# Mathematical modelling of daily computer network traffic

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#### Abstract

The paper substantiates the mathematical model structure of daily computer network traffic as a periodically correlated random process, which makes it possible to develop efficient methods for effective future forecasting of network congestion to optimize the network resource allocation among users.

#### **Keywords**

Daily traffic, computer network, forecasting, mathematical modelling, periodically correlated random process

## 1. Introduction

In the current conditions, it is difficult to imagine any sphere of human activity without the use of modern Internet technologies. This became especially relevant when moving on to distance work and study during quarantine measures in 2020-2021. Therefore, it is important to provide a high-quality and reliable Internet connection to all users.

One of the most important factors of a computer network is network traffic, which allows you to assess the activity and behavior of its users, as well as to monitor and analyze its functioning. Therefore, the intensity of information exchange in computer networks, both local and global, determines the relevance of the procedure for optimizing the allocation of network resources and their dynamic management in order to minimize the likelihood of overload. The most effective way to avoid network congestion is to predict the level of network traffic intensity over time, which will provide future optimization of network resources and their parameters, and this is impossible without the use of mathematical modeling. Today, a number of mathematical models are known (Wold and Hawks model [1], Poisson model [2], Gaussian self-similar processes (fractional Brownian motion and fractional Gaussian noise) [3-5], diffusion equations [6], differential equations of oscillatory processes [7], models based on the technique of "dynamic Markov modeling" [8], model based on diffusion theory [9], queue models [10], Markov and diffusion models [11], hidden Markov models [12], diffusion approximation model [13], the model based on diffusion approximation [14], a differential equation describing oscillating motion with low perturbation [15]) on the basis of which various algorithms and software for computer network traffic analysis, etc. are built.

Not taking into account the correlation between different iterations of the same implementation of daily network traffic in the structures of known models does not allow tracking the dynamics of variability of its phase-time structure in order to predict the congestion of computer network traffic in the future.

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In [16, 17], computer network traffic is treated as a model of a periodically correlated random process in general terms, which requires further development and specification of its use.

# 2. Experimental data of computer network traffic

The general view of the computer network traffic of UFONet provider in Ternopil, registered within seven days (from 01.09.2021 to 07.09.2021) (Residential complex "Park complex"), is shown in Fig.1.



Figure 1: Computer network traffic from 01.09.2021 to 07.09.2021 (Residential complex "Park complex", Ternopil)

## 3. Analysis of the computer network traffic structure

It should be noted that each day has a distinct peak of network traffic (see Fig. 2), which is characterized by daily variations relative to each other over time and the amplitude values of the peaks (Fig. 2).



**Figure 2**: Variation of parameters of computer network traffic peaks from 01.09.2021 to 07.09.2021 (Residential complex "Park complex", Ternopil)

Quantitative information generated by computer systems for predicting the state of the computer network in the future is not very informative for tracking the dynamics of changes in its load as time and amplitude indicators (Fig. 2), since they do not quantitatively reflect variations in the phase-time structure of the investigated networks traffics, which are shown in Fig.2.

In Fig. 3, the phase  $\varphi_n$  means an n-non-numerical indicator, which is a measure of the time variation of the onset of fluctuations of each *n*-th day of network traffic relative to the In Fig. 3, the phase  $\varphi_n$  means an n-non-numerical indicator, which is a measure of the time variation of the onset of fluctuations of each *n*-th day of network traffic relative to the value of the (n-1)T value of the day.

Based on the analysis of the phase-time structure of network traffic (Fig. 3) it is established that each *n*-th day, which is within the time period of traffic *T*, has a variability of phase values  $\varphi_1 - \varphi_7$  of traffic components  $\xi_1(t) - \xi_7(t)$  in relation to the initial value of the period (n-1)T in the implementation of network traffic  $\xi(t)$  over time:

$$\varphi_1 \neq \varphi_2 \neq \dots \neq \varphi_n = var, \tag{1}$$

where n is the iteration number of the network traffic or the phase number of the n-th component of the network.



Figure 3: Phase-time structure of network traffic

Based on the analysis of the phase-time structure of network traffic (Fig. 3), it is found that its mathematical model should provide an opportunity to study the dynamics of this structure (its time interdependence) for each day to predict the variable behavior of the computer network in the future.

To study the variation of amplitude-time indicators of computer network traffic (Fig. 2) and identify their interdependence over time, it is necessary to apply mathematical modeling, namely, to describe the mathematical model of network traffic, which makes it possible to study these variable interrelations in network traffic for different days.

To determine the structure of the mathematical model of network traffic, the parameters of the real signal are analyzed. Considering the implementation of traffic within the stationary model, it is noticed that the functions of the distribution density (Fig. 4) are transformed in time space, which indicates the fact of non-stationary implementation of network traffic. The probability density of instantaneous values of network traffic for the k-th days, which allows to set the law of their changes over time, calculated by the formula:

$$p(\xi,t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{X} e^{-\frac{(t-m_{\xi})^2}{2D_{\xi}}} dt$$
 (2)

where  $m_{\xi}$  – the mean of *X*;  $D_{\xi}$  – the dispersion of *X*.



**Figure 4**: Density of probability distribution of traffic implementations  $p(\xi_k, t_n)$ 

According to the results of correlation analysis of traffic, it is found that its correlation function  $r_{\xi}(t,u)$  as an ensemble of implementations is periodic  $r_{\xi}(t,u) = r_{\xi}(t+T,u+T)$  (Fig. 5) and periodically disappearing (Fig. 6) as a continuous implementation  $R_{\xi}(u)$ , indicating the signal iterations and its finiteness  $R_{\xi}(u) < \infty$ . Autocorrelation function (AF) of energy-finite network traffic for an ensemble of implementations  $\xi_{\mu}(t)$  calculated by the formula:

$$r_{\xi_{k}}(t,u) = \frac{1}{T} \int_{0}^{T} \xi(t-kT) \xi(t-kT-u) dt , \quad u,t \in [0,T), \quad k = \overline{0,(K-1)}.$$
(3)

where K – the number of days  $\xi_k(t)$ , which is multiplied on each *k*-th day  $[\xi_1(t), \xi_2(t), ..., \xi_k(t)]$  form a continuous implementation  $\xi(t)$ .

For continuous implementation of network traffic AF is calculated by the formula:  $r_{\varepsilon}(u) = \int_{0}^{T} \xi(t)\xi(t-u)dt$ .  $t \in \mathbb{R}$ 

(4)

Figure 5: Implementation of correlation functions due to the ensemble of traffic implementations



Figure 6: Implementation of autocorrelation function of traffic

Based on the results of traffic analysis, it is established that its adequate mathematical model should consider the properties of randomness (Fig. 2) and iterations (Fig. 4) in its structure. It should belong to the class of finite processes (Fig. 5) and take into account relationships between different iterations of the same implementation to study the dynamics of changes in the phase-time structure of the traffic (Fig. 3).

The model being periodically correlated random process allows one to take into account the periodicity (iteration) of the day, time and amplitude variability of daily network traffic in its structure and has methods and algorithms for studying their interdependence as in the following expression:

$$\xi(t) = \sum_{k \in \mathbb{Z}} \xi_k(t) e^{j 2\pi k/T} , \ t \in \mathbb{R}$$
(5)

where  $\xi_k(t)$  – stochastic (variational) component of daily computer network traffic in the form of stationary components;  $e^{j2\pi k/T}$  – periodic (daily) component of the daily traffic of the computer network with the period *T*, which is equal to the length of the day (24 hours).

The model in the form of (5) has in its arsenal in-phase (with and without taking into account the relationships between components) component and filter methods for analysis (processing) of daily computer network traffic in order to obtain results that quantitatively reflect estimates of amplitude time indicators of traffic.

### 4. Conclusions

The study of variational relationships between computer network traffic as a periodically correlated random process on different days of observation will provide a procedure for a priori determining the modes of operation of network resources in order to optimize them in providing quality services to network users.

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