. F. Procacci, C. E. Phelan, T. Aste, Market structure dynamics during covid-19 outbreak, 2020. arXiv: 2003.10922.
- [6] A. A. Toda, Susceptible-infected-recovered (sir) dynamics of covid-19 and economic impact, 2020. arXiv:2003.11221.
- [7] R. Anderson, J. Heesterbeek, D. Klinkenberg, T. Hollingsworth, Comment how will country-based mitigation measures influence the course of the covid-19 epidemic?, The Lancet 395 (2020) 921–1010. doi:10.1016/S0140-6736(20)30567-5.
- [8] B. M. Pavlyshenko, Regression approach for modeling covid-19 spread and its impact on stock market, 2020. arXiv:2004.01489.
- [9] M. Costola, M. Iacopini, C. R. M. A. Santagiustina, Public concern and the financial markets during the covid-19 outbreak, 2020. arXiv: 2005.06796.
- [10] K. Arias-Calluari, F. Alonso-Marroquin, M. Nattagh-Najafi, M. Harré, Methods for forecasting the effect of exogenous risk on stock markets, 2020. arXiv: 2005.03969.
- [11] M. Garcin, J. Klein, S. Laaribi, Estimation of time-varying kernel densities and chronology of the impact of COVID-19 on financial markets, Working Papers hal-02901988, HAL, 2020. URL: https://ideas.repec.org/p/hal/wpaper/hal-02901988.html.
- [12] A. Ammy-Driss, M. Garcin, Efficiency of the financial markets during the COVID-19 crisis: time-varying parameters of fractional stable dynamics, Working Papers hal-02903655, HAL, 2020. URL: https://ideas.repec.org/p/hal/wpaper/hal-02903655.html.

- [13] A. F. Colladon, S. Grassi, F. Ravazzolo, F. Violante, Forecasting financial markets with semantic network analysis in the covid-19 crisis, 2020. arXiv:2009.04975.
- [14] N. Courtois, M. Grajek, R. Naik, Optimizing sha256 in bitcoin mining, Communications in Computer and Information Science 448 (2014) 131–144. doi:10.1007/ 978-3-662-44893-9\_12.
- [15] L. Kristoufek, Grandpa, grandpa, tell me the one about bitcoin being a safe haven: New evidence from the covid-19 pandemic, Frontiers in Physics 8 (2020) 296. URL: https:// www.frontiersin.org/article/10.3389/fphy.2020.00296. doi:10.3389/fphy.2020.00296.
- [16] D. Broomhead, G. P. King, Extracting qualitative dynamics from experimental data, Physica D: Nonlinear Phenomena 20 (1986) 217 – 236. URL: http:// www.sciencedirect.com/science/article/pii/016727898690031X. doi:https://doi.org/ 10.1016/0167-2789(86)90031-X.
- [17] M. Rajkovic, Extracting meaningful information from financial data, Physica A: Statistical Mechanics and its Applications 287 (2000) 383–395. doi:10.1016/S0378-4371(00) 00377-0.
- [18] V. Ponomarenko, M. Prokhorov, Extracting information masked by the chaotic signal of a time-delay system, Physical review. E, Statistical, nonlinear, and soft matter physics 66 (2002) 026215. doi:10.1103/PhysRevE.66.026215.
- [19] M. Henry, G. Judge, Permutation Entropy and Information Recovery in Nonlinear Dynamic Economic Time Series, Econometrics 7 (2019) 1–16. URL: https://ideas.repec.org/ a/gam/jecnmx/v7y2019i1p10-d213039.html.
- [20] H. Sigaki, M. Perc, H. Valentin Ribeiro, Clustering patterns in efficiency and the comingof-age of the cryptocurrency market, Scientific Reports 9 (2019) 1440. doi:10.1038/ s41598-018-37773-3.
- [21] S. Pincus, R. E. Kalman, Irregularity, volatility, risk, and financial market time series, Proceedings of the National Academy of Sciences 101 (2004) 13709–13714. URL: https: //www.pnas.org/content/101/38/13709. doi:10.1073/pnas.0405168101.
- [22] A. Delgado-Bonal, Quantifying the randomness of the stock markets, Scientific Reports 9 (2019). doi:10.1038/s41598-019-49320-9.
- [23] S. Çalik, K. A. Metin, Entropy approach for volatility of wind energy, Thermal Science 23 (2019) 1863–1874.
- [24] D. T. Pele, M. Mazurencu, Using high-frequency entropy to forecast bitcoin's daily value at risk, Entropy 21 (2019) 102. doi:10.3390/e21020102.
- [25] A. Belinskyi, V. Soloviev, S. Semerikov, V. Solovieva, Detecting stock crashes using levy distribution, in: A. Kiv, S. Semerikov, V. Soloviev, L. Kibalnyk, H. Danylchuk, A. Matviychuk (Eds.), Proceedings of the 8th. International Conference on Monitoring, Modeling & Managment of Emergent Economy, volume 2422 of *POPL '79*, CEUR Workshop Proceedings, Odessa, Ukraine, 2019, pp. 226–236. doi:10.1145/567752.567774.
- [26] A. Bielinskyi, S. Semerikov, V. Solovieva, V. Soloviev, Levy's stable distribution for stock crash detecting, SHS Web Conf. 65 (2019) 06006. URL: https://doi.org/10.1051/shsconf/ 20196506006. doi:10.1051/shsconf/20196506006.
- [27] V. Derbentsev, S. Semerikov, O. Serdyuk, V. Solovieva, V. Soloviev, Recurrence based entropies for sustainability indices, E3S Web Conf. 166 (2020) 13031. URL: https://doi.org/ 10.1051/e3sconf/202016613031. doi:10.1051/e3sconf/202016613031.

- [28] V. Soloviev, A. Belinskiy, Complex Systems Theory and Crashes of Cryptocurrency Market: 14th International Conference, ICTERI 2018, Kyiv, Ukraine, May 14-17, 2018, Revised Selected Papers, 2019, pp. 276–297. doi:10.1007/978-3-030-13929-2\_14.
- [29] V. Soloviev, A. Belinskij, Methods of nonlinear dynamics and the construction of cryptocurrency crisis phenomena precursors, in: V. Ermolayev, M. C. Suárez-Figueroa, V. Yakovyna, V. Kharchenko, V. Kobets, H. Kravtsov, V. Peschanenko, Y. Prytula, M. Nikitchenko, A. Spivakovsky (Eds.), Proceedings of the 13th International Conference on ICT in Education, Research and Industrial Applications. Integration, Harmonization and Knowledge Transfer, volume 2104, CEUR Workshop Proceedings, Kyiv, Ukraine, 2018, pp. 116–127.
- [30] V. Soloviev, O. Serdiuk, Quantum econophysical precursors of cryptocurrency crashes, Cherkasy University Bulletin: Applied Mathematics. Informatics (2020) 3–16. doi:10. 31651/2076-5886-2019-1-3-16.
- [31] V. Soloviev, A. Bielinskyi, V. Solovieva, Entropy analysis of crisis phenomena for djia index, in: V. Ermolayev, F. Mallet, V. Yakovyna, V. Kharchenko, V. Kobets, A. Korniłowicz, H. Kravtsov, M. Nikitchenko, S. Semerikov, A. Spivakovsky (Eds.), Proceedings of the 15th International Conference on ICT in Education, Research and Industrial Applications. Integration, Harmonization and Knowledge Transfer, volume 2393, CEUR Workshop Proceedings, Kherson, Ukraine, 2019, pp. 434–449.
- [32] V. Soloviev, O. Serdiuk, S. Semerikov, O. Kohut-Ferens, Recurrence entropy and financial crashes, in: Proceedings of the 2019 7th International Conference on Modeling, Development and Strategic Management of Economic System (MDSMES 2019), Atlantis Press, 2019/10, pp. 385–388. URL: https://doi.org/10.2991/mdsmes-19.2019.73. doi:https: //doi.org/10.2991/mdsmes-19.2019.73.
- [33] V. Soloviev, S. Semerikov, V. Solovieva, Lempel-ziv complexity and crises of cryptocurrency market, in: Proceedings of the III International Scientific Congress Society of Ambient Intelligence 2020 (ISC-SAI 2020), Atlantis Press, 2020, pp. 299–306. URL: https://doi. org/10.2991/aebmr.k.200318.037. doi:https://doi.org/10.2991/aebmr.k.200318. 037.
- [34] V. Soloviev, A. Bielinskyi, O. Serdyuk, V. Solovieva, S. Semerikov, Lyapunov exponents as indicators of the stock market crashes, in: O. Sokolov, G. Zholtkevych, V. Yakovyna, Y. Tarasich, V. Kharchenko, V. Kobets, O. Burov, S. Semerikov, H. Kravtsov (Eds.), Proceedings of the 16th International Conference on ICT in Education, Research and Industrial Applications. Integration, Harmonization and Knowledge Transfer. Volume II: Workshops, Kharkiv, Ukraine, October 06-10, 2020, volume 2732 of *CEUR Workshop Proceedings*, CEUR-WS.org, 2020, pp. 455–470. URL: http://ceur-ws.org/Vol-2732/20200455.pdf.
- [35] V. Soloviev, V. Solovieva, A. Tuliakova, Visibility graphs and precursors of stock crashes, Neuro-Fuzzy Modeling Techniques in Economics (2019) 3–29. doi:10.33111/nfmte. 2019.003.
- [36] A. Bielinskyi, S. Semerikov, O. Serdyuk, V. Solovieva, V. Soloviev, L. Pichl, Econophysics of sustainability indices, CEUR Workshop Proceedings 2713 (2020) 372–392.
- [37] R. Clausius, The Mechanical Theory of Heat: With its Applications to the Steam-Engine and to the Physical Properties of Bodies, J. Van Voorst, 1867.
- [38] J. W. Gibbs, Elementary Principles in Statistical Mechanics: Developed with Especial Ref-

erence to the Rational Foundation of Thermodynamics, Cambridge Library Collection - Mathematics, Cambridge University Press, 2010. doi:10.1017/CB09780511686948.

- [39] L. Boltzmann, Weitere studien über das wärmegleichgewicht unter gasmolekülen, Wiener Berichte 66 (1872) 275–370.
- [40] C. Chakrabarti, I. Chakrabarty, Boltmann-shannon entropy: Generalization and application, Modern Physics Letters B 20 (2006) 1471–1479. URL: https://doi.org/10.1142/ S0217984906011529. doi:10.1142/S0217984906011529.
- [41] C. E. Shannon, A mathematical theory of communication, The Bell System Technical Journal 27 (1948) 379–423. doi:10.1002/j.1538-7305.1948.tb01338.x.
- [42] T. Constantino, Introduction to Nonextensive Statistical Mechanics: Approaching a Complex World, 1st ed., Springer-Verlag New York, 2009.
- [43] R. Sole, S. Valverde, Information Theory of Complex Networks: On Evolution and Architectural Constraints, volume 207, 2004, pp. 189–207. doi:10.1007/ 978-3-540-44485-5\_9.
- [44] J. S. Richman, J. R. Moorman, Physiological time-series analysis using approximate entropy and sample entropy, American Journal of Physiology-Heart and Circulatory Physiology 278 (2000) H2039-H2049. URL: https://doi.org/10. 1152/ajpheart.2000.278.6.H2039. doi:10.1152/ajpheart.2000.278.6.H2039. arXiv:https://doi.org/10.1152/ajpheart.2000.278.6.H2039, pMID: 10843903.
- [45] S. M. Pincus, Approximate entropy as a measure of system complexity, Proceedings of the National Academy of Sciences 88 (1991) 2297-2301. URL: https://www.pnas.org/content/88/6/2297. doi:10.1073/pnas.88.6.2297. arXiv:https://www.pnas.org/content/88/6/2297.full.pdf.
- [46] C. Bandt, B. Pompe, Permutation entropy: a natural complexity measure for time series., Physical review letters 88 17 (2002) 174102.
- [47] J. Amigó, Permutation Complexity in Dynamical Systems: Ordinal Patterns, Permutation Entropy and All That (Springer Series in Synergetics), 2010th ed., Springer, Reading, MA., 2010.
- [48] M. Zanin, L. Zunino, O. Rosso, D. Papo, Permutation entropy and its main biomedical and econophysics applications: A review, Entropy 14 (2012) 1553. doi:10.3390/e14081553.
- [49] H. Kantz, T. Schreiber, Nonlinear Time Series Analysis, 2 ed., Cambridge University Press, 2003. doi:10.1017/CB09780511755798.
- [50] R. Gu, Multiscale shannon entropy and its application in the stock market, Physica A: Statistical Mechanics and its Applications 484 (2017) 215 – 224. URL: http://www.sciencedirect.com/science/article/pii/S0378437117304740. doi:https: //doi.org/10.1016/j.physa.2017.04.164.
- [51] F. Takens, Detecting strange attractors in turbulence, volume 898, 1981, p. 366. doi:10. 1007/BFb0091924.
- [52] C. L. Webber, J. P. Zbilut, Dynamical assessment of physiological systems and states using recurrence plot strategies, Journal of Applied Physiology 76 (1994) 965–973. URL: https://doi.org/10.1152/jappl.1994.76.2.965. doi:10.1152/jappl.1994.76.2.965. arXiv:https://doi.org/10.1152/jappl.1994.76.2.965, pMID: 8175612.
- [53] J. P. Zbilut, C. L. Webber, Embeddings and delays as derived from quantifica-

tion of recurrence plots, Physics Letters A 171 (1992) 199 – 203. URL: http:// www.sciencedirect.com/science/article/pii/037596019290426M. doi:https://doi.org/ 10.1016/0375-9601(92)90426-M.

- [54] N. Marwan, N. Wessel, U. Meyerfeld, A. Schirdewan, J. Kurths, Recurrence-plot-based measures of complexity and their application to heart-rate-variability data, Physical Review E 66 (2002) 026702. doi:10.1103/PhysRevE.66.026702.
- [55] H. Rabarimanantsoa, L. Achour, C. Letellier, A. Cuvelier, J.-F. Muir, Recurrence plots and shannon entropy for a dynamical analysis of asynchronisms in noninvasive mechanical ventilation, Chaos (Woodbury, N.Y.) 17 (2007) 013115. doi:10.1063/1.2435307.
- [56] M. Little, P. Mcsharry, S. Roberts, D. Costello, I. Moroz, Exploiting nonlinear recurrence and fractal scaling properties for voice disorder detection, Biomedical engineering online 6 (2007) 23. doi:10.1186/1475-925X-6-23.
- [57] G. Corso, T. L. Prado, G. Z. dos S. Lima, S. R. Lopes, A novel entropy recurrence quantification analysis, 2017. arXiv:1707.00944.
- [58] S. Lopes, T. Prado, G. Corso, G. dos S. Lima, J. Kurths, Parameter-free quantification of stochastic and chaotic signals, Chaos, Solitons & Fractals 133 (2020) 109616. URL: http://www.sciencedirect.com/science/article/pii/S0960077920300151. doi:https: //doi.org/10.1016/j.chaos.2020.109616.
- [59] H. Jang, J. Lee, An empirical study on modeling and prediction of bitcoin prices with bayesian neural networks based on blockchain information, IEEE Access 6 (2018) 5427– 5437.
- [60] L. Alessandretti, A. ElBahrawy, L. M. Aiello, A. Baronchelli, Anticipating cryptocurrency prices using machine learning, Complexity 2018 (2018) 1–16. URL: http://dx.doi.org/10. 1155/2018/8983590. doi:10.1155/2018/8983590.
- [61] L. Zheng, H. He, Share price prediction of aerospace relevant companies with recurrent neural networks based on pca, 2020. arXiv: 2008.11788.
- [62] N. Gandal, H. Hałaburda, Can we predict the winner in a market with network effects? competition in cryptocurrency market, SSRN Electronic Journal (2016). doi:10.2139/ ssrn.2832836.
- [63] T. Guo, A. Bifet, N. Antulov-Fantulin, Bitcoin volatility forecasting with a glimpse into buy and sell orders, 2018 IEEE International Conference on Data Mining (ICDM) (2018). URL: http://dx.doi.org/10.1109/ICDM.2018.00123. doi:10.1109/icdm.2018.00123.
- [64] T. Guo, N. Antulov-Fantulin, Predicting short-term bitcoin price fluctuations from buy and sell orders, ArXiv abs/1802.04065 (2018).
- [65] A. Bielinskyi, I. Khvostina, A. Mamanazarov, A. Matviychuk, S. Semerikov, O. Serdyuk, V. Solovieva, V. Soloviev, Predictors of oil shocks. Econophysical approach in environmental science, IOP Conference Series: Earth and Environmental Science 628 (2021). doi:10.1088/1755-1315/628/1/012019, 8th International Scientific Conference on Sustainability in Energy and Environmental Science, ISCSEES 2020; Conference Date: 21 October 2020 Through 22 October 2020.