

Automatic Construction of (L-R)-functions by Experimental Data

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Abstract

An approach to the automatic construction of membership functions of fuzzy numbers and intervals is proposed. The construction of triangular, trapezoidal and Gaussian membership functions is considered. This approach is based on experimental data that reflect a person's representation of the classes' boundaries of numbers, approximately equal to a certain number. The proposed mechanisms also make it possible to directly specify fuzzy numbers of the (L-R) -type based on fuzzy linguistic statements. This makes it possible to use this approach when estimating the parameters of models, building fuzzy time series, forming databases, in fuzzy inference systems and information retrieval systems, as well as in many other applied aspects.

Keywords ¹

fuzzy numbers and intervals, membership function, fuzzy linguistic statements, automatic construction, experimental data, fuzzy modeling.

1. Introduction

Recently, fuzzy modeling has been one of the most active and promising areas of applied research in the field of management and decision making [1]. In fuzzy modeling, fuzzy numbers are most commonly used to represent fuzzy sets. They are the basis for building mathematical models using linguistic variables and performing arithmetic operations.

Arithmetic operations for fuzzy numbers and intervals can be defined using Zadeh's generalization principle. However, such operations are very laborious. Therefore, in practice, operations on fuzzy numbers and intervals of (L-R)-type have become widespread, the use of which reduces the amount of computation.

Fuzzy numbers and intervals of (L-R)-type are specified by the corresponding membership functions (MF). Moreover, the main difficulty is that the membership function is defined outside the theory of fuzzy sets and, therefore, its adequacy cannot be verified by means of this theory [2].

The construction of the MF of continuous fuzzy sets is mainly based on the use of some well-known analytical model of the membership function, with the subsequent adjustment of its parameters (manual or automatic) [3]. In situations where it is impossible to obtain all the necessary information from an expert, or when the problem under consideration is complex, automatic methods are more effective for adjusting the MF parameters, which implement the generation of membership functions based on statistical (experimental) data.

Currently, a large number of methods for automatic adjustment (modification) of MF parameters have been developed, each of which formulates its own requirements and justifications for the choice of such an approach [4-9]. So in [4], an overview of the most important methods of automatic construction of membership functions of both type 1 and interval type 2 is given, and the main characteristics of each approach are highlighted. In [5], an algorithm for automatic determination of the parameters of the membership functions for the input variables and the output result of a fuzzy inference system of the Mamdani type for solving approximation problems based on training examples is proposed. In [6,7], methods for setting the MF parameters using training methods and

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optimization methods (gradient method and genetic algorithms) are considered. The article [8] considers a method for automatically constructing membership functions of students' assessments, and in [9], a method for automatic construction fuzzy Gaussian membership functions and fuzzy rules based on training representative data using a histogram of each function is considered.

In these works, the automatic adjustment of MF parameters is mainly based on training and optimization methods. Moreover, the setting of parameters is understood as their adjustment during the operation of the system. At the same time, not enough attention has been paid to the issues of automatic determination of MF parameters at the stage of their construction.

The article proposes one of the possible approaches to the automatic construction of the MF of fuzzy numbers and intervals. This approach is based on experimental data that reflect the human representation of the values of the parameters of these functions.

2. Requirements for membership functions

A more or less substantiated construction of the MF is possible only if the semantic interpretation of this set is taken into account. Let us present the requirements for the form of MF, which take into account such an interpretation [10].

Let $T = \{T_i | i = \overline{1, n}\}$ be the set of basic terms of the linguistic variable $\langle \beta, T, X \rangle$; $\langle T_i, X, \tilde{C}_i \rangle$ is a fuzzy variable corresponding to the term $T_i \in T$; $\tilde{C}_i = \{(\mu_{\tilde{C}_i}(x)/x) | x \in X\}$; S_i is a support \tilde{C}_i . Let also $X \subseteq R^1$, $|T| = n$, $\inf_{x \in X} x = x_1$ and $\sup_{x \in X} x = x_2$. Let us arrange the set T in accordance with the expression

$$(\forall T_i \in T)(\forall T_j \in T)[(i > j) \Leftrightarrow (\exists x \in S_i)(\forall y \in S_j)(x > y)],$$

meaning that the term, which has a support located to the left, gets a lower number. Then any linguistic variable must satisfy the following conditions:

$$\mu_{\tilde{C}_1}(x_1) = 1, \mu_{\tilde{C}_n}(x_2) = 1, \quad (1)$$

$$(\forall T_i, T_{i+1} \in T)(0 < \sup_{x \in X} \mu_{\tilde{C}_i \cap \tilde{C}_{i+1}}(x) < 1), \quad (2)$$

$$(\forall T_i \in T)(\exists x \in X)(\mu_{\tilde{C}_i}(x) = 1), \quad (3)$$

$$(\forall \beta)(\exists x_1 \in R^1)(\exists x_2 \in R^1)((\forall x \in X)(x_1 < x < x_2)). \quad (4)$$

Let's comment on these expressions using Figure 1 for $n = 5$.

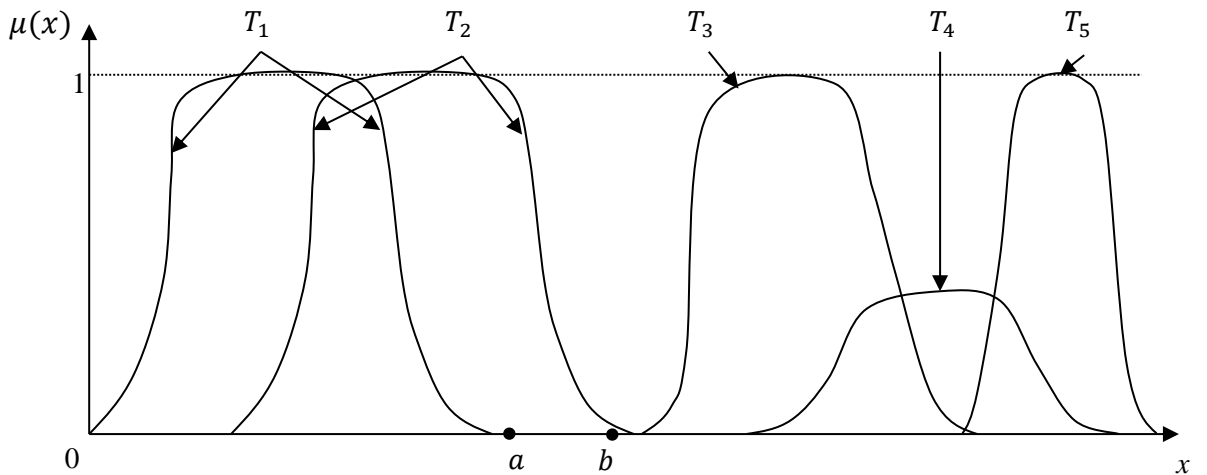


Figure 1: Forbidden basic terms of a linguistic variable

Condition (1) forbids the MF extreme terms (in this case, T_1 and T_5) to have the shape of bell-shaped curves, which is associated with the location of these terms in the ordered set T . Condition (2)

prohibits the presence of pairs of terms of the type T_1, T_2 and T_1, T_3 in the base set T , since in the first case there is no natural distinction between the concepts approximated by the terms, and in the second case, no concept corresponds to interval $[a, b]$ of the domain of definition. Since each concept has at least one typical object denoted by this concept, condition (3) prohibits the presence of terms of type T_4 in the set. Condition (4) limits the domain of X either to a finite set of points (with a discrete nature of the domain of definition) or to some segment or interval (with a continuous trend of the domain X). This condition sets physical restrictions on the numerical values of the parameters present in any control problem.

3. Membership functions of fuzzy numbers and intervals

From a linguistic point of view, a fuzzy number is a fuzzy quantity, interpreted as an imprecise, indefinite numerical value of some measurable quantity. Such numbers arise as a result of evaluating the parameters of the model, when there is no complete and accurate information about its characteristics.

Therefore, in conditions of uncertainty, it is psychologically easier for a person to give a fuzzy linguistic assessment to such parameters.

Fuzzy linguistic assessment is understood as a numerical assessment, which is expressed using the modality "approximately" [11].

Such estimates are expressed by statements of the form "the value of the parameter p is approximately equal to c " or "the value of the parameter is approximately in the range from c to d " and are represented, respectively, by fuzzy numbers and fuzzy intervals.

For the representation of fuzzy numbers and intervals in solving practical problems of fuzzy modeling, the greatest application is received by the triangular, trapezoidal and Gaussian membership functions [12].

They provide flexibility and simplicity of fuzzy models, as well as their good interpretability and adequacy. At the same time, when constructing these functions, certain difficulties arise with the determination of some of their parameters. So in triangular and trapezoidal functions (Figure 2) such parameters are fuzziness coefficients α and β , and in Gaussian functions (Figure 3: Gaussian membership functions: a) - standard; b) - combined (double)Figure) - distances b, b_1 and b_2 between their transition points.

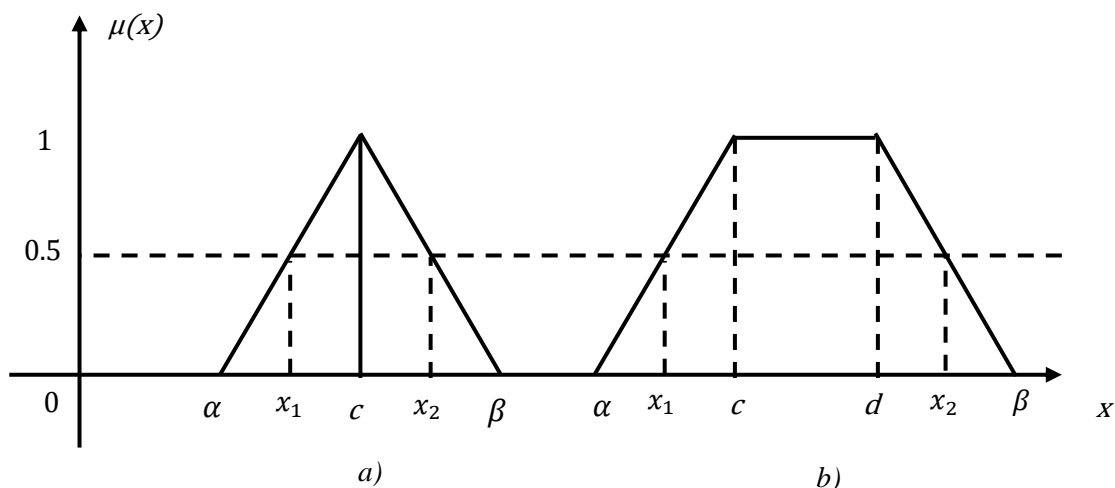


Figure 2: Membership functions: a) - triangular; b) – trapezoidal

These parameters are determined mainly by experts. Wherein, the task of psychological measurements is complicated by the fact that, as a rule, a person has uncertainty about the correctness of the estimates of these parameters.

This difficulty is to some extent eliminated by group expertise. However, carrying out such examinations is associated with significant costs (material and time). Given that the fuzziness coefficients of triangular and trapezoidal MF can be determined using the distance between their transition points, therefore, an approach can be used which excludes such examinations [13].

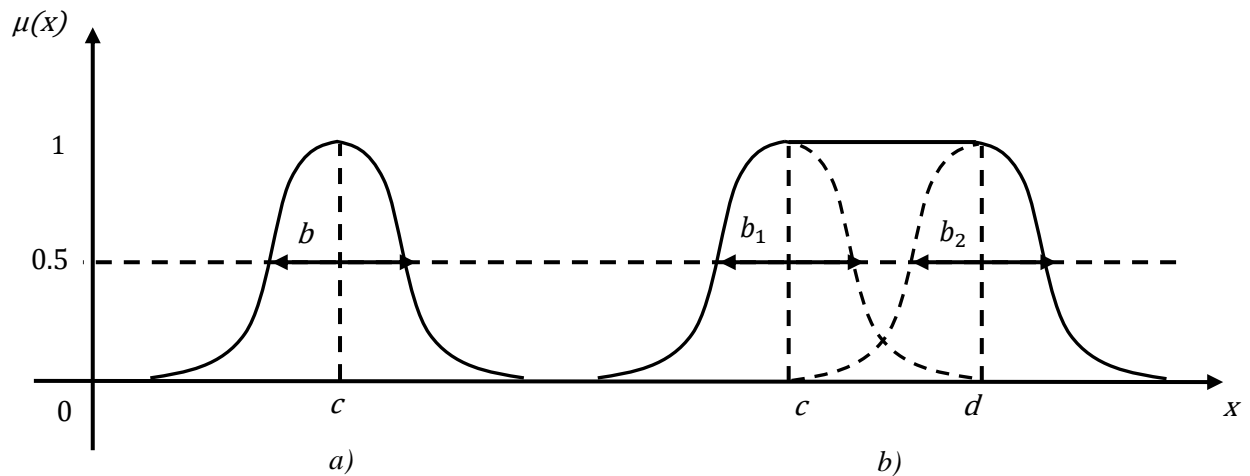


Figure 3: Gaussian membership functions: a) - standard; b) - combined (double)

4. Calculating the distance between transition points

To determine this distance, we use the algorithm considered in [13,14]. This algorithm is based on experimental data, which, according to experts, reflect the transition points for numbers approximately equal to T . Based on this data, formulas were obtained to calculate the distance between the transition points for each number $T \in [1, 99]$. The results are shown in Table 1 [13].

Table 1

The distance between the transition points

Number x	The distance between the transition points $b(x)$
1, 2, 3, 4, 6, 7, 8, 9	$0,46 x$
10, 20, 30, 40, 60, 70, 80, 90	$(0,357 - 0,00163x)x$
35, 45, 55, 65, 75, 85, 95	$(0,213 - 0,00067x)x$
5	2,8
15	6,48
25	6,75
50	24
Other two-digit numbers	$\frac{1}{2} (b(\left[\frac{x}{10}\right] \cdot 10 + 5) + b(x - \left[\frac{x}{10}\right] \cdot 10))$

Let a fuzzy linguistic quantity “the number is near the number T ” be given, where T is a natural number. If $T \in [1, 99]$, then $b(T)$ could be found according to the Table 1, in which $[a]$ is an integer part of the number a . Otherwise, let its least significant digit has an order of q . We divide the possible values into the residue classes modulo 3. As a result, we obtain three classes M_d , $d \in \{0, 1, 2\}$, where $d = q \bmod 3$. In this case the value $b(T)$ also depends on the class M_d , to which the number T belongs.

Let r_q be the numeral that is in the q th place of the number T . Then:

1. If $T \in M_0$ (for example, 300, 300000 etc.), then $b(T) = b(x) \cdot 10^{q-2}$, where $x = r_q \cdot 10$ and $b(x)$ is taken from Table 1.
2. If $T \in M_1$ (for example, 101, 202000, 15000 etc.), then two options are possible:
 - a) if $r_{q+1} = 0$, then $b(T) = b(x) \cdot 10^{q-1}$, where $x = r_q$;
 - b) if $r_{q+1} \neq 0$, then $b(T) = b(x) \cdot 10^{q-1}$ where $x = r_{q+1} \cdot 10 + r_q$.
3. If $T \in M_2$ (for example, 2030, 2140 etc.), then two options are also possible:
 - a) if $r_{q+1} = 0$, then $x = r_q \cdot 10$; $b(T) = b(x) \cdot 10^{q-2}$;
 - b) if $r_{q+1} \neq 0$, then $x = r_{q+1} \cdot 10 + r_q$; $b(T) = b(x) \cdot 10^{q-1}$;

As a result, the value $b(T)$ will be obtained. Then the transition points of the MF of the fuzzy set "the number is near the number T " are found from the relations $k_1 = T - \frac{b(T)}{2}$ and $k_2 = T + \frac{b(T)}{2}$. These values define the bounds of the confidence interval of numbers, approximately equal to the specified number T .

5. Construction of membership functions

5.1 Construction of triangular and trapezoidal functions

For triangular functions, firstly, the distance $b(c)$ is calculated and the transition points x_1 and x_2 are determined (Figure 2 a): $x_1 = c - \frac{b(c)}{2}$, $x_2 = c + \frac{b(c)}{2}$.

Then the equations of straight lines $\mu = f_1(x)$ and $= f_2(x)$ passing through points $(x_1, 0.5)$, $(c, 1)$ and $(c, 1)$, $(x_2, 0.5)$, respectively, are constructed.

After that, the coefficients α and β are calculated: $\alpha = f_1^{-1}(r)$, $\beta = f_2^{-1}(r)$, where $r = 0.01$. This number, as a rule, is taken as the value of the boundaries of the carrier of the considered fuzzy sets. As a result the MF

$$\mu_1(x) = \begin{cases} 0, & x < \alpha \\ \frac{x - \alpha}{c - \alpha}, & \alpha \leq x \leq c \\ \frac{\beta - x}{\beta - c}, & c \leq x \leq \beta \\ 0, & x > \beta \end{cases} \quad (5)$$

will be determined.

For trapezoidal functions, distances $b(c)$, $b(d)$ are also calculated and points x_1 and x_2 are determined (Figure 2 b): $x_1 = c - \frac{b(c)}{2}$, $x_2 = d + \frac{b(d)}{2}$.

Then the equations of straight lines $\mu = f_1(x)$ and $= f_2(x)$ passing through the points $(x_1, 0.5)$, $(c, 1)$ and $(d, 1)$, $(x_2, 0.5)$ are constructed, and the coefficients α and β are calculated similarly. In this case, the function

$$\mu_1(x) = \begin{cases} 0, & x < \alpha \\ \frac{x - \alpha}{c - \alpha}, & \alpha \leq x \leq c \\ 1, & c \leq x \leq d \\ \frac{\beta - x}{\beta - d}, & d \leq x \leq \beta \\ 0, & x > \beta \end{cases} \quad (6)$$

becomes defined.

5.2 Construction of Gaussian functions

The standard Gaussian function is used to define fuzzy sets $\tilde{A} \triangleq$ "the number is approximately equal to c ". We will use the Gaussian function of the form [13]:

$$\mu_{\tilde{A}}(x) = \exp(-a(x - c)^2), \quad (7)$$

where $\alpha = -\frac{4 \ln 0.5}{b^2(c)}$, and $b(c)$ is the distance between the transition points.

Such points are points like $c \pm \frac{b(c)}{2}$, that define the boundaries of the confidence interval of numbers approximately equal to c . Taking this into account, the fuzzy number that corresponds to this function is constructed as follows.

The Gaussian function has an unbounded support, since it tends to zero asymptotically on the left and right. However, in practice, the carrier of this function can be considered limited by points $x = c \pm 3\sigma$, at which its value is approximately equal to 0.01. Therefore, it can be assumed that the value of the function equal to 0.01 corresponds to the complete non-belonging of the element to the fuzzy set \tilde{A} . If we go from σ to $b(c)$ then the boundaries of this interval will be equal to $c \pm \frac{k \cdot b(c)}{2}$, where $k \approx 2.5$ is the scaling factor [15]. These boundaries will be the coefficients α and β of the fuzzy number $M_1 = (c, \alpha, \beta)$.

The combined function describes fuzzy set $\tilde{A} \triangleq$ "the number is approximately in the range from c to d ". This function has the form:

$$\mu_{\tilde{A}}(x) = \begin{cases} \mu_{\tilde{B}}(x), & x < c \\ 1, & c \leq x \leq d \\ \mu_{\tilde{C}}(x), & x > d \end{cases} \quad (8)$$

where $\mu_{\tilde{B}}(x)$ is the membership function of the fuzzy set $\tilde{B} \triangleq$ "the number is near the number c ", and $\mu_{\tilde{C}}(x)$ is the membership function of the fuzzy set $\tilde{C} \triangleq$ "the number is near the number d ". These functions are built in a similar way.

In this case, the transition points of function $\mu_{\tilde{A}}(x)$ are found from the relations $k_1 = c - \frac{b(c)}{2}$, $k_2 = d + \frac{b(d)}{2}$. These values determine the boundaries of the confidence interval of numbers, which are approximately in the range from c to d .

The fuzzy interval, described by this function, is constructed in a similar way. In this case, the indistinctness coefficients will be equal to $\alpha = c - \frac{k \cdot b(c)}{2}$ and $\beta = d + \frac{k \cdot b(d)}{2}$. As a result, we get a fuzzy interval $M_2 = (c, d, \alpha, \beta)$.

Setting task parameters in the form of a fuzzy interval is a convenient form for formalizing imprecise values. It is psychologically easiest to give a fuzzy interval assessment, and the carrier of a fuzzy interval is guaranteed to contain the value of the parameter under consideration.

At the same time, in problems of fuzzy modeling, arithmetic operations are easier to perform with fuzzy triangular numbers. It is possible to convert a fuzzy interval $M_2 = (c, d, \alpha, \beta)$ into a fuzzy number if we put $a = \frac{c+d}{2}$. Then the fuzzy number will look like $M = (a, \alpha, \beta)$.

Note, when it is necessary to estimate a certain parameter of the model or build a fuzzy time series, fuzzy numbers $M = (a, \alpha, \beta)$ and intervals $M = (c, d, \alpha, \beta)$ can be directly specified by fuzzy linguistic statements. In addition, such a construction of fuzzy numbers and intervals can be applied in the construction of object-oriented programs using fuzzy databases [16,17], as well as in determining the weighting coefficients of experts [18].

In conclusion, consider an example that illustrates this approach. Let it be necessary to construct a membership function of a fuzzy set $\tilde{A} \triangleq$ "the number is approximately in the range from 90 to 137". Let us construct a trapezoidal and combined Gaussian function.

To set the trapezoidal function (6), it is necessary to determine the parameters c, d, α, β . According to the set \tilde{A} we have $c = 90$ and $d = 137$. The value $b(90) \approx 32$ is determined according to the Table 1. Then $x_1 = 90 - 16 = 74$. After that, we determine the point x_2 . As $d > 99$, therefore this point will be calculated by the above algorithm.

The least significant digit of 137 is in the discharge of units ($q = 1$), therefore $r_q = r_1 = 7$, $r_{q+1} = r_2 = 3$ is a digit whose order is unit higher than the order of the least significant digit of the number 137. After dividing q by 3, the remainder gives 1, therefore, the number 137 belongs to the equivalence class M_1 .

Since r_{q+1} , then according to item 2b we have that $x = r_{q+1} \cdot 10 + r_q = r_2 \cdot 10 + r_1 = 37$ and $b(137) = b(37)$, and $b(37)$ is calculated by the formula

$$b(37) = \frac{1}{2} \left(b \left(\left[\frac{37}{10} \right] \cdot 10 + 5 \right) + b \left(37 - \left[\frac{37}{10} \right] \cdot 10 \right) \right) = \frac{1}{2} (b(35) + b(7)),$$

where $b(35)$ and $b(7)$ are found in Table 1: $b(35) = 6.63$ and $b(7) = 3.22$. Then $b(137) = \frac{1}{2} (6.63 + 3.22) \approx 5$, and $x_2 = 137 + 2.5 = 139.5$.

Then the equations of straight lines $\mu = f_1(x)$ and $\mu = f_2(x)$ passing through the points $(74, 0.5)$, $(90, 1)$ and $(139.5, 0.5)$, $(137, 1)$: $\mu = \frac{x-58}{32}$ and $\mu = \frac{142-x}{5}$. For $\mu = 0.01$ we obtain $x = 58.32 = \alpha$ and $x = 141.95 = \beta$, respectively. As a result, the MF will have the form:

]

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, & x \leq 58.32 \\ \frac{x-58}{32}, & 58.32 \leq x < 90 \\ 1, & 90 \leq x \leq 137, \\ \frac{142-x}{5}, & 137 < x \leq 141.95 \\ 0, & x \geq 142 \end{cases}$$

and the corresponding fuzzy interval $M = (90, 137, 58.32, 141.95)$.

Using the obtained data, the combined Gaussian function (8) of the fuzzy set \tilde{A} has the form:

$$\mu_{\tilde{A}}(x) = \begin{cases} \mu_{\tilde{B}}(x), & x < 90 \\ 1, & 90 \leq x \leq 137, \\ \mu_{\tilde{C}}(x), & x > 137 \end{cases},$$

where $\mu_{\tilde{B}}(x) = e^{-\frac{4 \ln 0.5 (x-90)^2}{32^2}}$, $\mu_{\tilde{C}}(x) = e^{-\frac{4 \ln 0.5 (x-137)^2}{5^2}}$ are the MF of fuzzy sets $\tilde{B} \triangleq$ "the number is about 90" and $\tilde{C} \triangleq$ "the number is about 137".

6. Construction of S- and Z-shaped functions

The considered approach can be used to construct S- and Z-shaped functions. We will consider only linear functions, since this approach can be used and for constructing quadratic S and Z membership functions.

S-functions are used to define fuzzy sets with the modalities "large, high, significant". The linear S-shaped membership function is as follows:

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, & x < \alpha \\ \left(\frac{x - \alpha}{\beta - \alpha}\right), & \alpha \leq x \leq \beta. \\ 1, & x > \beta \end{cases}$$

Z-functions are used to define fuzzy sets with the modalities "small, little, significant, low". The linear Z-shaped membership function has the form:

$$\mu_{\tilde{A}}(x) = \begin{cases} 1, & x < \beta \\ \left(\frac{\alpha - x}{\alpha - \beta}\right), & \beta \leq x \leq \alpha. \\ 0, & x > \alpha \end{cases}$$

In these formulas, parameters α and β set the boundaries of the carrier of the fuzzy set \tilde{A} , i.e. $\mu_{\tilde{A}}(\alpha) = 0.01$, $\mu_{\tilde{A}}(\beta) = 1$. It is assumed that the value of FP equal to 0.01 corresponds to the complete non-belonging of element $x = \alpha$ to set \tilde{A} . The graphs of these functions are similar to the left and right parts of the graph of the trapezoidal membership function (Figure . b).

Let it be necessary to construct an S-shaped membership function of a fuzzy set \tilde{A} . Since it is psychologically difficult for a person to directly set the numerical values of parameters α and β , therefore, the boundaries of the carrier \tilde{A} are given a fuzzy estimate "the carrier of the set \tilde{A} is approximately in the range from c to d ".

First, confidence intervals are found $[c_1, c_2]$, $[d_1, d_2]$ for numbers c and d , respectively. Then the interval $[c_1, d_2]$ will be an extension of the interval $[c, d]$. In this case $\mu_{\tilde{A}}(c_1) = 0$, $\mu_{\tilde{A}}(d_2) = 1$. Next, the parameter β will be equal to d_2 . Afterward, the equation of the straight line $\mu_{\tilde{A}}(x)$ passing through the points $(c_1, 0)$ and $(d_2, 1)$ is constructed. After that, the parameter $\alpha = \mu_{\tilde{A}}^{-1}(0.01)$ is calculated.

Let $\tilde{A} \triangleq$ "high pressure". The carrier of this set is given a fuzzy estimate, for example, "high pressure is approximately in the range from 70 to 90". For these numbers, the confidence intervals will be the intervals $[57.5, 82.5]$ and $[74, 106]$, and the extended interval is $[57.5, 106]$, at the boundary points of which the membership function takes on the values 0 and 1. In this case $\beta = 106$ and the equation of the straight line passing through points $(57.5, 0)$ and $(106, 1)$ will have the form $y = \frac{x-57.5}{48.5}$. Then equation $\frac{x-57.5}{48.5} = 0.01$ is solved and $x = 58$ is found. This value is the parameter α . As a result, we get the following S-shaped membership function:

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, & x < 58 \\ \frac{x - 57.5}{48.5}, & 58 \leq x \leq 106 \\ 1, & x > 106 \end{cases}$$

6.1 Coordination of expert assessments

When constructing linear S- and Z-shaped functions, the key point is to assign fuzzy linguistic estimates. In order to increase the objectivity of expert assessments, a group examination is carried out. The results of the examination are considered reliable if there is good consistency of assessments of experts.

A lot of research has been devoted to the issues of coordinating assessments of group expertise, among which the work [11] can be noted. In this paper, a mechanism for reconciling interval estimates is presented.

Moreover, as a measure of consistency of estimates, the coefficient of variation is used. This coefficient is determined separately for the left and right boundaries of the intervals by the formula $V = s/\bar{x}$, where s is the sample standard deviation of the estimates; \bar{x} is their average value.

Let $[a_1, b_1], \dots, [a_k, b_k]$ be estimates of the carrier of the set \bar{A} , which are given k by experts. Then the coefficients of variation of the boundaries of these intervals are determined as follows [14]:
for left borders according to the formula $V_L = s_L/\bar{x}_L$, where

$$s_L = \sqrt{\frac{1}{k-1} \sum_{j=1}^k (a_j - \bar{x}_L)^2 r_j}, \bar{x}_L = \sum_{j=1}^k a_j r_j;$$

for the right borders according to the formula $V_R = s_R/\bar{x}_R$, where

$$s_R = \sqrt{\frac{1}{k-1} \sum_{j=1}^k (b_j - \bar{x}_R)^2 r_j}, \bar{x}_R = \sum_{j=1}^k b_j r_j.$$

Here r_j is the weight coefficient of the j -th expert, moreover

$$\sum_{j=1}^k r_j = 1.$$

The practice of applying expert methods shows that the results of the examination can be considered as satisfying if $0,2 \leq V \leq 0,3$ and good, if $V < 0,2$. These conditions can be used as a criterion for the consistency of estimates and the basis for their clarification.

7. Conclusion

An approach to the automatic construction of membership functions of fuzzy numbers and intervals is proposed. The construction of triangular, trapezoidal and Gaussian membership functions is considered.

This approach is based on experimental data that reflect a person's representation of the classes' boundaries of numbers, approximately equal to a certain number. The proposed mechanisms also make it possible to directly specify fuzzy numbers of the (L-R) -type based on fuzzy linguistic statements.

This makes it possible to use this approach when estimating the parameters of models, building fuzzy time series, forming databases, in fuzzy inference systems and information retrieval systems, as well as in many other applied aspects.

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