Behavioral Properties of Bounded Solutions for a Weakly Nonlinear Impulse System that Describe the Dissemination of Information on Social Networks

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Abstract

Processes with instantaneous (abrupt) changes are observed in radio engineering (pulse generation), in biology (heart work, cell division, signal transmission by neurons), in control theory (work of industrial robots), in social systems (social communications, information dissemination). Therefore, the qualitative study of impulse systems in this work is an urgent task in modern theory of mathematical modeling.

The work is devoted to the study of the existence of bounded solutions along the entire real axis (on the half-axis) of weakly nonlinear systems of differential equations with impulsive perturbations at fixed moments of time. The notion of a regular and weakly regular system of equations for the class of weakly nonlinear impulse systems of differential equations is introduced.

Sufficient conditions for the existence of a bounded solution for an inhomogeneous system of differential equations in the case of weak regularity of the corresponding homogeneous system of equations are obtained. The conditions for the existence of the unique bounded solution on the whole axis for weakly nonlinear impulse systems are established. The obtained results are applied to the study of bounded solutions of impulsive SIR model that can be considered as a model which describes the dissemination of information on social networks.

Keywords

Differential equations, impulse system, bounded solution, Green-Samoilenko function, SIR model.

1. Introduction

The information space in the modern world is constantly under the influence of various destabilizing factors of various nature. The problem of dissemination of reliable information became especially acute with the emergence and rapid development of social networks. In recent years, we have seen a comprehensive penetration of the impact of information through social networks in almost all areas of activity. It should be noted that along with the dynamic dissemination of useful and important information of a mass nature, the replication of openly harmful information messages acquires. It is not uncommon for the spread of malicious information messages to provoke serious consequences. In particular, in the context of the SARS-CoV-2 pandemic, entire information wars are being waged on social networks to discredit the efforts of the world community to take control of the spread of the virus. A clear example is interference in the electoral process in many countries. It was the massive spread of malicious

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information that significantly affected the results of the US elections, the results of the Brexit referendum, and so on.

In this sense, it is extremely important to study mathematical models that make it possible to obtain constructive conditions for the existence of bounded solutions of nonlinear systems, which are described by differential equations with momentum. After all, such systems allow to model the behavior of "viral impressions" both in biological systems and the circulation of information messages on social networks. And the conditions for the existence of a single limited solution on the entire axis for weakly nonlinear pulse systems that allow for qualitative analysis of the solutions of the respective systems provides an opportunity to develop strategies to reach the target audience with socially significant information messages and build protection against population "infection" social networks with harmful information fakes.

The modern natural sciences and technology development contributes to the emergence of problems described by differential equations systems with discontinuous trajectories and, in particular, to the development of mathematical impulse systems theory. In mathematical modeling of these processes, such perturbations duration can often be conveniently neglected, believing that they have the character of an impulse.

This idealization leads to the need to study of differential equations systems, the solutions of which change abruptly. But not only the idealization of the replacement of short-term perturbations by "instantaneous" leads to differential equations with discontinuous trajectories. Often breaks of certain dependencies in the studied system are its essential characteristic.

The theory of systems with impulse influence has a wide range of applications. Such systems arise in the study of automatic control impulse systems, in mathematical modeling of various mechanical, physical, biological and other processes.

For example, processes with abrupt changes are observed in mechanics (movement of a spring at impact action, functioning of the clockwork, change of the rocket speed at steps separation), in radio engineering (generation of impulses), in biology (heart work, cell division, signaling by neurons), in control theory (work of industrial robots). Therefore, the qualitative study of impulse systems in this paper is an actual problem in modern mathematical modeling theory.

The theory of nonlinear differential equations systems with impulse influence, to which a number of natural science and technology problems are reduced, has been enriched with significant results in recent decades. Among the studied systems there are systems with impulse action with weak nonlinearity. The complexity of the mathematical formulation of the problem for the analytical study of this system type is due to the corresponding dynamic processes nonsmoothness. This leads to the need to develop methods for studying weakly nonlinear systems of differential equations with impulse effects. Therefore, the study of this type systems solutions is an urgent task today.

2. Analysis of literature sources and problem statement

The most important and effective impulse systems research have been conducted in the last decade. In [1] the conditions guaranteeing the hyperbolicity of differential equations systems with impulse action are established. The obtained hyperbolicity conditions allow us to investigate the existence of bounded solutions for inhomogeneous multidimensional differential equations systems with momentum perturbation. In [2], sufficient conditions for the existence of an asymptotically stable invariant toroidal manifold of linear extensions of a dynamic system on a torus in the case when a matrix of a system commuting with its integral are obtained. The proposed approach is applied to the study of the stability of invariant sets of some class of discontinuous dynamical systems.

In [3] the review of the most modern research methods for impulse differential equations solutions stability and their application to problems of impulse control is carried out. In [4], the trivial torus exponential stability for one class of nonlinear extensions of dynamical systems on a torus is proved. The obtained results are applied to the study of the toroidal sets stability for impulse dynamical systems. In [5,6] the problem of constructing approximate adaptive control, including the case of impulse control, is considered for one infinite-dimensional problem with a target functional of Nemytsky type. The method of averaging for obtaining approximate adaptive control is substantiated. In [6] the concept of a impulsed non-autonomous dynamical system is introduced. For it the existence and properties of a impulse attracting set are investigated. The obtained results are applied to the study of the two-dimensional impulse-perturbed Navier-Stokes system stability. In [7] the recursive properties of almost periodic motions of impulsed dynamical systems are studied. The obtained results are applied to the study of discrete systems qualitative behavior. In [8] the properties of stability in relation to external (control) perturbations for differential equations systems with impulse action at fixed moments of time are considered. Necessary and sufficient stability conditions for classes of impulsive systems having a Lyapunov type function are obtained. In [9], a non-autonomous evolutionary inclusion with impulse influences at fixed moments of time is considered. A corresponding non-autonomous multivalued dynamical system is being constructed, for which the existence of a compact global attractor in phase space is proved. In [10–13], the existence of global attractors in multi-valued discontinuous infinite-dimensional dynamical systems, which can have trajectories with an infinite number of impulsed perturbations, was proved. The obtained results are applied to the asymptotic behavior study of the weakly nonlinear impulsed-perturbed parabolic equations and inclusions.

The properties of optimal sets of practical stability of differential inclusions and maximum sets of initial conditions in problems of practical stability were studied in [16-18]. In papers [19-20], the practical stability of discrete systems of discrete inclusions with spatial components is investigated.

In all the above works the bases of the qualitative theory of differential equations with impulse action are stated. In essence, the foundations of the qualitative theory of impulse systems based on the qualitative theory of differential equations, methods of asymptotic integration for such equations, the theory of difference equations and generalized functions were laid. However, the question of the solutions existence for weakly nonlinear impulse systems has not yet been fully investigated.

At the same time, the works in which important results in the field of information technologies and social communications were obtained deserve attention. In works [21–22] it is investigated Modification of the algorithm (OFM) S-box, which provides increasing crypto resistance in the post-quantum period and detection of slow DDoS attacks based on user's behavior forecasting.

Ways to improve the quality of signal detection by taking into account interference and the method of signal detection of covert means of obtaining information were studied in [23]. The model of the accuracy of the localization of the hidden transmitter based on multi-position distance measurement and the method of obtaining estimates of the parameters of the radio signals of the hidden means of obtaining information are described in [24–25]. The work [26–27] investigated the system of indicators and criteria for assessing the level of functional stability of information heterogeneous networks and special purpose networks. In works [28-29] studied applied control algorithm functionally sustainable production processes industry.

3. The purpose and objectives of the research

The aim of the research is to find the conditions for the existence of bounded solutions along the entire real axis for weakly nonlinear differential equations systems with momentum perturbations at fixed moments in time. The found conditions allow to model and study dynamic systems of various evolutionary processes, the parameters of which can change under the influence of external perturbations.

To achieve this goal the following tasks are solved:

- to find sufficient conditions for the existence of bounded solutions for a weakly nonlinear multidimensional differential equations system with momentum action;
- to establish the conditions for the existence and the uniqueness of the bounded solution on the entire axis for weakly nonlinear impulse systems;
- use the obtained conditions for the theoretical study of inhomogeneous impulse systems bounded solutions;

to test the possibility of studying the solutions on the example the model of "impulse vaccination" SIR.

4. Finding bounded solutions on the entire real axis for weakly nonlinear systems of differential equations with momentum perturbations at fixed moments of time

4.1. Sufficient conditions of the bounded solutions existence for a weakly nonlinear system with impulse action

Consider differential equations system with impulse perturbations

$$\frac{dx}{dt} = A(t)x + f(t,x), \quad t \neq \tau_i, \qquad (1)$$
$$\Delta x|_{t=\tau i} = B_i x + I_i(x),$$

where $x \in R^n$, the matrix $A(t)$ is continuous and bounded for all $t \in R$, the matrices B_i are uniformly on $i \in Z$ bounded and such that

$$\inf_{i \in Z} |\det(E + B_i)| > 0. \qquad (2)$$

The function $f(x,t)$ is piecewise continuous on t with first kind discontinuities at points the $t = \tau_i$, and satisfies the Lipschitz condition on x uniformly with respect to $t \in R$. The same condition is satisfied the functions $I_i(x)$:

$$\|f(t,x) - f(t,y)\| < L\|x - y\|, \qquad (3)$$
$$\|I_i(x) - I_i(y)\| < L\|x - y\|.$$

for all $t \in R$, $i \in Z$ and some $L > 0$.

The sequence of moments of impulse perturbation $\{\tau_i\}$ is numbered by integers so that when $i \to -\infty$ and $\tau_i \to +\infty$ when $i \to +\infty$. We also consider that uniformly with respect to $t \in R$ a finite boundary exists

$$\lim_{T \to \infty} \frac{i(t, t+T)}{T} = p < \infty. \qquad (4)$$

We are interested in the question of the existence of bounded on the whole axis solutions of equations (1) under the assumption that the corresponding linear system

$$\frac{dx}{dt} = A(t)x, \quad t \neq \tau_i, \qquad (5)$$
$$\Delta x|_{t=\tau_i} = B_i x,$$

weakly regular on R.

Consider the corresponding inhomogeneous equation

$$\frac{dx}{dt} = A(t)x + f(t), \quad t \neq \tau_i, \qquad (6)$$
$$\Delta x|_{t=\tau_i} = B_i x + a_i$$

and give the necessary definitions.

Definition 1. *We call a homogeneous system of equations* (5) *weakly regular on the whole axis R if corresponding equation* (6) *for every bounded vector-function $f(t)$ has at least one bounded on R solution, and regular on R, if this system has exactly one bounded on R solution for every fixed bounded function $f(t)$.*

According theorem 1 [1], for arbitrary bounded on R functions $f(t)$ and sequence $\{a_i\}$ system of equations has only one solution bounded on all axis.

Theorem 1. *Let system of equations* (5) *is weakly regular on whole numerical line and functions $f(t,x)$ and $I_i(x)$ satysfy for everyone $t \in R$ and $i \in Z$ in some ball $S_r = \{x \in R^n, \|x\| \le r\}$ conditions*

$$C\|f(t,x)\| \le r, \; C\|I_i(x)\| \le r, \qquad (7)$$

where C is a constant of weakly regularity. Then system of equation (1) *has at least one solution bounded on all axis R.*

4.2. Existence of a single bounded solution for weakly nonlinear pulse systems

Theorem 2. *Let system of equation* (5) *is weakly nonlinear on all numerical axis R, functions $f(t,x)$ and $I_i(x)$ satisfy inequalities* (3) *with the Lipschitz constant $L < \frac{1}{C}$ (C is constant of weakly regularity of equations* (5)*) in the ball B_r, where $r > 0$ satisfies inequalities*

$$C \cdot \max\left\{\sup_{t \in R}\|f(t,0)\|, \; \sup_{i \in Z}\|I_i(0)\|\right\} + CLr \le r. \qquad (8)$$

When equation (1) *has unique bounded on whole axis solution $x = \phi(t)$, that satisfies condition $P\phi(0) = 0$, and $\sup_{t \in R}\|\phi(t)\| \le r$.*

We can be used Theorem 2 in the study of the existence of bounded overall axis solutions for differential equations systems with impulse influence in the form

$$\frac{dx}{dt} = A(t)x + f(t) + g(t, x, \varepsilon), \quad t \neq \tau_i \qquad (9)$$
$$\Delta x|_{t=\tau_i} = B_i x + a_i + I_i(x, \varepsilon).$$

In (9) matrixes $A(t)$, B_i and moments of time τ_i such as in equations (1); $f(t)$ is bounded overall axis continuous (piecewise continuous with breaks of the first kind at $t = \tau_i$) function; $t = \tau_i$ – is bounded sequence; function $g(t,x,\varepsilon)$ is continuous (piecewise continuous with breaks of the first kind at $t = \tau_i$) on t, continuous on x and ε, moreover satisfyes the Lipschitz condition according to x; functions $I_i(x,\varepsilon)$ are also continuous over the set of their variables and satisfy the Lipschitz condition according to x, ε is a small positive parameter.

Suppose also that

$$\sup_{t\in R}\|g(t,0,\varepsilon)\| \le L(\varepsilon), \quad (10)$$

$$\|g(t,x_1,\varepsilon)-g(t,x_2,\varepsilon)\| \le \le l(\varepsilon)\|x_1-x_2\|,$$

$$\|I_i(x_1,\varepsilon)-I_i(x_2,\varepsilon)\| \le l(\varepsilon)\|x_1-x_2\|, \quad (11)$$

for all x_1, x_2 such that $\|x_1\| \le r$, $\|x_2\| \le r$, where $L(\varepsilon)$ and $l(\varepsilon)$ are non-negative non-decreasing functions of the parameter ε, moreover $L(\varepsilon) \to 0$, $l(\varepsilon) \to 0$ when $\varepsilon \to 0$.

Theorem 3. *If system of equation* (5) *is weakly regular over the whole real line and number*

$$r > C \cdot \max\left\{\sup_{t\in R}\|f(t)\|, \sup_{i\in Z}\|a_i\|\right\},$$

where C is a constant of weakly regularity of equations (5), *then we can specify positive number ε_0 that for any $\varepsilon \in [0,\varepsilon_0]$ system of equation* (9) *has unique bounded on the whole axis solution $\phi(t,\varepsilon)$ that satisfies conductions $P\phi(0,\varepsilon)=0$ and $\sup_{t\in R}\|\phi(t,\varepsilon)\| \le r$.*

Also, a function $\phi(t,\varepsilon)$ continuous on ε and $\lim_{\varepsilon\to 0}\phi(t,\varepsilon)=\phi_0(t)$, where $\phi_0(t)$ is bounded on whole axis solution of equation (6), that satisfies condition.

4.3. SIR-model of "impulse vaccination" as a model of the dissemination of information on social networks

There are many mathematical models for effective diagnosis of infectious diseases, prediction and study of the pathological process dynamics, which modern medicine actively uses. One such epidemiological model is the SIR model proposed by W. O. Kermack and A. G. McKendrick [30]. It divides the population into three groups:

- healthy individuals who are at risk and can catch the infection (denoted as S - susceptible);
- infected persons who are carriers of the virus (denoted as I - infected);
- recovered persons who have acquired permanent immunity to this disease (denoted as R - recovered).

Consider the SIR model for the “impulse vaccination” strategy the percentage of susceptible patients is below the threshold required to start an epidemic (Figure 1).

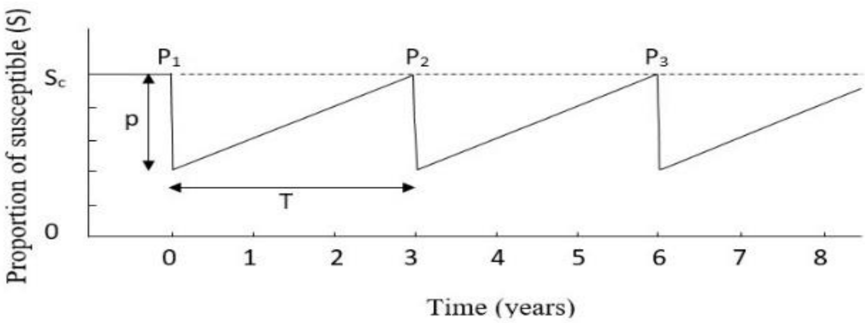

Figure 1: The graph shows the scheme of impulse vaccination for susceptible populations at some time interval. Here the impulses of vaccination, which are carried out every year are the threshold of the epidemic

From the point of view of the SIR-model, the mathematical component is described by a system of differential equations with impulse influence:

$$\frac{dS}{dt} = m-(\beta I+m)S,$$

$$\frac{dI}{dt} = \beta IS-(m+g)I,$$

$$\frac{dR}{dt} = gI-mR,$$

$$S(t_n) = (1-p)S(t_n-0),$$

$$t_{n+1} = t_n + T;$$

where t_n is moment of times in which we apply n-th impulse of vaccination; $t_n - 0$ is moment of time immediately before the application of the n-th impulse; p is the proportion of the susceptible population to which the vaccine is currently administered in the time moment $t=t_n$; T is period between two consecutive vaccinations.

A typical solution of the SIR-model with an impulse vaccination strategy is shown on Figure 2. We can observe how the part of favorable $S(t)$ fluctuates in a stable cycle with the use of impulse vaccine ($p = 0.5$ and $T = 2$). Favorable ones are attracted to the periodic solution "without infection". The line $S_C \approx 0.0556$ means "epidemic threshold".

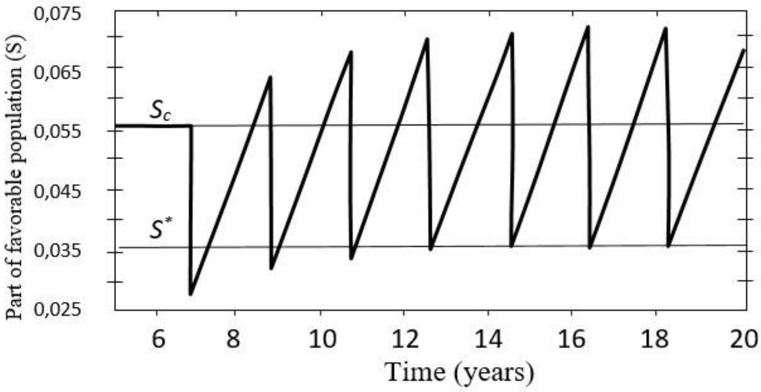

Figure 2: A typical solution $S(t)$ of the SIR-model with an impulse vaccination

In contrast, the part of the infected population decreases rapidly to zero, as shown in Figure 3.

In real conditions the classical SIR-model can not always accurately describe the results of the real situation. The coefficients of the model, depending on the input data, may deviate from the classical condition. Therefore, there is a need to study appropriate systems that contain weak nonlinearity. The obtained results allow to determine and analyze the solutions of the respective systems.

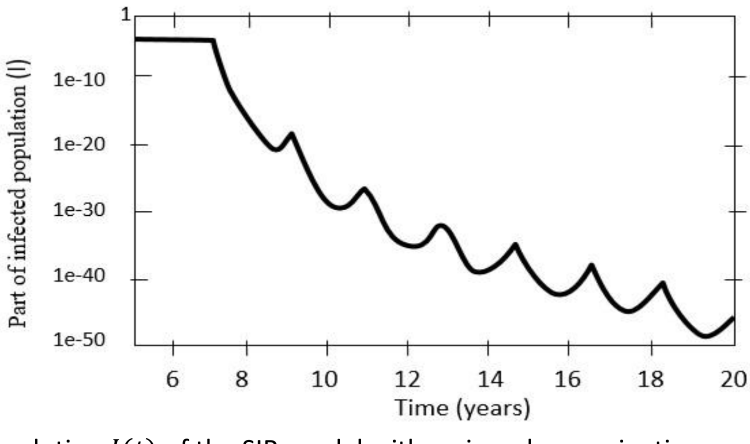

Figure 3: A typical solution $I(t)$ of the SIR-model with an impulse vaccination

Therefore, if we have enough vaccines to fight an infectious disease, it would be reasonable for the susceptible population to be vaccinated each time to be proportional to the number of susceptible individuals (here p). However, such an approximation cannot reflect the real case. Typically, the number of susceptible people that need of vaccination may exceed local medical conditions due to a shortage of vaccines and doctors, especially in rural areas of many developing countries, where reaching the entire target population can be difficult.

Now we explain how SIR model can be considered as a model which describes the dissemination of information on social networks. As in SIR model we divide the population into three groups:

- susceptible will mean uninformed subjects, to whom an information attack can be directed;
- infected will mean persons who are carriers of information and actively disseminate information (harmful or useful);
- recovered will mean informed persons who are information carriers and are not vulnerable to information attacks (will not change their attitude to the object of information).

"Impulse vaccination" will mean "Information stuffing", i.e., "Informing the population of the social network".

5. Discussion of the results of finding bounded solutions for weakly nonlinear systems with impulse action

The problem of finding and investigating bounded solutions for nonlinear differential equations with impulse action is quite important and little studied in the general case. Recent work in this direction has focused mainly on one-dimensional and two-dimensional systems of equations. When considering multidimensional systems, a method based on the analysis of qualitative properties (regularity) of the corresponding linear homogeneous systems is used. These properties, on the one hand, can be effectively tested for wide classes of impulse systems, and on the other hand, make it possible to prove a number of properties of a qualitative nature for inhomogeneous pulse-perturbed systems. Thus, the conditions for the existence of bounded solutions of linear differential equations can be extended to classes of weakly nonlinear impulse systems (Theorem 1). Also obtained are the conditions (Theorem 2) under which the existence and uniqueness of bounded solutions for weakly nonlinear systems of differential equations with momentum action at fixed moments of time is guaranteed. The results of Theorem 2 are applied to the case of piecewise continuous functions with discontinuities of the first kind (Theorem 3).

6. Conclusions

The problem of existence of bounded solutions on the whole real axis (on the half-axis) of weakly nonlinear systems of differential equations with impulse perturbations at fixed moments of time is investigated.

Sufficient conditions for the existence of bounded solutions of a weakly nonlinear multidimensional system of differential equations with momentum actions are obtained. It is important that the found conditions were formulated through the coefficients of the original problem.

Conditions of existence and uniqueness of a limited solution on the whole axis for weakly nonlinear pulse systems, which allow to make a qualitative analysis of the solutions of the respective systems.

The obtained results for the coefficients of the SIR-model allow to determine and analyze the solutions of the corresponding systems that contain weak nonlinearity depending on the input data. In this way, a tool for modeling the circulation of information messages in social networks is obtained.

Further research in this area will provide qualitative results for the development of strategies for mass information of various target groups of users of socially significant information and develop strategies to combat the spread of unreliable and harmful information through social networks.

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