Modelling of Ultrasonic Testing and Diagnostics of Materials by Application of Inverse Problems

Vladyslav Khaidurov[a], Tamara Tsiupii[b] and Tetiana Zhovnovach[c]

[a] *Institute of Engineering Thermophysics of NAS of Ukraine, Maria Kapnist (Zhelyabova) st., 2a, Kyiv, 03057, Ukraine*
[b] *National University of Life and Environmental Sciences of Ukraine, Heroyiv Oborony st., 15, Kyiv, 03041, Ukraine*
[c] *Cherkasy Branch of the European University, Smilyanska st., 83, Cherkasy, 18000, Ukraine*

Abstract
Mathematical models of inverse problems of mathematical physics have been built for ultrasonic diagnostics of complex systems in order to study the structure of the material of the systems under study, as well as to detect material defects in the form of cracks and chips.

Keywords 1
Defects in materials, ultrasonic testing, inverse problems, applied software packages for modelling.

1. Introduction

Most modern processes in science and technology are described by nonlinear mathematical models. It is obvious that the solution of such models cannot always be written in analytical form, for example, in the form of functions or functional series. In such cases, computational methods for solving nonlinear equations and their systems, methods of global optimization, methods for solving ordinary differential equations, equations of mathematical physics and their systems, as well as other methods for studying complex dynamical systems in general are used.

An example of complex processes can be called physical and technical processes, which are described by models of mathematical physics in the reverse mathematical formulation. Such problem statements imply the identification of the parameters of the processes under consideration. The problems that reduce to such models are called inverse problems of mathematical physics. Inverse problems of mathematical physics, like all inverse problems, are experimental problems [1; 13; 14]. The desired parameters in such problems can be the characteristics or modes of the system under study, for example, the geometric dimensions and shape of the system object, boundary conditions, initial conditions, internal conditions and others [1; 2]. The input values of the parameters of such mathematical models are statistical data about the object or process itself. The more there is experimental data, the easier it is to obtain information about him in general.

Nowadays, the scientific direction on the study of ultrasonic processes is very developed in order to diagnose elements of building structures, as well as to increase their functionality and reduce wear. Ultrasonic elastic vibrations and waves obey general laws that characterize processes in liquids and solids in general. The speed of propagation of waves in a material environment is determined by various physical and technical parameters, one of which is elasticity and density. However, with an increase in some limiting values of the considered medium of ultrasound intensity during the transition from the propagation of oscillations of small amplitude to oscillations of finite amplitude, nonlinear effects arise

ITTAP'2021: 1nd International Workshop on Information Technologies: Theoretical and Applied Problems, November 16–18, 2021, Ternopil, Ukraine
EMAIL: allif01111@gmail.com (A. 1); ts.tamara19@gmail.com (A. 2); z.ta@ukr.net (A. 3)
ORCID: 0000-0002-4805-8880 (A. 1); 0000-0003-2206-2897 (A. 2); 0000-0003-1037-4383 (A. 3)

that can violate the principle of superposition of the wave process in a certain medium. In this regard, new physical phenomena arise, to which pressure can be attributed.

The search for a numerical solution to the inverse problem is a rather complicated process that requires an integrated approach. Regardless of the structure of the physical and technical process, practical scientists develop new effective methods for solving inverse problems. A very popular formulation of inverse problems in general is the optimization formulation. Therefore, to obtain a solution to the inverse problem, fast methods of multivariate global optimization are needed. Deterministic optimization methods make it possible to quickly get a good approximation to the optimal solution. But the resulting solution may be suboptimal. To obtain the global optimal solution to the inverse problem, stochastic methods of multidimensional global optimization are also used. They do not require any information about the differentiability of the function or functional under study. Such methods work longer than deterministic ones. To obtain the golden mean in terms of speed and optimality of the obtained solution, deterministic and stochastic methods of multivariate global optimization are combined.

The combination of deterministic and stochastic methods of multidimensional global optimization is especially effective for the study of mathematical models of systems with distributed parameters, for example, models that describe the processes of heat and mass transfer, hydrodynamics and wave processes.

2. Formulation of the problem

Inverse problems in most cases are incorrect, that is, those in which one or more of the following conditions are violated: the existence of a solution; uniqueness of the solution; the condition under which small changes in the required parameters correspond to small perturbations for the corresponding solutions.

The simplest and most urgent formulation of the parameter identification problem is the optimization formulation, in which it is required to find the global extremum of some function or functional:

$$J(c_1, c_2, \dots, c_n) = F\big(U(c_1, c_2, \dots, c_n)\big) \to min, \quad (1)$$

where U – the required function that delivers the minimum; n – is the number of parameters to be identified; $c_1, c_2, \dots, c_n$ – the parameters of some process on which the function depends U.

Constraints are imposed on (1) in the form of functional dependencies, for example, equations of the form:

$$\{L_j\big(U(c_1, c_2, \dots, c_n)\big) = 0\}, j = \overline{1, m} \quad (2)$$

and inequalities of the form

$$\begin{gathered} \{S_k\big(U(c_1, c_2, \dots, c_n)\big) \le 0\}, k = \overline{1, p}, \\ \{P_m\big(U(c_1, c_2, \dots, c_n)\big) = 0\}, m = \overline{1, d}, \end{gathered} \quad (3)$$

where $L_j, j = \overline{1, m}$ – some differential operators that describe some process in an object, and S_k – functional dependencies, which are constraints on a function U. Boundary, boundary, internal conditions in (2) are set depending on the process under consideration [1; 2]. They can be specified on a part of the area of the process under consideration or on the entire area. They can also act as the required parameters c_i, $i = \overline{1, n}$.

Problems of the form (1)–(3) in the optimization formulation are very resource-intensive. The amount of computational costs increases significantly with the dimension of the problem. It is known that to solve one inverse problem, a multiple call of the procedure for solving the direct problem is used. In this regard, it is possible to use applied software packages to solve direct problems, which quickly and efficiently cope with the task at hand. These packages include MATLAB, COMSOL Multiphysics, and ANSYS. MATLAB has a wide library for working with matrix calculations, which are actively used in the process of solving equations of mathematical physics and their systems using the finite element method [1]. In this environment, it is very convenient to carry out calculations for objects of arbitrary 2D-shape. MATLAB quite efficiently uses memory when solving sparse systems of algebraic equations, to which discrete analogs of equations of mathematical physics are reduced. COMSOL Multiphysics, ANSYS are significantly improved applied software packages, which, in addition to all of the above for MATLAB, have a fairly advanced constructor for creating 3D object geometry. They also have the ability to represent intermediate solutions, slices of solutions, solutions on subdomains, etc., which gives an overall picture of the process under study.

2.1. A Mathematical Model of the Process

The propagation of ultrasonic waves can be mathematically described by a wave equation (linear and nonlinear) of the second order [1; 5; 12]. In this description, the body is considered to be elastic, the deformation is elastic and negligible compared to the geometric dimensions of the object itself. The technical task is to determine the geometric position of the oscillator, provided that in specific positions of the studied area of the object are sensors that receive the signal generated by the source (exciter). The physical and technical characteristics of the object under study, such as the speed of propagation and attenuation of the waves, are known. The characteristics of the source, such as frequency and amplitude of oscillations also are known. In this case, the mathematical model of the problem has an optimization statement [6–8], in which you need to find a minimum:

$$F(x^{source}, y^{source}) = \frac{1}{8}\sum_{i=1}^{8}\left[\int_{0}^{1,5}\left(s_i(x^{source}, y^{source}, t) - s_i^{experimental}(t)\right)^2 dt\right] \to \min \qquad (4)$$

with restrictions in the form

$$\begin{cases} s_i(x^{source}, y^{source}, t) = U(x_i^{sensor}, y_i^{sensor}, t), \\ \dfrac{\partial^2 U}{\partial t^2} = \Delta U \text{ on } \Omega, t \in [0; 1,5], \\ \Omega = (x, y) \in [0; 1]^2, i = \overline{1,8}, \\ U(x, y, 0) = U_t(x, y, 0) = 0, \\ \dfrac{\partial U}{\partial \overline{n}} = 0, t \in [0; 1,5], \\ U(x^{source}, y^{source}, 0) = A\, sin(2\pi f t), \end{cases} \qquad (5)$$

where

n	–	total number of sensors;
τ_i^{beg}	–	the start time of the observation on the *i*-th sensor;
τ_i^{end}	–	end time of observation on the *i*-th sensor;
$s_i^{experimental}(t)$	–	the data taken by the *i*-th sensor for a certain period of time $[\tau_i^{beg}; \tau_i^{end}]$;
T	–	the total time of the experiment;
Ω	–	calculation area of the problem (object under study);
A, f	–	amplitude and frequency;
$(x_i^{sensor}, y_i^{sensor})$	–	position of the *i*-th sensor;
(x^{source}, y^{source})	–	position of the oscillator.

Source characteristics is $A = 30^{-1}$, $f = 9$. The sensors positions are shown in Table 1.

Table 1
Geometric positions of sensors

x_i	y_i	x_i	y_i
0,5333	0,0667	0,3167	0,8500
0,7167	0,9167	0,1000	0,2833
0,4500	0,1167	0,8667	0,5167
0,1500	0,3500	0,4167	0,9000

To find a numerical solution to this optimization problem (4)–(5), a classical genetic algorithm was applied. The results of mathematical modelling and a discussion of the results are given below.

2.2. Numerical Results of Computer Experiments

The following are the results of computational experiments performed using the MATLAB 2021a application software environment. The mathematical model of problem (4)–(5) is solved using the finite difference method. Estimated grid – 60x60 grid nodes, $w = 32$ – the number of bits for encoding by a genetic algorithm.

Figure 1 shows the values of the objective function (4) of the mathematical model (4)–(5) using the classical genetic algorithm in the first twenty and in the last twenty iterations.

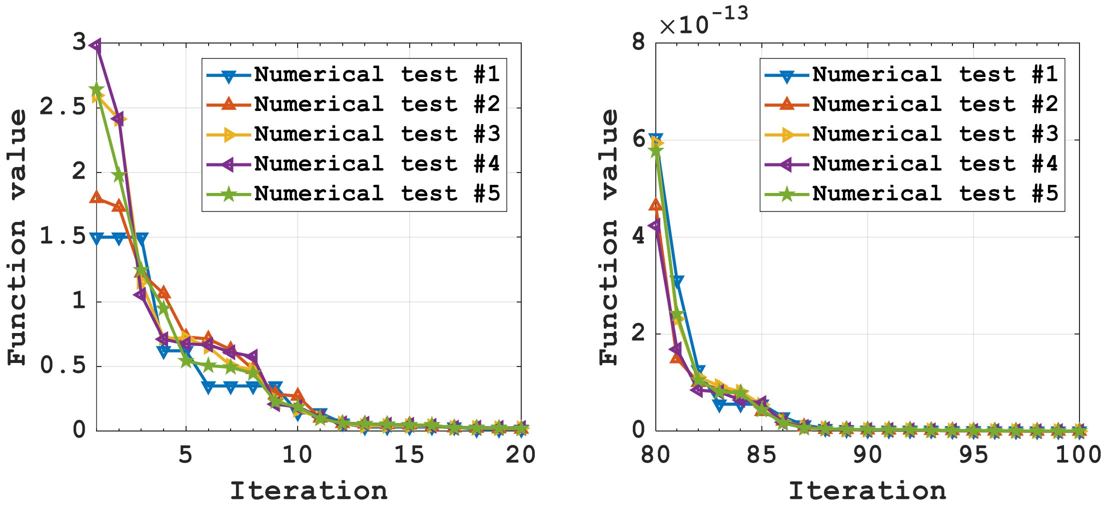

Figure 1: Convergence of the optimization mathematical model (3)–(4) by the classical genetic algorithm

The accuracy of the obtained solution was 1,0E–011. When sensors receive real data, the accuracy will be lower due to the error in obtaining indicators from sensors.

3. The Discussion of the Results

The mathematical model of the form (4)–(5) belongs to the models of the class of nonlinear global optimization [6; 10; 11], in which the restriction on the objective function (1) is the wave equation, which describes the process of wave propagation in a particular medium. The function (1) is nonlinear. Thus, stochastic or population methods can be used to minimize it, as local minima can be obtained [9–12]. The model of problem (4)–(5) is an inverse problem, because the results of data from sensors (indirect measurements – consequence) need to establish the position of the pathogen (cause).

As a result of computer and mathematical modeling, it is established that various modifications of the mathematical model of the form (4)–(5) can be used to determine defects in inhomogeneous two- and three-dimensional bodies.

4. Conclusions

The choice of applied software packages when solving resource-intensive problems, such as the problem of identifying the parameters of physical and technical processes using the example of inverse problems of mathematical physics, is an important factor, due to which the time of searching for a solution to the problem is significantly saved, as well as the total computational costs of a personal computer when solving such problems are reduced.

The obtained mathematical model can be modified for research of more complex objects and processes that arise in production. It is recommended to use genetic algorithms as a tool for finding the optimal solution to such problems, since in such processes the sought solutions in the form of functions can be nondifferentiable. The presence of nondifferentiability of the sought functions makes some classical deterministic methods of searching for the global extremum unusable.

Obviously, the complication of this mathematical model can be carried out by searching for some parameters, which are represented as multidimensional functions of time. Obviously, when solving the problem numerically, the required functions are sought in the form of a table of values. In this case, the number of calculations increases markedly. To reduce computations, it is recommended to use interpolation and approximation, as well as apply multigrid methods.

[1] Yerofeyev V.A., Miller S.S. Ul'trazvukovoy monitoring mikrodefektov v metalle. Zhurnal «Izvestiya Tul'skogo gosudarstvennogo universiteta», tekhnicheskiye nauki, mashinostroyeniye i mashinovedeniye. 2014, S. 211–217.

[2] Kolesnikov A.Ye. Ul'trazvukovyye izmereniya. M.: Standarty, 1982. 178 s.

[3] Klyuyev V.V. Nerazrushayushchiy kontrol'. Tom 3. Spravochnik. V 7-i knigakh. M.: Mashinostroyeniye, 2004.

[4] Zuyev L.B., Semukhin B.S., Bushmeleva K.I., Zarikovskaya N.V. Skorost' rasprostraneniya ul'trazvuka v polikristallakh Al s raznym razmerom zerna. FMM, 2000. T. 89. No 4.

[5] Solov'yanova I.P., Shabunin S.N. Teoriya volnovykh protsessov: akusticheskiye metody: ucheb. posobiye. Yekaterinburg: GOU VPO UGTU-UPI, 2004. 142 s.

[6] Andreyeva Ye.A. Variatsionnoye ischisleniye i metody optimizatsii. M.: Vysshaya shkola, 2006. 584 s.

[7] Vasil'yev F.P. Metody optimizatsii v 2-kh knigakh. Kn. 1. M.: MTSNMO, 2011. 619 s.

[8] Vasil'yev F.P. Metody optimizatsii v 2-kh knigakh. Kn. 2. M.: MTSNMO, 2011. 433 s.

[9] Yemel'yanov S.V., Korovin S.K., Bobylev N.A. Metody nelineynogo analiza v zadachakh upravleniya i optimizatsii. M.: URSS, 2002. 120 s.

[10] Zolotarev A.A. Metody optimizatsii raspredelitel'nykh protsessov. Vologda: Infra-Inzheneriya, 2014. 160 s.

[11] Zubarev YU.M. Spetsial'nyye metody optimizatsii: Uchebnoye posobiye. SPb.: Lan', 2014. 384 s.

[12] Khaidurov V. Nonlinear Mathematical Model for Determining the Geometric Position of Point Signals in Homogeneous Materials. Abstracts of X International Scientific and Practical Conference Liverpool, United Kingdom 27–29 May 2020.

[13] David Matajira-Rueda, Jorge Mario Cruz-Duarte, Juan Gabriel Aviña-Cervantes, Mario Alberto Ibarra-Manzano, Rodrigo Correa: Optimization-Based Strategies for Optimal Inverse Parameters Estimation for Heat Transfer Systems. IEEE Access 9: 71328-71343 (2021)

[14] Lin Qiu, Minghui Zhang, Qing-Hua Qin: Homogenization function method for time-fractional inverse heat conduction problem in 3D functionally graded materials. Appl. Math. Lett. 122: 107478 (2021)