Development of "Mathematical Models in Economics" Software Package and its Application in Educational Process

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Abstract

Digitalization of the educational environment and digital learning are important components of modern education. In the context of digital transformation, there is a need to create electronic resources for each specialty of a bachelor's degree, formation of the electronic information, and educational environment of the university. The article describes the process of developing the «Mathematical Models in Economics» software package, its purpose, components of the windows, functions of buttons, and fields. MATLAB application software package, MATLAB Application Designer is the main tool of package development. The digital resource is used for training bachelor students of «Applied Mathematics and Informatics», «Pedagogical Education, profile Mathematics and Informatics» specialties. It enables them to implement the stages of laboratory practicum. With the use of the package students build skills of individual and research work, perform a check of the results obtained in the course of mathematical research, compliment them, make visualization. The use of the software package within the laboratory practicum is carried out with the help of the case method. The article suggests variants of using the package for organizing individual work, pair, and team research work. While organizing pair and teamwork networking is used: cloud services and videoconferences. The suggested learning software package and the method of its application allow the more efficient building of students' research skills and skills of independent work.

Keywords [1]

Digital resource, software package, MATLAB, mathematical models, economic models, individual work, research work, case, bachelor students, applied mathematics and informatics, pedagogical education.

1. Introduction

Nowadays the educational process is dynamically digitalized. Transition to digital learning in the sphere of education is quite difficult, but essential in the present context. This process must be carried out step by step, not only from the viewpoint of technical training of the participants but also professional training and retraining of the teaching staff, who provide methodical support of the digital learning [1].

Digital transformation of education is primarily aimed to achieve the required educational results. Authors V.N. Kormakova, A.G. Klepikova, M.A. Lapina, J. Rugelj say that a teacher must understand ICT possibilities both for enlargement and improvement of the educational process, and for using digital technologies for improvement and introduction of innovations in the sphere of education [2].

The authors K.A. Markelov, E.V. Polyanskaya, O.K. Mineva, V.N.Taran of the article [3] prove that the future of education lies in the formation of a hybrid educational environment that permeates the entire vertical and horizontal of the individual trajectory of development of a human of the future.

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Moodle is a tool for exchange educational materials, means of the organization of training and education is the most popular educational environment. Many authors describe their experience of work in this environment while implementing different courses and types of educational activity [4, 5, 6]. So, students' research work is one of the activities. Yu. Bogatyreva, A. Privalov, V. Romanov, E. Konopko suggest using a systemic-activity approach in the organization of students' research work in the institute's digital educational environment [7].

Panyukova S.V. in the manual [8] speaks of the emerging need to create digital learning tools, united into a single package. She distinguishes the main approaches to the creation of educational content, electronic educational resources:

1. using of the computer programming languages;
2. using of special and universal applied programming tools;
3. using of digital tools and web services;
4. formation of educational content from the information presented on educational channels, platforms, portals, and sites.

The relevance of the work lies in the existing need for the formation of electronic information and educational environment of the university; the need to create digital teaching aids in various disciplines, based on which it is possible to carry out the educational process, taking into account the peculiarities of the specialties, to organize independent, research work of students.

The objective of the article is to describe the development process of the digital resource used in Astrakhan State University for training within «Applied Mathematics and Informatics», «Pedagogical Education, profile Mathematics and Informatics» specialties, and application in the educational process.

2. The Main Content

The development process of the software package «Mathematical Models in Economics» is the following. It is intended for use in the educational process of bachelors of «Applied Mathematics and Informatics» specialty within the discipline «Dynamic Models» and «Pedagogical Education, Profiles - Mathematics and Informatics» specialty within the discipline «Differential Equations». In her article [9] M.V. Kolomina described the learning process in the discipline «Dynamic Models» in stages. It contains two blocks: theoretical and laboratory. The first block studies the theory associated with the study of systems of differential equations: singular points, local and global phase portraits, motion stability. Practical tasks are considered. In this block, students develop the ability to apply the fundamental knowledge gained while studying the previous disciplines of the curriculum: «Mathematical Analysis», «Algebra and Geometry», «Differential Equations» [10], «Applied Software». Within the framework of the second block, students perform a laboratory workshop related to the study of mathematical models from various fields, in particular, economics. The models are represented by nonlinear systems of second-order ordinary differential equations. The workshop allows bachelor students to develop the ability to apply and modify mathematical models to solve problems in the professional field, skills of independent work, forms the research skills.

Laboratory practicum has five stages:

1. collection of information on the model under study;
2. mathematical investigation of the model;
3. use of software packages, MATLAB package to supplement mathematical research, graphing;
4. transfer of the obtained results to the considered phenomenon, which describes the model, predicting the behavior of the model;
5. report on the laboratory work.

When performing laboratory work, the software package «Mathematical Models in Economics» is used. The purpose of its application is to systematize students' knowledge and enable them to implement some stages of the laboratory practice independently.

Considering the first stage, the software package should contain information about mathematical models:

1) information about the phenomenon described by the mathematical model;
2) a system of differential equations that describes a mathematical model;
3) the value of variables and coefficients of the system of differential equations.

Considering the third stage, the following functions should be implemented in the software package:

1) building a phase portrait taking into account the constraints on the coefficients and variables imposed by the model;
2) building a complete phase portrait without taking into account the constraints on the coefficients and variables imposed by the model;
3) plotting the dependence of the variables of the system of differential equations on time.

To prepare a report on a laboratory practicum (fifth stage), the software package should allow unloading graphs in jpeg format.

The software package should include several mathematical models of different levels (with a different number of singular points of the system of differential equations).

With the help of this software package, the bachelor must

1) check the results obtained in the course of mathematical research;
2) supplement the results of mathematical research in cases where there are exceptions to the theorems used;
3) build phase portraits, visualize the mathematical results of the study.

The model of the software package is presented in Figure 1.

3. Development of the Software Package

For the software package, four mathematical models were selected from the field of economics of different levels of complexity:

1) a market model with predicted prices (one singular point);
2) Phillips business cycle model (one singular point);
3) model of the energy market (two singular points);
4) model of inflationary expectations (three singular points).

The main tool for developing a software package for the study of mathematical models in economics is the MATLAB software package. The MATLAB Application Designer tool offers using a wide range of components to create a graphical interface, and the MATLAB Compiler module allows you to compile graphical applications into a single independent application. The components of the user interface used in the software package are presented in the table.

The software package consists of five windows. When starting the software package, the Main.mlapp introductory window opens (Figure 2). The interface of the introductory window is bright and intuitive, which attracts the user's attention and interest.

Table 1
Components of MATLAB Application Designer user interface

Components of the user interface	Description
UIAxes	Cartesian graph
Button	Button
Edit Field (Number)	Number field
Image	Image
Panel	Control panel
Label	Text label

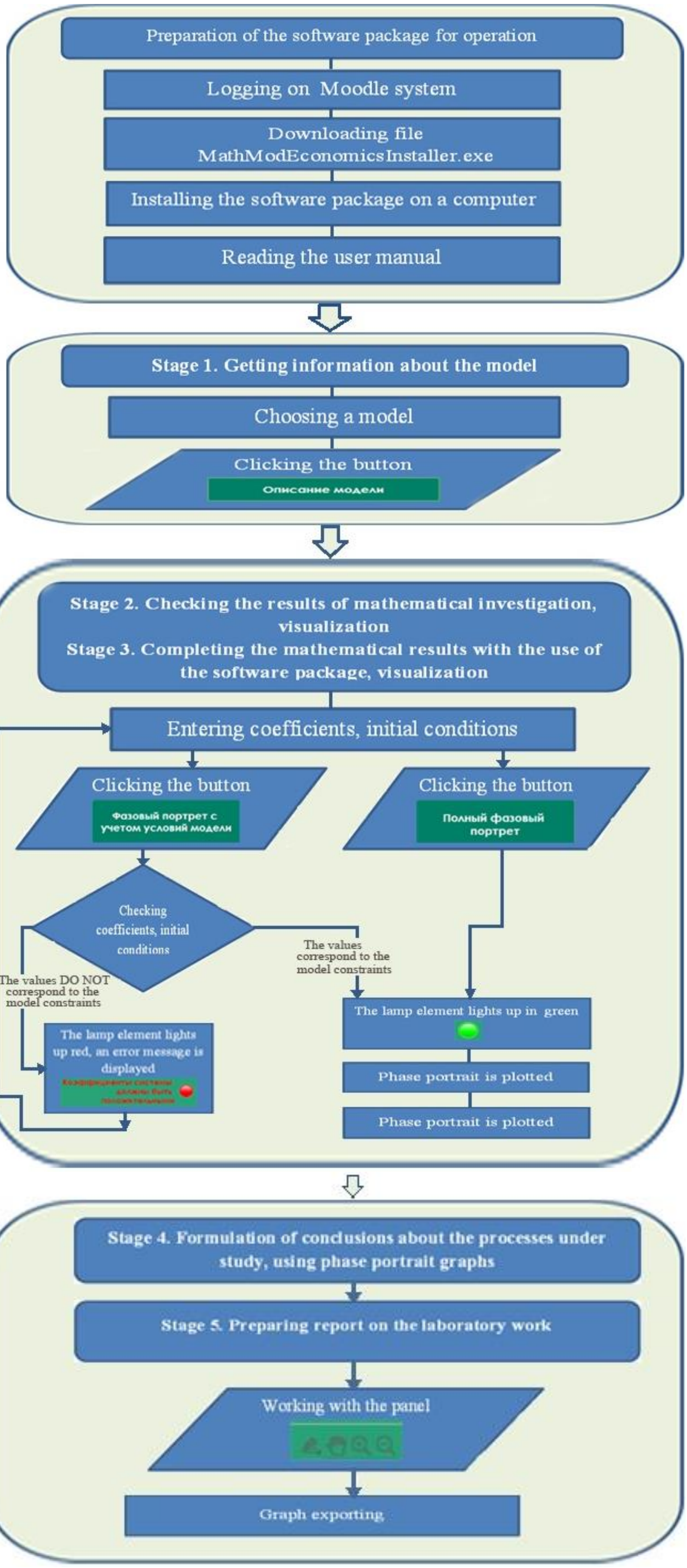


Figure 1: The model of the software package

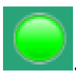
Figure 2: Starting screen of the software package

The introductory page of the software package contains a background image, the name of the software package, and four buttons. Each of the buttons provides access to the corresponding window for a specific model:

1) market model with predicted prices (MathMod1.mlapp);
2) Phillips business cycle model (MathMod2.mlapp);
3) models of the energy market (MathMod3.mlapp);
4) model of inflationary expectations (MathMod4.mlapp).

Let us consider a market model with predicted prices. When clicking on this button, the MathMod1.mlapp window opens (Figure 3).

At the top of the window we see:

1) a text label with the name of the mathematical model;
2) the «?» button;
3) a text label with a system of differential equations that describes the mathematical model;
4) «Model Description» button;
5) «System coefficients» control panel, which contains text labels with the names of the system coefficients and the corresponding fields for entering the coefficients;
6) the control panel «Initial conditions», which contains text labels with the names of the coordinates of the initial conditions and the corresponding fields for entering the initial conditions;
7) button «Clear the graph»;
8) button «Phase portrait taking into account the conditions of the model»;
9) button «Complete phase portrait»;
10) lamp .

At the bottom of the window, there is a control panel with a phase portrait and a graph «Functions versus time».

Figure 3: «Market Model with Predicted Prices» Window

Clicking on the «?» opens a pdf file with user instructions using the function

function Button_InstructionPushed(app, event)
open Инструкция_пользователя.pdf (User's instruction);
end

Clicking on the «Model Description» button calls the function

function Button_DescriptionPushed2(app, event)
open Модель_рынка_с_прогнозируемыми_ценами.pdf (Market Model With Predicted Prices);
end

which opens a .pdf file that contains information about the mathematical model.

When the «Market Model with Predicted Prices» window is launched, several actions take place:

1) numerical fields on the "System coefficients" control panel are filled with certain coefficients:

```
app.a1EditField.Value=1;
app.a2EditField.Value=7;
app.b1EditField.Value=2;
app.b2EditField.Value=2;
app.c1EditField.Value=1;
app.c2EditField.Value=1;
app.sEditField.Value=2;
app.dEditField.Value=6;
```

2) numerical fields on the "Initial conditions" control panel are filled with certain values:

```
app.x0EditField.Value=2.5;
app.y0EditField.Value=0.5;
```

Clicking the «Phase portrait based on model conditions» button, the code calls the function ButtonPushed (app, event). Let's consider the main actions implemented in this function.

First of all, the coefficients of the system and the initial conditions are read from the numerical fields into the variables:

```
a1 = app.a1EditField.Value;
a2 = app.a2EditField.Value;
b1 = app.b1EditField.Value;
b2 = app.b2EditField.Value;
c1 = app.c1EditField.Value;
c2 = app.c2EditField.Value;
s = app.sEditField.Value;
d = app.dEditField.Value;
x0 = app.x0EditField.Value;
```

```
        y0 = app.y0EditField.Value;
```
Then the entered coefficients and initial conditions are checked for correctness:
```
        if (a1<=0 || a2<=0 || b1<=0 || b2<=0 || c1<=0 || c2<=0 || ...
        s<=0 || d<=0)
               app.ErrorLabel.Text = 'System coefficients must be
               positive';
               app.ErrorLabel.FontColor = [1.00 0.00 0.00];
               app.Lamp.Color = [1.00 0.00 0.00];
               return
        elseif (x0<=0 || y0<=0)
               app.ErrorLabel.Text = 'Initial conditions must be
positive';
               app.ErrorLabel.FontColor = [1.00 0.00 0.00];
               app.Lamp.Color = [1.00 0.00 0.00];
               return
        else
               app.ErrorLabel.Text = '';
               app.ErrorLabel.FontColor = [1.00 1.00 1.00];
               app.Lamp.Color = [0.00 1.00 0.00];
        end
```
In case when the system coefficients and/or initial conditions do not correspond to the model constraints, the lamp element lights up in red (app.Lamp.Color = [1.00 0.00 0.00]) and a corresponding error message is displayed, after which the ButtonPushed function is exited (Figure 4).

Figure 4: Error message

If the system coefficients and the initial conditions are entered correctly, the lamp element will turn green (app.Lamp.Color = [0.00 1.00 0.00]) and no error message is displayed, the execution of the ButtonPushed function continues.

At the next step, the coordinates of the singular point are calculated (in this model, one singular point). A preliminary mathematical study was carried out, formulas for the coordinates of a singular point were obtained, they are immediately written into the code:
```
        RestPointX=0;
        RestPointY=(d-s)/(c1+c2);
```
To construct the trajectories of a system of differential equations, it is necessary to solve this system. The ode23 function is used to solve a system of differential equations. The arguments of this function are the given system of differential equations, the time interval, the initial conditions, and the accuracy with which the system will be solved. These arguments must be predefined:
```
        opt = odeset('RelTol', 1e-6);
        tmin=0;tmax=10;
        f=@(t,y)[((b1+b2)*y(1))/(a1-a2)+((c1+c2)*y(2))/(a1-a2)+(s-
d)/(a1-a2);y(1)];
        [time,z]=ode23(f,[tmin, tmax],[x0 y0], opt);
```
Using the plot function, a trajectory is plotted on the phase portrait with respect to the entered initial conditions and a singular point is marked, and using the quiver function, a vector field is drawn on the phase portrait:
```
        plot(app.UIAxes,z(:,1),z(:,2),'LineWidth', 1);
        plot(app.UIAxes,RestPointX,RestPointY, 'r*');
```

```
x1dot  =  ((b1+b2)*x1)/(a1-a2)+((c1+c2)*x2)/(a1-a2)+(s-d)/(a1-a2);
x2dot = x1;
quiver(app.UIAxes,x1,x2,x1dot,    x2dot,'color',[0.65    0.65 0.65]);
```

Using the plot function on the graph «Dependence of functions on time», two curves are plotted corresponding to the constructed phase trajectory:

```
plot(app.UIAxes2, time, z(:,1), 'b', time, z(:,2), 'r');
```

The legend function on the graph «Functions versus time» places the legend (Figure 5):

```
legend(app.UIAxes2,{'x','y'});
```

The display of the type of a singular point on the legend of the phase portrait is carried out using the function

```
function restpoint(app,rx, ry)
            a1 = app.a1EditField.Value;
            a2 = app.a2EditField.Value;
            b1 = app.b1EditField.Value;
            b2 = app.b2EditField.Value;
            c1 = app.c1EditField.Value;
            c2 = app.c2EditField.Value;
            l1=(b1+b2+sqrt((b1+b2)^2+4*(a1-a2)*(c1+c2)))/(2*(a1-a2));
            l2=(b1+b2-sqrt((b1+b2)^2+4*(a1-a2)*(c1+c2)))/(2*(a1-a2));
            P1=plot(app.UIAxes,rx,ry, 'r*');
            if (l1>0 && l2>0)
                 legend(app.UIAxes,P1,{ ‘unstable nodal point'});
            end
            if (l1<0 && l2<0)
                 legend(app.UIAxes,P1,{ ‘stable nodal point'});
            end
            if ((l1>0 && l2<0) || (l1<0 && l2>0))
                 legend(app.UIAxes,P1,{'saddle'});
            end
            if ((b1+b2)^2+4*(a1-a2)*(c1+c2))<0
                 if ((b1+b2)/(2*(a1-a2)))==0
                     legend(app.UIAxes,P1,{'center'});
                 elseif ((b1+b2)/(2*(a1-a2)))>0
                     legend(app.UIAxes,P1,{'unstable focus'});
                 elseif ((b1+b2)/(2*(a1-a2)))<0
                     legend(app.UIAxes,P1,{stable focus'});
                 end
            end
         end
P1=plot(app.UIAxes,RestPointX,RestPointY, 'r*');
legend(app.UIAxes,P1,{'Особая точка'});
```

When you hover the cursor over the graphs to the right of the graph name, a panel of elements is displayed (Figure 6):

```
app.UIAxes.Toolbar.Visible = 'on';
axtoolbar(app.UIAxes,{'export','zoomin','zoomout','pan'});
app.UIAxes2.Toolbar.Visible = 'on';
axtoolbar(app.UIAxes2,{'export','zoomin','zoomout','pan'});
```

This toolbox allows exporting and scaling graphs.

When the initial conditions are changed and the button «Phase portrait taking into account the model conditions» is pressed again, a new trajectory is added to the phase portrait due to the NextPlot = add a parameter. Since the NextPlot = replace children parameter is set on the graph «Functions versus time», then when the initial conditions are changed and the button «Phase portrait taking into account the model conditions» is pressed again on this graph, the constructed curves are replaced with new ones.

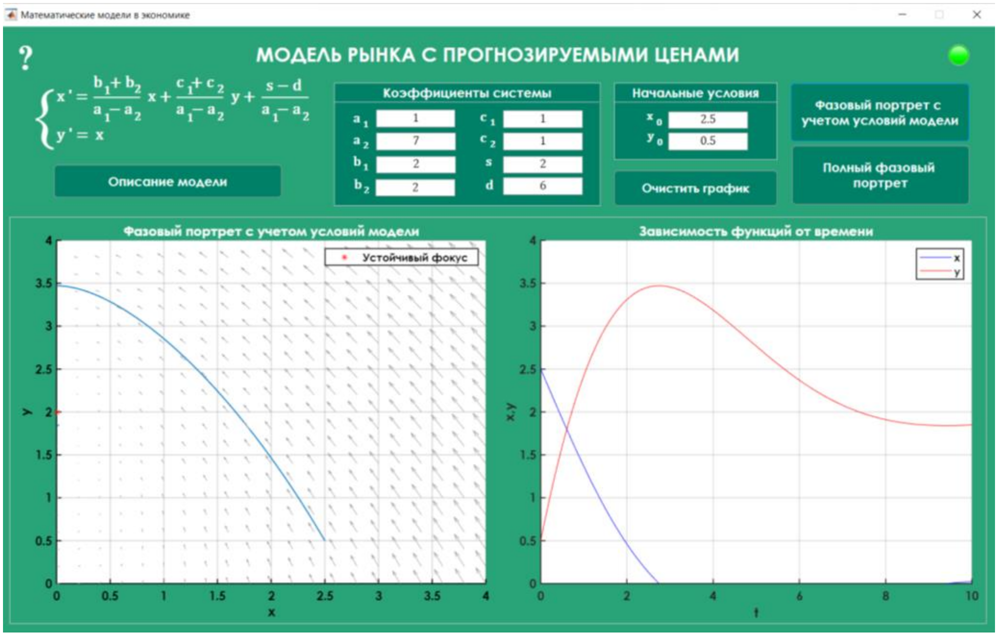


Figure 5: Graph «Phase portrait taking into account the conditions of the model»

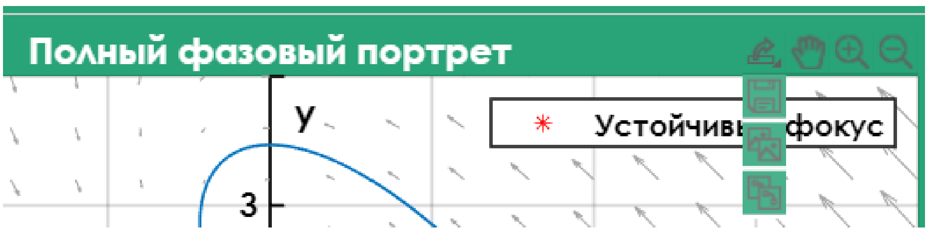


Figure 6: Graph control panel

When changing the coefficients of the system and pressing the button «Phase portrait taking into account the model conditions» again, the phase portrait is cleared using the cla(app.UIAxes1) function and new trajectories are built.

By clicking on the «Complete Phase Portrait» button, the code calls the function Button_PhasePortraitALLPushed2 (app, event). Its action is similar to the function ButtonPushed (app, event) (pressing the button «Phase portrait taking into account the model conditions»), but at the same time a different scale is set on the graphs, and there is no check for correctly entered coefficients (Figure 7).

Clicking on the «Clear Graph» button, the function is called

```
function Button_ClearPushed(app, event)
app.ErrorLabel.Text = '';
app.ErrorLabel.FontColor = [1.00 1.00 1.00];
app.Lamp.Color = [0.00 1.00 0.00];
cla(app.UIAxes);
cla(app.UIAxes2);
legend(app.UIAxes, 'off');
legend(app.UIAxes2, 'off');
end
```

This function clears the phase portrait and the function versus time graph cancels the display of legends and turns the lamp green.

In the remaining windows for the Phillips business cycle models (MathMod2.mlapp), the energy market (MathMod3.mlapp), and inflation expectations (MathMod4.mlapp), the interface and code remain the same.

The operating model of the software package can be presented in the form of a scheme.

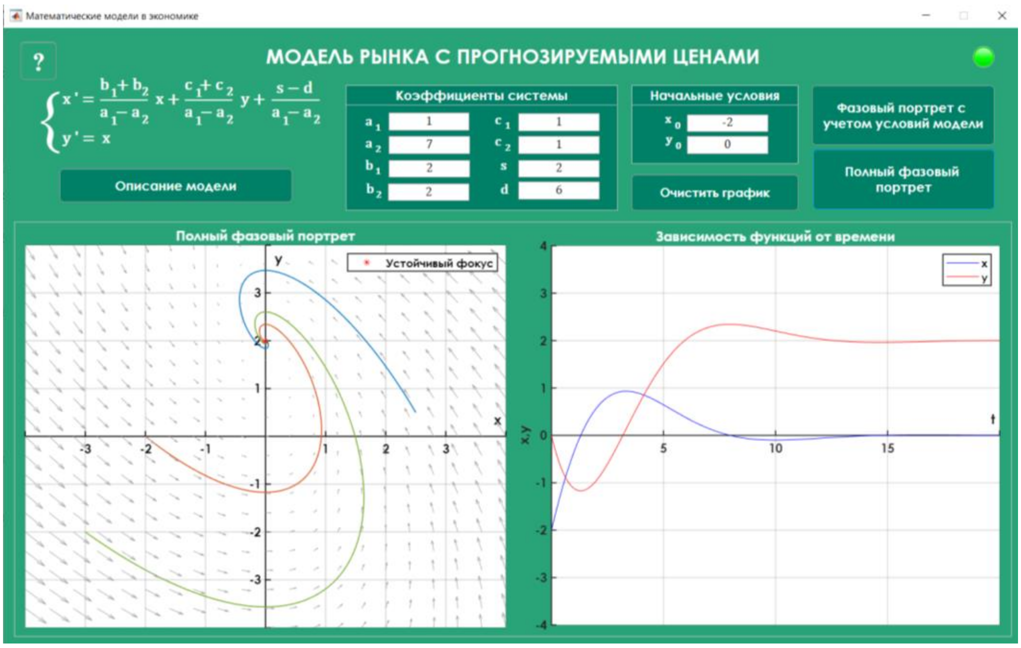


Figure 7: Graph «Complete phase portrait»

4. Application of the Educational Software Package

The use of the digital resource in the framework of a laboratory practicum is carried out using the case method. For the formation of the research skills of the trainees, it is advisable to present the case in the form of 3 blocks: information-coordinating, practical, and control. This makes it possible to differentiate different types of activity, as well as to determine the level of knowledge and skills building for each type.

Information-coordinating: familiarization with the situation, identification of the problem, analysis.

Practical: search and formulation of alternative solutions, analysis of alternative solutions, substantiation of the solution to the problem.

Control block: presentation of results employing IT, evaluation of results.

A general approach has been developed for all cases.

Case. Situation: a mathematical model of the economic process is given. It is necessary, based on the results of a mathematical study, to make a forecast about the possible development of the economic situation. Determine the conditions under which this or that outcome occurs. Note: the software package «Mathematical Models in Economics», the Word text editor, the services Google+, MOODLe, and ZOOM are available.

The implementation of the case takes place in five stages. Stage 1: collecting information about the model under study. Stage 2: the mathematical study of the model. Stage 3: selection of coefficients of the system of differential equations according to the conditions obtained in the course of mathematical investigation. Stage 4: transfer of the obtained mathematical results to the considered economic process. Stage 5: preparation of a case report.

When performing cases, individual, pair, and group independent work of students is possible. In individual work, a market model with predicted prices, Phillips business cycle are considered. These systems have one singular point. The energy market model (two singular points) can be used for work in pairs. Each student examines one of the points, then the results are combined, and they are jointly checked using the software package. In group work, the model of inflationary expectations (three singular points) is considered.

The preparation of the report for group work is carried out using Google+ documents, the report is posted in the MOODLe system, where it is evaluated. In addition, it is possible to use other digital tools and services described in the article by the authors E.A. Konopko, O. P. Pankratova, D.A. Abdullaeva, A.M. Edieva, V.N. Taran [11].

5. Conclusion

25 students who studied using the developed digital resource and the use of the case method when performing a laboratory workshop were surveyed. Students noted that they gained experience in:

- organization and planning of their research activities (60%),
- optimal distribution of time (50%);
- work with information, analysis of assigned tasks, their solution (70%);
- correct formulation of their thoughts when defending their point of view (40%);
- the ability to listen to others (30%);
- presentation of accurate, well-structured reports (40%);
- communication (60%);
- teamwork (50%);
- coordination of their own and others' work (30%);
- team leadership (20%).

All students noted that the gained experience contributed to the formation of their research skills that will be useful for preparing a bachelor’s thesis.

The software package «Mathematical Models in Economics» complements the electronic information and educational environment of Astrakhan State University for bachelors of the specialties «Applied Mathematics and Informatics», «Pedagogical Education, Profiles Mathematics and Informatics», allows forming research skills, skills of independent work of students. The complex can be used in the educational process of other bachelor majors, studying applications of differential equations in the field of economics.

6. References

[1] T. V. Shushara, N. A. Delvig, S. M. Abraamyan, Digitalization of Education: Distant Learning Prospects and Realities. CEUR Workshop Proceedings. Distance Learning Technologies 2019 URL: http://ceur-ws.org/Vol-2834/Paper48.pdf526-532

[2] V. N. Kormakova, A. G. Klepikova, M. A. Lapina, J. Rugelj, ICT Competence of a Teacher in the Context of Digital Transformation of Education. CEUR Workshop Proceedings. Distance Learning Technologies 2020 URL: http://ceur-ws.org/Vol-2914/paper12.pdf138-150

[3] K. A. Markelov, E. V. Polyanskaya, O. K. Mineva, V. N.Taran, Paradigm Transformation of Education System in Digital Reality. In the V International Scientific and Practical Conference «Distance Learning Technologies» (DLT 2020).Yalta, Crimea, September 22-25, 2020. PP. 186-198. http://ceur-ws.org/Vol-2914/paper16.pdf

[4] T. N. Filimonenkova, Experience in the Development and Application of an Electronic Learning Resource in the Discipline «Internet Programming» Based on Moodle. CEUR Workshop Proceedings. Distance Learning Technologies 2019 URL: http://ceur-ws.org/Vol-2834/Paper11.pdf

[5] A. Luˇstek, S. Jedrinovi´c, J. Rugelj, Supporting Teachers in Higher Education for Didactic Use of the Learning Environment Moodle. CEUR Workshop Proceedings. In 2019 International Conference on Innovative approaches to the application of digital technologies in education and research SLET-2019. URL: http://ceur-ws.org/Vol-2494/paper_20.pdf

[6] I. Bubenschikova, M. Kolomina, M. Litvinova, T. Vezirov, Use of Project Technologies in Implementation of Activity Approach in Training First-Year Bachelor´s Degree Students at Modern Stage of Education Digitalization. CEUR Workshop Proceedings. In 2019 International Conference on Innovative approaches to the application of digital technologies in education and research SLET-2019. URL: http://ceur-ws.org/Vol-2494/paper_8.pdf

[7] Y. Bogatyreva, A. Privalov, V. Romanov, E. Konopko, The Organization of Scientific Research Work of Students in the Conditions of the Digital Environment in the Modern Universities. CEUR Workshop Proceedings. In 2019 International Conference on Innovative approaches to the application of digital technologies in education and research SLET-2019. URL: http://ceur-ws.org/Vol-2494/paper_7.pdf

[8] S.V. Panyukova Digital tools and services in teacher's work. Study guide. Moscow, «Pro-Press», 2020. 33 p.
[9] M.V. Kolomina. Organization of research activities of students in the study of systems of differential equations, using applied software. Happy to be a teacher!. Proceedings of the 2nd All-Russian Youth school conference [Schastliv byt' uchitelem!: materialy II Vserossijskoj molodezhnoj shkoly-konferencii], April 27, 2019. Ryazan. 2019. P. 53-57.
[10] M.V. Kolomina Differential Equations. The course of lectures: for students studying in the specialty 010200 Applied Mathematics and Informatics [Differencial'nye uravnenija. Kurs lekcij: dlja studentov, obuchajushhihsja po special'nosti 010200 Prikladnaja matematika i informatika]. Federal Agency for Education, Astrakhan State University. Astrakhan, 2007, 172 p.
[11] E. A. Konopko, O. P. Pankratova, D. A. Abdullaev, A. M. Ediev, V. N. Taran, Digital Education Toolkit and an Overview of Distance Learning Resources CEUR Workshop Proceedings. DLT 2020 Distance Learning Technologies 2020 http://ceur-ws.org/Vol-2914/paper35.pdf