IoT Control Systems base on Fuzzy PWM-controller

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Abstract

The paper investigates the use of fuzzy data converters based on intelligent systems of fuzzy inference in the Internet of Things systems. An approach has been developed for converting fuzzy input parameters read from sensors into a PWM signal based on the Takagi-Sugeno fuzzy algorithm. Based on the suggested method an IoT-based smart fan was developed to demonstrate the characteristics of the proposed method for intelligent control of Internet of Things devices that support a PWM signal. Finally, the findings of the experimental data were utilized to simulate the performance of the fuzzy model, using three parameters: temperature, relative humidity, and carbon dioxide concentrations (CO_2) vis-à-vis the PWM signal output. The proposed method shows the simplicity of training a smart fan control system and makes the possibility of efficient energy consumption.

Keywords [1]

Control systems, fuzzy inference, Internet of Things, fuzzy PWM-controller, Smart fan.

1. Introduction

Nowadays intelligent systems continue to rapidly flood the world. Although IoT technology is still thought to be about smart homes and greenhouses, actually an IoT system can be any object that contains sensors, software, or other technologies to connect and communicate with other objects over the Internet. So, the range of such objects is huge - from ordinary household items to complex industrial tools. According to Oracle, there are already more than 7 billion devices connected to the IoT, and that number continues to grow. And by 2025, experts predict that this number will cross the mark of 22 billion devices [1].

Among the devices connected to the IoT, which are quite popular in household use, are smart ventilation systems. It is quite an outdated notion that fans may be inefficient in comparison with air conditioners. However, the ceiling fans are much more healthier and energy-efficient. Additionally, they constantly improve and become smarter allowing remote control by your smartphone. For instance, Google and Amazon successfully created a line of smart fans which work with voice assistants and can be controlled remotely from the smartphone through the Internet. They allow users to control the temperature and humidity of the air while away from home, as well as customize climate control to their liking. Therefore, today there is a great need to develop and improve effective methods of intelligent control for IoT ventilation systems. One of the methodologies for the intellectualization of control systems is the development of fuzzy inference (FIS) systems, which are based on fuzzy logic, the founder of which is L. Zade [2]. It is to FIS-systems will be the main attention in this work in the development of intelligent ventilation systems.

It should also be noted that such systems, in addition to the higher identified advantages, have disadvantages, one of which is energy consumption [3–5]. Heating, ventilation, and air-conditioning systems predominate in energy usage in commercial buildings. The numbers variate between 40 percent and 70 percent of the total building electricity consumption [6]. The International Energy Agency predicts that the demand for rooms with cooling systems will increase three-fold between

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2010 and 2050 [7]. Therefore, the development of IoT-based platforms in the field of smart ventilation needs to be improved in terms of resource consumption. We must develop new techniques for improving the energy efficiency of ventilation systems to achieve the reduction in overall building energy consumption and as a result decrease operating electricity costs.

IoT smart fan systems still rely on the traditional methods of control strategies such as cycling or on/off control, staging, modulation, or proportional control [8]. These may be quite obsolete, inaccurate, and inefficient for energy consumption in smart fan systems. The novel idea is to apply fuzzy logic in early IoT-based fans to assess the environmental factors like temperature, humidity, carbon dioxide concentrations, etc and convert them by Takagi-Sugeno [9] algorithm into Pulse width modulation (PWM) signal to control the speed of motors, heat output of heaters, and the other parameters in an energy-efficient and quieter manner [10]. Thus, the proposed smart inverter fan based on fuzzy logic will provide comfortable levels of cooling and optimized electricity consumption.

The main contribution of the paper is:

- To consider the Takagi-Sugeno algorithm for input data conversion in IoT-based fans with a fuzzy expert system.
- To propose the use of Takagi-Sugeno systems for converting input data from sensors into an output PWM signal for efficient energy consumption in smart ventilation systems.
- The experimental results demonstrate dependencies between input and output variables of a smart fan controller based on the fuzzy converter, which transforms quality indicators into PWM signal, which assume its high performance and cost-effectiveness.

The paper is structured as follows: Section 1 introduces the existing methods of control strategies in smart ventilation systems and describes the energy consumption problem. Section 2 introduces the fundamentals of fuzzy control systems and describes the Takagi-Sugeno algorithm. In Section 3, Takagi-Sugeno system for converting output data from sensors into an output PWM signal is proposed to provide efficient energy consumption in smart air ventilation systems. In Section 4, the experiment results are demonstrated. Finally, Section 5 concludes the research paper.

2. Fundamentals of fuzzy control systems

Intelligent fuzzy systems are actively used to solve a wide class of problems in many areas of industry and life (medicine, production, safety, management, etc.). Belonging to the class of intellectual, fuzzy systems are designed to solve highly specialized tasks of the creative direction. Such tasks may include decision-making systems, expert systems, artificial intelligence systems, testing, assessment, classification systems, etc. [11-16].

2.1. Fuzzy inference

Control systems based on fuzzy converters of input data into output data are used in cases where the following features take place:

- The system operates with qualitative values and characteristics
- There is incomplete data about the modeled object and its environment
- The investigated object is extremely difficult to model and find ideal solutions
- There is a non-linear dependence of the input-output data
- Decision making by the system is based on the knowledge and experience of experts in a specific problem area

Figure 1 shows a general scheme of a fuzzy inference system, which is equally suitable for all fuzzy algorithms [11] (Takagi-Sugeno, Mamdani, Larsen ...). As can be seen in the figure, many input values $X = \{x_i : i = \overline{1,n}\}$ are converted into a set of output conclusions $Y = \{y^j : j = \overline{1,m}\}$, using an inference algorithm on a fuzzy knowledge base. The knowledge base, typical for any fuzzy inference algorithm, consists of blocks of fuzzy rules $B = \{R_w, w = \overline{1,L}\}$ products of IF-THEN statements (1).

$$R_w:\ IF\ \ \mathrm{x}\ is\ A\ \ \ THEN\ \ \ \mathrm{y}\ is\ B, \tag{1}$$

where x ⊆ X, y ⊆ Y, A – input linguistic variable, B – output linguistic variable respectively.

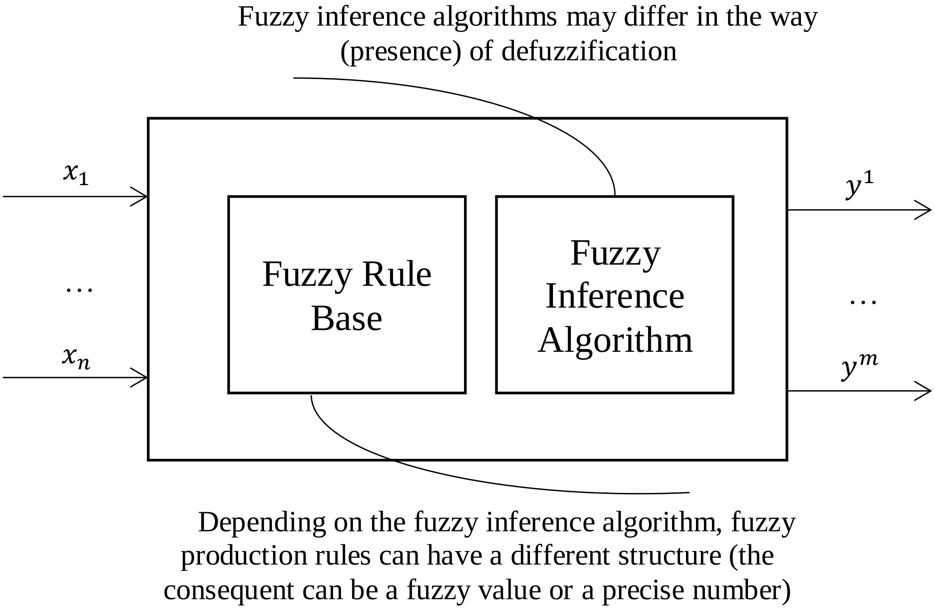

Figure 1: The general scheme of fuzzy data converter

In general, the functioning of fuzzy systems consists of the following stages [11]:

1. Fuzzification of input variables
2. Activation of fuzzy production rules
3. Aggregation of rule subconclusions (in consequent)
4. Accumulation of sub-conclusions of the consequent of fuzzy rules (carried out only for those systems, the consequents of which are fuzzy values)
5. Defuzzification of output values (or a procedure similar to defuzzification, if the consequents of the rules are clear numbers)

Let us take a closer look at the fuzzy logical conclusion of Takagi-Sugeno [9], since for research in this work it is of the greatest interest in the context of fuzzy data converters in IoT systems. The main feature of Takagi-Sugeno systems is the ability to transform qualitative indicators (fuzzified values) into quantitative ones since the subconclusions of fuzzy rules consist of functional dependencies on a set of input data (primary, non-fuzzified) and generate precise numbers.

The Takagi-Sugeno fuzzy rule is:

$$R_w^j:\ IF\ \ \mathrm{x}\ is\ A_w^j\quad THEN\quad y_w^j = k_{w0}^j + \sum_{i=1}^{n} k_{wi}^j x_i, \tag{2}$$

where j – system-generated output number, w – fuzzy rule number, n – number of input variables, $\boldsymbol{k_{w0}^j}$ – free coefficient. The consequent of a rule is essentially a weighted summation of non-fuzzified input prerequisites.

An output conclusion for every y^j (see Figure 1) according to the Takagi-Sugeno algorithm, it is the finding of the weighted average of rule subconclusions in a fuzzy knowledge base (3):

$$y^j = \frac{\sum_{w=1}^{L}\left[\bigwedge_{i=1}^{n}\mu_w^j(x_i)\right]\left(k_{w0}^j + \sum_{i=1}^{n}k_{wi}^j x_i\right)}{\sum_{w=1}^{L}\left[\bigwedge_{i=1}^{n}\mu_w^j(x_i)\right]} = \frac{\sum_{w=1}^{L}\mu_w^j(\mathrm{x})y_w^j}{\sum_{w=1}^{L}\mu_w^j(\mathrm{x})}, \tag{3}$$

where $\wedge -$ is the operation of taking the minimum, $\mu(\mathrm{x})$ – is the membership function of the input value to a fuzzy term.

3. Control of the output PWM signal in IoT systems based on intelligent fuzzy converters (fuzzy PWM-controller)

PWM-signal [10, 17] (pulse width modulation) – one of the varieties of a digital signal and is designed to control the power (speed) on output devices based on periodic on / off voltage on the port (Figure 2). One of the tasks related to the operation of the microcontroller in IoT systems is to control the PWM signal, and often its value depends on the input parameters read from other sensors (temperature, smoke sensor, proximity, etc.).

The total power consumption at the PWM output is calculated as:

$$P_{out} = \frac{E_r + E_{dr} + E_f + E_{rc}}{T_{pwm}}, \tag{4}$$

where E_r – the energy consumption during the time of transition from Low to High mode, E_{dr} – the energy consumption during the current supply time to maintain High mode, E_f – the energy consumption during the time of transition from High to Low mode, E_{rc} – the energy consumption during the time for current regeneration in order to maintain Low mode, T_{pwm} – total operating time of the PWM signal (0..255).

Thus, there is often a non-linear relationship between the input parameters and the PWM output signal in such systems. It should be noted that the read-out data from the sensors are very often qualitative indicators and require the introduction of fuzzy linguistic values for their further operation and classification.

The paper proposes the use of Takagi-Sugeno systems for converting input data from sensors into an output PWM signal. A characteristic feature of this approach is the ability to convert input data from several sensors, realizing a nonlinear input-output dependence (a model of a converter of quality data from sensors into a PWM signal is shown in Figure 3). This approach allows the implementation of intelligent IoT systems with high energy savings due to the use of PWM as a control signal.

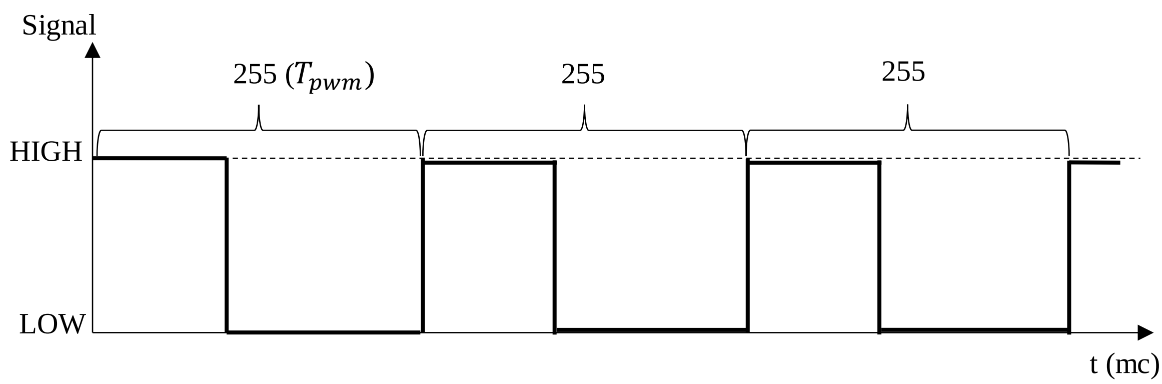

Figure 2: Scheme of the digital PWM output signal

In Figure 3, the input parameters are represented by a vector of fuzzy (fuzzified) values, which are qualitative characteristics read from the sensors:

$$\tilde{X} = \{\tilde{x}_i, i = \overline{1, n}\},$$

$$\tilde{x}_i = \left\{ \frac{x_i}{\mu_{term\,1}(x_i)} + \frac{x_i}{\mu_{term\,2}(x_i)} + \dots + \frac{x_i}{\mu_{term\,Q}(x_i)} \right\},$$

where $+$ is a union operation, $\tilde{X}$ – vector of fuzzy input values.

Subconclusions of fuzzy rules (Takagi-Sugeno fuzzy rules), form the weighted values of the PWM output signal from 0 to 255, which then go through the procedure for finding the weighted average value, which, accordingly, will also lie in the range [0..255].

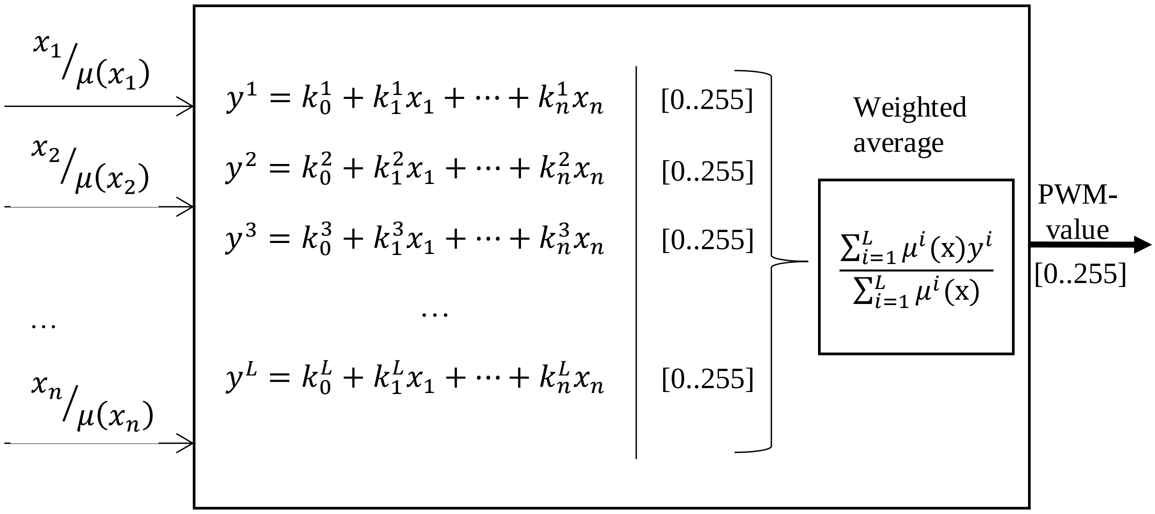

Figure 3: Model of the converter of fuzzy (quality) data from sensors to a PWM signal (range of values [0..255])

4. Smart fan based on the fuzzy PWM-controller for convert of quality indicators into PWM signal

4.1. Problem definition of smart ventilation systems

In this paper, a model of an intelligent fan based on fuzzy control has been developed using the proposed in this article method of fuzzy transformation of quality indicators into a PWM signal. The use of a fuzzy controller allows the ventilation system to work in different modes of operation, adjusting to the external environment, and also taking into account the individual requirements of the customer, the climate of a particular region, medical contraindications, etc. Based on a fuzzy knowledge base, this system makes it possible to formalize the customer's requirements in the form of an expert set of rules, makes it possible to make the operating modes more varied.

It is also worth mentioning that efficient energy consumption, sustainability, environment-friendly – terms that should be taken into consideration in the modern scientific world. The heating, ventilation, and air conditioning systems can be the largest energy consumers in the building. The different approaches to modeling these systems and providing them with additional controllers can change the situation for the better.

The most common methods to solve the above problems are the classic use of proportional-integral-differential (PID) controllers and Computational Intelligence techniques [18].

The advantage of using intelligent control methods in ventilation systems over the classic ones is the ability to regulate the room temperature at partial load, minimize system steady-state error. In recent studies [19–20] there was an attempt to use fuzzy logic to model the cooling process of ventilation systems. However, existing systems do not take into account that the resistance of transistors and resistors also leads to additional energy loss, because they burn part of it as a heat. Such systems are constantly running at full speed, making a lot of noise and consuming a lot of energy. The solution may be to use PWM, which varies the speed of the motors of the devices, so they consume only as much energy as they need [10].

4.2. The architectural design of the Smart ventilation system

Basic fuzzy-based architectural model of smart ventilation system consists of (Figure 4):

1. Sensor components:

1.1. Sensor temperature
1.2. Sensor relative humidity
1.3. Gas sensor

2. Micro Controller Unit – intermediate component, which receives transmitted information from the sensors for processing of the collected data
3. Cloud API

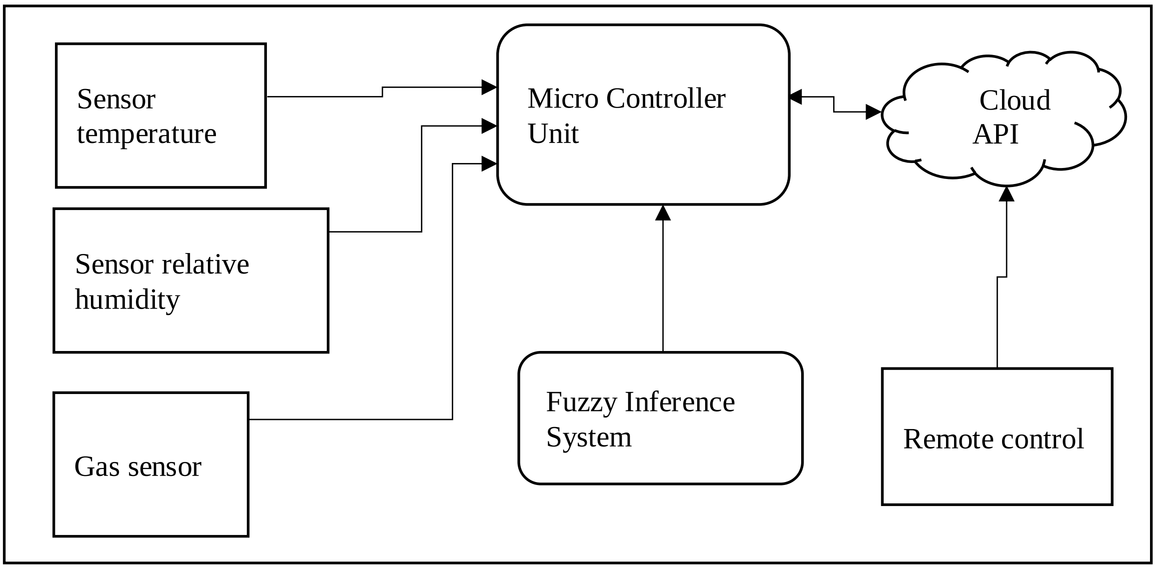

Figure 4:Block diagram of the fuzzy-based smart ventilation system

4.3. Fuzzy Smart Fan Controller Model

To model the edge values (terms) of linguistic variables, we use sigmoidal membership functions:

$$\mu(x) = \frac{1}{1 + \exp[-b(x - c)]}, \tag{5}$$

where b – the number characterizing the slope of the graph (the larger the b, the greater the slope), c – inflection point of the function ($\mu(c) = 0{,}5$).

Furthermore, we will use the bell-shaped membership functions to model the mean values of fuzzy terms:

$$\mu(x) = \frac{1}{1 + \frac{|x - c|^{2b}}{\alpha}}, \tag{6}$$

where c – central (modal) value at which $\mu(c) = 1$, $b \geq 0$ – number characterizing the slope of the graph (similar to sigmoidal membership functions), $\alpha > 0$ – the distance from the center c to the inflection points of the function, where at $b = 0{,}5$ the $\mu(c \pm \alpha) = 0{,}5$ is fulfilled.

Methods for constructing fuzzy membership functions are considered in [21].

Linguistic variable "temperature" and its membership function graph (Figure 5):

$$T = \left\{ \mu_{LOW}(x) = \frac{1}{1 + e^{0{,}5x - 6{,}25}} + \mu_{MEDIUM}(x) = \frac{1}{1 + \frac{|x - 25|^2}{30}} + \mu_{HIGH}(x) = \frac{1}{1 + e^{-0{,}5x + 18{,}75}} \right\}.$$

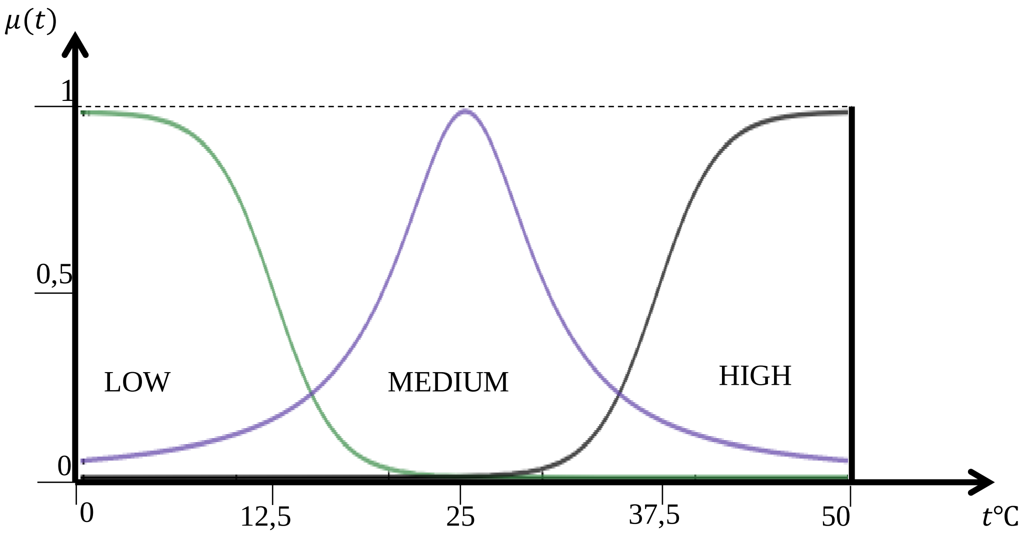

Figure 5: Membership functions of the linguistic variable "temperature"

Linguistic variable "carbon dioxide concentration" and its membership function graph (Figure 6):

$$CO2 = \left\{ \mu_{EX}(x) = \frac{1}{1 + e^{0,03x-24}} + \mu_{GOOD}(x) = \frac{1}{1 + \frac{|x-850|^{1,5}}{1000}} + \mu_{HEAVY}(x) = \frac{1}{1 + e^{-0,05x+50}} \right\}.$$

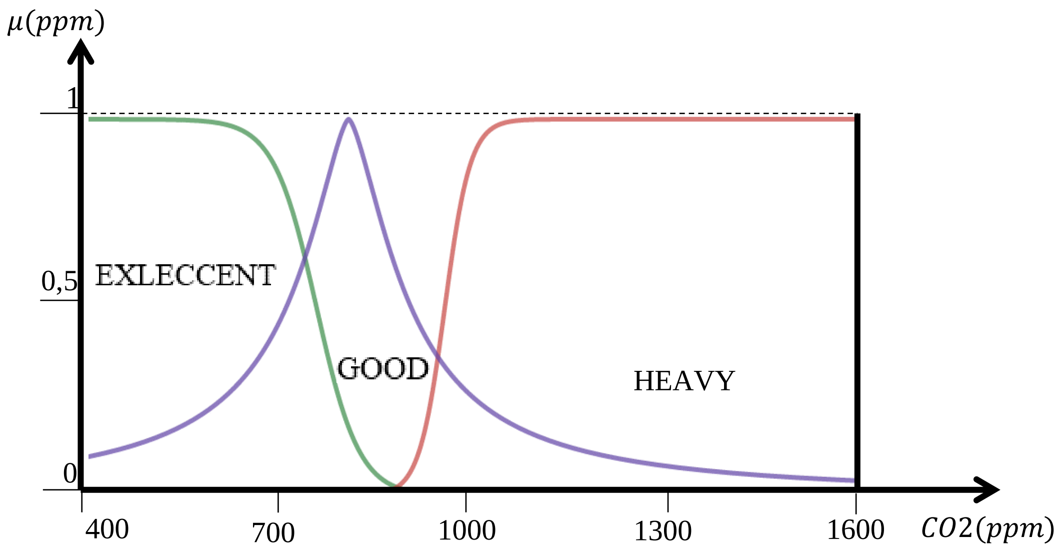

Figure 6: Membership functions of the linguistic variable "carbon dioxide concentration"

Linguistic variable "relative humidity" and its membership function graph (Figure 7):

$$RH = \left\{ \mu_{LOW}(x) = \frac{1}{1 + e^{0,1x-5}} + \mu_{HIGH}(x) = \frac{1}{1 + e^{-0,1x+5}} \right\}.$$

Table 1 presents a block of Takagi-Sugeno fuzzy production rules for controlling fan power through a PWM signal, consisting of 18 fuzzy rules. It is also important to note that the maximum number of rules of a fuzzy system can be found according to (7).

$$RuleMaxNum = \prod_{i=1}^{N} terms(L_i), \tag{7}$$

where $terms$ – returns the number of fuzzy terms of a linguistic variable L_i, N – number of input variables.

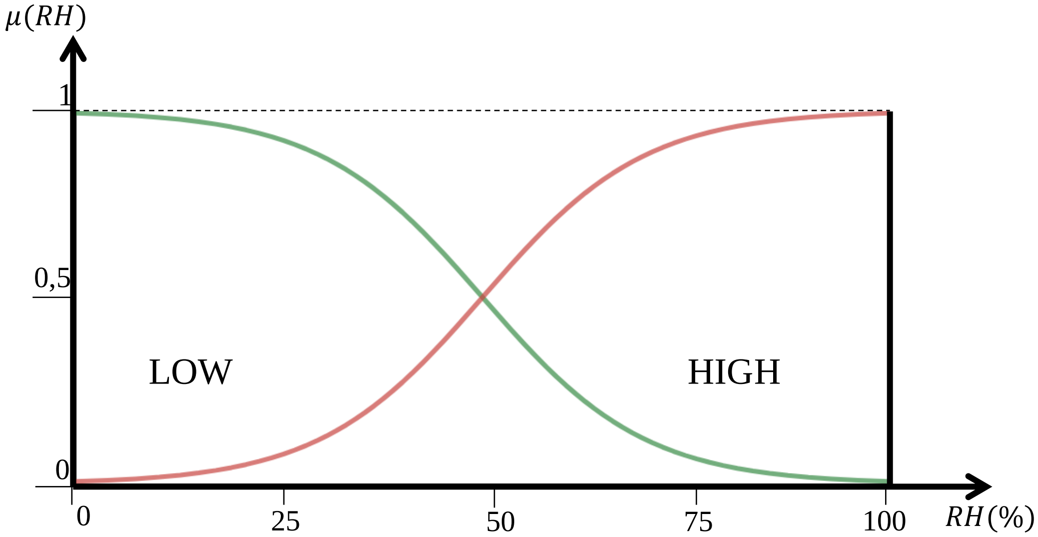

Figure 7: Membership functions of the linguistic variable "relative humidity"

Table 1
Takagi-Sugeno fuzzy rule block for Smart fan control

R_i	x_1 (t°C)	x_2 (ppm)	x_3 ($RH(\%)$)	y_i (PWM)
R_1:	LOW	EXCELLENT	LOW	$y_1 = 0{,}5x_1 + 0{,}1x_2 + 1{,}5x_3$
R_2:	LOW	EXCELLENT	HIGH	$y_2 = 0{,}25x_1 + 0{,}1x_2 - 0{,}15x_3$
R_3:	LOW	GOOD	LOW	$y_3 = 0{,}6x_1 + 0{,}15x_2 + 1{,}7x_3$
R_4:	LOW	GOOD	HIGH	$y_4 = 0{,}5x_1 + 0{,}12x_2 + 0{,}2x_3$
R_5:	LOW	HEAVY	LOW	$y_5 = 0{,}7x_1 + 0{,}1x_2 + 1{,}6x_3$
R_6:	LOW	HEAVY	HIGH	$y_6 = 0{,}5x_1 + 0{,}1x_2 + 0{,}7x_3$
R_7:	MEDIUM	EXCELLENT	LOW	$y_7 = 0{,}6x_1 + 0{,}15x_2 + 1{,}4x_3$
R_8:	MEDIUM	EXCELLENT	HIGH	$y_8 = 0{,}3x_1 + 0{,}12x_2 - 0{,}1x_3$
R_9:	MEDIUM	GOOD	LOW	$y_9 = 0{,}7x_1 + 0{,}17x_2 + 1{,}7x_3$
R_{10}:	MEDIUM	GOOD	HIGH	$y_{10} = 0{,}4x_1 + 0{,}14x_2 + 0{,}3x_3$
R_{11}:	MEDIUM	HEAVY	LOW	$y_{11} = 0{,}7x_1 + 0{,}11x_2 + 1{,}8x_3$
R_{12}:	MEDIUM	HEAVY	HIGH	$y_{12} = 0{,}8x_1 + 0{,}1x_2 + 0{,}9x_3$
R_{13}:	HIGH	EXCELLENT	LOW	$y_{13} = 0{,}8x_1 + 0{,}2x_2 + 1{,}7x_3$
R_{14}:	HIGH	EXCELLENT	HIGH	$y_{14} = 0{,}7x_1 + 0{,}15x_2 + 0{,}8x_3$
R_{15}:	HIGH	GOOD	LOW	$y_{15} = 0{,}9x_1 + 0{,}14x_2 + 2x_3$
R_{16}:	HIGH	GOOD	HIGH	$y_{16} = 0{,}7x_1 + 0{,}14x_2 + 0{,}5x_3$
R_{17}:	HIGH	HEAVY	LOW	$y_{17} = x_1 + 0{,}11x_2 + 1{,}7x_3$
R_{18}:	HIGH	HEAVY	HIGH	$y_{18} = 0{,}95x_1 + 0{,}11x_2 + 0{,}34x_3$

4.4. Simulation Experimental Setup

Table 2 and Figure 8 show the experimental results obtained during the simulation of the intelligent fuzzy Smart Fan system, which converts the output metrics from the sensors into a PWM signal in the range from 0 to 255. Among the advantages of this system, it is worth to mention the high operating speed (no defuzzification), as well as the simplicity of training the system, which essentially boils down to finding the weights of the input parameters in the consequent of fuzzy rules and which, if necessary, can also be produced by a subject matter expert.

Table 2
Experimental results of operation of the Smart Fan fuzzy control system

№	t°C	ppm	$RH(\%)$	PWM
1	23	400	70	77
2	28	820	40	208
3	14	1200	85	198
4	37	940	29	203
5	40	730	50	201
6	15	570	65	130
7	25	1000	90	195
8	30	1300	80	234
9	42	700	45	198
10	21	1400	33	234

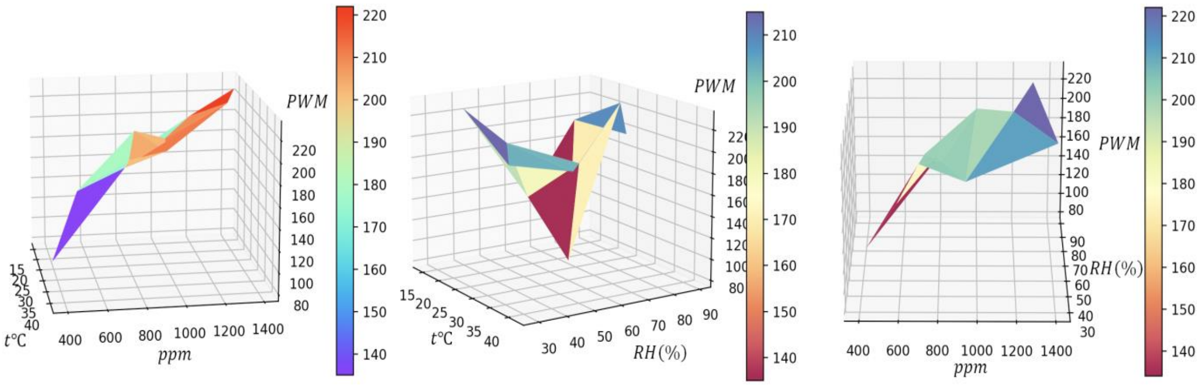

Figure 8: Graph of dependencies between input and output variables of a fuzzy Smart fan controller

5. Conclusions

A fuzzy PWM controller based on the Takagi-Sugeno fuzzy inference algorithm is proposed. In this paper, a method of fuzzy control of the output PWM signal based on the Takagi-Sugeno fuzzy inference algorithm is presented. An IoT system of an intelligent fan has been developed based on the approach of converting fuzzy input parameters read from sensors into a PWM signal to control the fan screw rotation speed. A mathematical model of an intelligent fan based on fuzzy control has been developed, as well as its hardware architecture. In this approach, the model operates with three input parameters, namely, temperature, relative humidity and carbon dioxide concentrations (CO_2) under different operating conditions vis-à-vis the PWM signal output. Experimental studies have been carried out that demonstrate the characteristics of the proposed methods for intelligent control of Internet of Things devices that support a PWM signal, using the example of a Smart Fan.

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