

Assessment of Possibility of Modernization of Hierarchy Code Structure of Multidimensional Signal to Increase the Efficiency of Functioning of Educational and Training Telecommunication Systems

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Abstract

In order to increase the efficiency of modern functioning of educational and training telecommunication systems, research is currently being conducted to increase the amount of information transmitted, its security and speed of transmission through communication channels. One of the directions of such work is the introduction of the approach to the use of multidimensional signals when using them in continuous information transmission channels of educational and training telecommunication systems. The results of research conducted in recent years show that to ensure high quality information transmission in continuous channels can be a method of joint demodulation and decoding operations in the process of performing a single procedure, which involves creating a code structure of multidimensional signal. In the given article the questions of an estimation of possibility of modernization of a hierarchical code design of a multidimensional signal by a method of variation of its parameter for increase of efficiency of work of a continuous channel of information transfer in educational and training telecommunication systems are considered. It is established that the hierarchical code construction of a multidimensional signal, when applied, has the potential to increase the speed of information transmission through a continuous channel. This can be done by upgrading the specified code structure of the signal by reducing the signal distance. The influence of the reduction of the signal distance on the efficiency of the hierarchical code construction is estimated. It was found that by reducing the signal distance of the hierarchical code structure of the signal from 2 or more times, the signal transmission rate can increase and reach up to twenty percent. The implementation of the modulation procedure has no fundamental difficulties, provided that for each code of the code structure known coding procedure using binary codes. The obtained results allow to build a sufficiently accepted procedure for demodulation according to the hierarchical code constructions of the signal with a simultaneous increase in the data rate in the continuous channel that will use such a code construct.

Keywords

Continuous transmission channel, multidimensional signal, hierarchical signal code structure, signal distance.

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1. Introduction

One of the ways to increase the efficiency of modern educational and training telecommunication systems is to improve existing and develop new methods of modulation and noise-tolerant coding for continuous signal transmission channels. The transition in the communication lines of educational and training telecommunication systems to ensembles of multidimensional signals increases the speed of information transmission and provides the transmission of large arrays of information. At the same time, the issues of ensuring a high probability of information transmission are solved by using perfect noise-tolerant coding.

The use of multidimensional (with a large base, components, complex) signals can significantly improve the quality of message transmission through communication channels training - training and other telecommunications systems [1–3].

2. Problem Statement

Constructive signal theory is developed mainly for a discrete, primarily binary channel, ie in the framework of coding theory. In coding theory, a discrete channel formed by a modulator of elementary signals, a continuous channel and a demodulator of elementary signals is considered to be given. At the same time, it is not possible to approach the potential characteristics of the continuous channel both due to the narrowing of the signal class and due to insufficient use when decoding information about the distortion of the signal in the continuous channel. The latter disadvantage is overcome by combining demodulation and decoding into a single reception procedure as a whole, the so-called soft (or analog) decoding (or solution) and reception in a semi-continuous channel [3, 4]. To overcome the first disadvantage, modulation, ie the conversion of a message word into a signal at the input of a continuous channel, should be considered as a single procedure that combines the coding and modulation of elementary signals. The number of known signal designs that reflect this approach is small. Basically, these are code constructions based on the construction of a hierarchical structure, ie hierarchical code constructions of the signal (HS) [4, 5].

In turn, regarding the construction of such code structures and the features of their use in multidimensional signals, it can be concluded that their certain parameters due to changes in values may affect the speed of information transmission in a continuous channel. That is, in general, to influence the effectiveness of such a channel [5, 6].

The question of estimating the possibility of the influence of the parameters of hierarchical code constructions of multidimensional signals on the efficiency of the continuous information transmission channel in training and other telecommunication systems is an urgent scientific task and is currently insufficiently studied.

3. Review of the Literature

A number of works are devoted to the analysis of code constructions of multidimensional signal and assessment of their possibilities for modernization in the direction of influencing the efficiency of continuous channel operation.

The paper [7] presents the results of research on the analysis and synthesis of code structures designed for use in modern and advanced telecommunications systems. Types and features of application of different types of code constructions are given. However, there was no direct assessment of the possibility of their modernization in the direction of the impact of changing the parameters to increase the efficiency of the continuous channel.

The work [8] is devoted to the research of the development of the theory of signal code constructions and code coding. In this paper the issues of application of different types of code constructions are considered at a high level and the directions of their improvement are determined. Direct questions of an estimation of efficiency of change of parameters of code designs in the direction of increase of efficiency of a continuous channel of information transfer are not presented in this work.

In article [9] the consideration of the question of construction of continuous channels of information transfer at application in them of various types of code designs is given. The issue of evaluating the efficiency of the channel and building a code structure is not considered in the paper. Accordingly, there are no questions in the paper to assess the impact of modernization of code structures on the efficiency of the continuous information transmission channel.

Article [10] investigates the construction of multidimensional information transmission signals using the hierarchical coding algorithm based on an integral system proposed in this paper. One of the types of code design of the signal is directly considered, but without the analysis of its efficiency and the impact on it of changes in the parameters of the code design.

Thus, the scientific task to which this article is devoted is the analysis and evaluation of the possibility of modernization of the hierarchical code structure of multidimensional signals in the direction of improving the efficiency of continuous information transmission channels. This modernization in general, while providing the properties of simplicity and versatility, should provide greater speed of information transmission in a continuous channel through its modernization by changing the parameters of the code design

4. Materials and Methods

The research is carried out in relation to the structural scheme of a single - channel information transmission system, which uses the code structure of a multidimensional signal for a continuous information transmission channel of a training telecommunication system.

The block diagram of a particular system is shown in Fig. 1 [7].

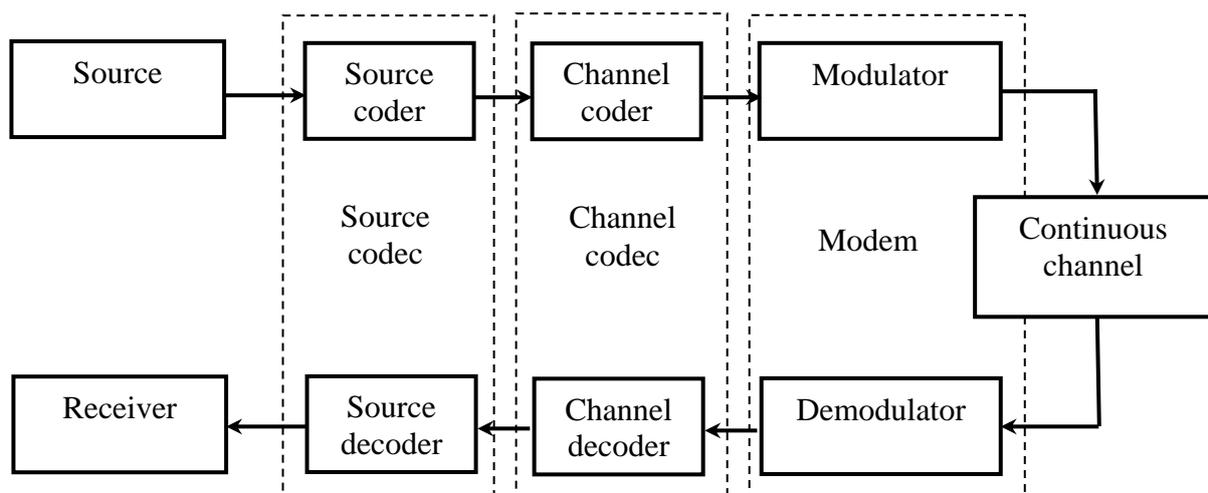


Figure 1: Block diagram of a single-channel information transmission system in a continuous channel

Methods of noise-tolerant coding theory with error correction, theory of radio signal redundancy were used to conduct research and evaluate the possibility of modernization of hierarchical code constructions of multidimensional signal for continuous information transmission channel. Operator methods of radio space transformation, statistical theory of communication, methods of determining the free distance of invariant signal-code structures were also used [11, 12,13].

5. Experiments

Let us denote for a multidimensional signal the M- dimensionality code of length N with words and the minimum Hamming distance d through $(N, M, d)_M$ or, at $M=MK$, through $[N, K, d]_M=(N, MK, d)_m$. The operator f of modulation of elementary signals is mapped to the symbol $q_n \in \{0, \dots, M-1\}$ of the word $q=(q_1, \dots, q_N) \in (N, M, d)_m$ elementary signal $x_n=f(q_n)$ from the set of elementary signals X of power $|X|=M$, contained in the full set of possible, at the input of a continuous channel, elementary signals. And! the coding operator φ to the word u of the source dictionary! U of the word q of the code.

A pair of mappings f and φ specifies the mapping of the dictionary onto the set of signals, determining the design! of the signal system, hereinafter referred to as code. Here is a constructive set of signals, presented in the form of a Cartesian power [13].

Suppose that for each pair of signals a measure of distinction $D(x', x'')$ is defined, hereinafter referred to as a signal distance or simply a distance if misunderstandings are excluded. The signal distance is not necessarily a metric but in some cases of interest is a monotonic function of the metric. For many (but not all) types of channels, the signal distance is additive, that is, represented in the form [1]:

$$D(x', x'') = \sum_{n=1}^N D_0(x'_n, x''_n). \quad (1)$$

An example is the Euclidean distance square (not the metric) or the distances of Hamming and Lee (metrics). When using a code construct, the relationship between the minimum signal distance on the set of signals and the Hamming's distance d is given in conditions (1) by the obvious ratio [1]:

$$D = \min_{x', x'' \in A, x' \neq x''} D(x', x'') \geq \delta d. \quad (2)$$

In the hierarchical code structure X is one-dimensional real (if the channel is low-frequency) or one-dimensional complex, that is, a two-dimensional real set (with amplitude and phase modulation of elementary signals in band channels). Fundamentally, the code construct is suitable at any dimensionality in the set of elementary X signals. However, it is successful only if all nonzero distances in X are the same, for example, when X is the correct simplex (in particular, consists of two signals) or a set of orthogonal signals with the same norms. Then the signal distance between the two signals from A is proportional to Hamming's distance between the words of the code – representations of these signals, and, with a good code, the dope is good. However, with a high power M of the set X , the increase of which is necessary to obtain high speed, the distances on X are significantly different. The code construct that replaces all nonzero distances $D_0(x'_n, x''_n)$ with the smallest of them, which can be interpreted as binary distance quantization, does not take into consideration these differences. At the same time, it has two important advantages – comparative simplicity and versatility. Under versatility, a fundamental possibility of obtaining signals systems of arbitrary dimensionality and with arbitrary signal distances is accepted. Simplicity is ensured by the regularity (for example, algebraic properties) of codes that combine the same-type elementary signals into multidimensional. Code structures retain these advantages in one way or another but make it possible to obtain more powerful signal systems due to more subtle accounting of the distribution of distances on X [8,13].

Hierarchical code structure are based on the split of many elementary signals into continuous subsets, in each of which, with a successful split, the signal distance between the nearest signals is greater than all X . The most convenient hierarchical structure (HS) is the one in which the ideas of the generalized cascade code [13–15] are adapted for the signal system with arbitrary additive signal distance [16]. The hierarchy means the set L of the breakdown of sets X into classes such that all classes of the same level (one partition) are equally powerful and can include classes of the previous level only entirely. That is, classes of the previous level are "nested" in classes of the next level, similar to the system of internal nested codes of generalized cascading code. The set of classes of the $(l-1)$ -level, included in the class of the 1st level, is mapped mutually and unambiguously onto the set of characters of the M_l -dimension code $N, M_l, d_l)M_l$ of the l -th level. This is the analog of the external code of the generalized cascading code, where $M_1 M_2 \dots M_L = M$. Since the signal distances between elementary signals of the l -level class increase with a decrease in l , the transition from a code structure with one M - dimensional code to a multicode HS makes it possible to increase the power of the set of signals without reducing the minimum signal distance. This is similar to when the transition from cascading to generalized cascading code makes it [1, 2, 13]:

$$\delta_l = \min_{q_{l+1, n}, \dots, q_{L, n} \in (q_{l+1, n}, \dots, q_{L, n})} D_0(x'_n, x''_n), \quad (3)$$

where $x'_n = f(q'_{1n}, q'_{1n}, q_{l+1, n}, \dots, q_{Ln})$, $x''_n = f(q''_{1n}, q''_{1n}, q_{l+1, n}, \dots, q_{Ln})$.

Possible to increase code power without reducing the minimum Hamming's distance [14, 15].

The totality of L-convolutional codes makes it possible to get a convolutional analog of HS signals for continuous or discrete channels with additive signal distance based on the same hierarchy.

First of all, in terms of assessing the effect of signal distance on the efficiency of code structure operation, we are interested in those designs in which the character encoding at all levels of the hierarchy is carried out through the signal distance [1, 3, 7].

Let's analyze and evaluate the influence of the signal distance on the efficiency of the hierarchical code structure.

Suppose that on a set of elementary signals X of power $M=M_1M_2\dots M_L$ a hierarchy is defined – a set L of division into disparate classes. Each class of the l-th level of the hierarchy (l-th division) includes M_l classes of the (l-1) level, that is, it consists of $\mu_l=M_1M_2\dots M_l$ signals. The numbering of the classes of the (l-1)-th level, which are included in the class of the l-th level, sets a mutually unique mapping of the set of classes of the (l-1)-th level onto the set of digits $\{0, \dots, M_l-1\}$. Therefore, the set (q_{l1}, \dots, q_{ln}) , where it determines the only value of the n-th elementary signal, where f is the rule (operator) of modulation of elementary signals. We compare the l-th level and the minimum l-th signal distance in the class [1, 2] The L level class is the same as X, so $\delta_L=\delta$. In the hierarchy, one can also include a zero level with M classes of one signal in each and at $\delta_0=\infty$. Since the class of the next level can include the class of the previous level only entirely, then one can combine two levels by disregarding the (l-1)-th partitioning, so that one can take into consideration that $\delta_1>\delta_2>\dots>\delta_L=\delta$ [13].

Let $q_1=(q_{11}, \dots, q_{1N})$ be the word of code $(N, M_l, d_l)M_l$ of the l-th level, the source dictionary is represented by the Cartesian product of the subsection of mutually unambiguous mapping onto the l-level code. Under the hierarchical structure, authentic authors understand the totality of the hierarchy on the set of elementary X signals, the mapping of the f sets of (q_{1n}, \dots, q_{Ln}) on the set of X, L level codes and L mapping ϕ_l . The scheme of the corresponding sequence of transformations (modulation) is as follows [1,2]:

$$u \rightarrow (u_1, \dots, u_L) \xrightarrow{\phi_l} q_l = (q_{l1}, \dots, q_{ln}) \xrightarrow{f} x_n.$$

Here, the left arrow means splitting a word into subsections (the word u can be a block in the sequence of characters of the source of information; if the power of the sets of blocks is less, then some dictionary words and signals are not used). Then each subword is encoded into the word code $(N_l, d_l)M_l$. The result is L code words of the same length N. The elementary signal modulator converts a set of n-characters of all words into the n-th elementary signal that enters the channel.

Assertion 1. The hierarchical structure sets the signal system with a power of the set of signals and with a minimum signal distance [3, 7, 11]:

$$D > \min_{i \leq l \leq L} (\delta_i, d_i). \quad (4)$$

The statement about the number of signals is obvious. It is also clear that HS sets the signal system, that is, the mutually unambiguous mapping of dictionary U on the set of signals A. To prove (4), consider a distance between the two signals $x'=(x'_1, \dots, x'_N)$, $x''=(x''_1, \dots, x''_N)$, where $x'_n = f(q_{1n}, \dots, q_{Ln})$, $x''_n = f(q'_{1n}, \dots, q'_{Ln})$. There are at least one l and one n such as $q_{ln} \neq q'_{ln}$ at $x' \neq x''$ and, then, there is $\lambda = \max\{l: q_{ln} \neq q'_{ln}, 1 < l < L, 1 < n < L, \}$. Then δ_λ is the least of the nonzero distances between the elementary signals included in x' and x'' any of such distances $D_0(x'_n, x''_n) \geq \delta_\lambda \delta(q_{ln}, q'_{ln})$, where $\delta(q_{ln}, q'_{ln})$ is the Kronecker symbol (0 or 1). Hence:

$$D(x', x'') \geq \delta_\lambda \sum_{n=1}^N \delta(q_{ln}, q'_{ln}) \geq \delta_\lambda d_\lambda, \quad (5)$$

Since the signals correspond to different words of the λ -th level code. Since such λ is found for any pair of different signals from, (4) follows from (5).

With the predefined minimum signal distance D, the minimum Hamming's distance of codes should be chosen equal to [1, 2]:

$$d_l = \lceil D/\delta_l \rceil, \quad (6)$$

where $\lceil D/\delta_l \rceil$ is the smallest integer, not less than D/δ_l .

Since at $l < L$, the Hamming's distance d_l necessary for codes of all levels, except the last, can be, especially for the first levels, significantly less than $d = dL$, which makes it possible to increase the power of the set of signals in HS compared to the code construct.

Note 2. 1: The hierarchy is set by any L equivalence relationship on X if each breaks X into equal-power classes and the class of the next equivalence relation includes either all or no elements of the class of the previous relation. Thus, if X is mutually unambiguously mapped onto group G (for example, [8, 11]) and $G_1 \subset G_2 \dots \subset G_L = G$ are its subgroups of orders μ_1, \dots, μ_L , where $\mu_L = M_1 \dots M_L$, then the adjacent class of group G on the subgroup G_l is mapped on the class of the l -level of the hierarchy. This type of HS with Hamming's distance as a signal includes generalized cascading codes. If X is given (as the region of values) by the function of integer arguments, then the equivalence ratio can be defined by the fixation of some arguments [13].

Let $(\lambda_1, \dots, \lambda_L)$ be a permutation of indexes and $q_{ln} = p_{\lambda_n}$. Any such order of indices corresponds to the L -level hierarchy with the minimum distances δ_l , conditioned by (3). Instead of permutation, one can use another arbitrary mutually unambiguous match of sets (p_{1n}, \dots, p_{Ln}) and (q_{1n}, \dots, q_{Ln}) , given by L functions. We do not know the general method of such variable recoding, which leads to a successful hierarchy. Some of the recoding techniques are given in examples 8–10.

Note 2. 2: The signal system given by the hierarchical structure can be matched with other HSs, as any of the $L!$ level permutations define any HS. Let q_{ln} becomes a symbol of the l -th level at the i -th permutation, at which, taking into consideration the corresponding changes in (3), the order of character fixation, the minimum distance equals $\delta_l^{(i)}$. From statement 1, the score from below follows $D \geq \min(\delta_l^{(i)} d_l)$. This score is true for all permutations, therefore [9,13]

Methods of mathematical modeling to assess the impact of changes in the signal distance on the efficiency of the continuous channel of information transmission for certain types of code structures of multidimensional signals.

For the hierarchical code structure we will perform the following calculations.

For a hierarchical code structure, the following calculations will be made. *Example 2:* Assume $v=2$, $N=3$, $X^{(0)}$ is the point at the coordinate origin (degenerate polytope), $X^{(1)}$ and $X^{(2)}$ are the mutually rotated hexagons with radii $\rho^{(1)} = \sqrt{2}$, $\rho^{(2)} = \sqrt{6}$ and angles at the vertices [5, 19]:

$$\alpha_n^{(i)} = (i-1)\pi/6 + s_n\pi/3 + t_n 2\pi/3,$$

where $i \in \{1,2\}$, $s_n \in \{0,1\}$, $t_n \in \{0,1,2\}$.

It should be borne in mind that the minimum energy distance between polytopes [3, 5]:

$$\delta_x(i, j) = (\rho^{(i)} - \rho^{(j)})^2 + 4\rho^{(i)}\rho^{(j)} \sin^2 \psi_{ij}, \quad (7)$$

where ψ_{ij} is the half of the smallest angle between the vertices of polytopes. The mapping $X_n = X^{(\rho_n)}$, $\rho_n \in \{0, \dots, R-1\}$ defines a polytope to which the n -th elementary signal x_n belongs and thereby matches the word $\rho = (\rho_1, \dots, \rho_N)$ of the code of polytopes P to a constructive class $X^{(\rho)}$.

As a result of (7), here $\delta_x(0,1) = \delta_x(1,2) = 2$, $\delta_x(0,2) = 6$.

Assume that to build using HS signals on the sphere it is necessary to satisfy the condition [2, 3, 5]:

$$\sum_{n=1}^N \rho_n^2 = N_v, \quad \rho_n = \|x_n\|, \quad (8)$$

The four words of the polytope code P satisfy (8): (1,1,1) – it corresponds to a set of radii $(\rho_1, \rho_2, \rho_3) = (\sqrt{2}, \sqrt{2}, \sqrt{2})$ and cyclic shifts of the word (2,0,0), that is, a set of radii $(\sqrt{6}, 0, 0)$. Let $D=6$. The energy distance between any pair of four constructive classes of signals $X^{(\rho)}$ is not less than $D=6$, which is checked by (7). Between the signals of each of the last three classes (with different dialing radii), the distance is also at least $D=6$, so all elements of these classes can be taken as signals. Such signals are $3 \cdot 6 = 18$. In the first constructive class, $6^3 = 216$ signals. Of these, using an HS example 4, one can select $332 = 54$ signal at $D=6$. The resulting set of $M=72$ signals corresponds to the most well-known 6-dimensional packing of equal spheres relating to their equal sphere [20]. Elements of this set can be encoded with four source characters $u = (u_1, u_2, u_3, u_4)$, from which u_1, u_2 are three, u_3 is binary, u_4 is

quaternary. If $u_4 \leq 2$, then $\rho_1 = \rho_2 = \rho_3 = \rho^{(1)} = \sqrt{2}$, that is, the word $p=(1,1,1)$ of the polytope code is passed, and the angles at the vertices of polytopes (hexagons) are equal to $\alpha_n = \alpha_n^{(1)} = s_n\pi/3 + t_n 2\pi/3$, where $t_1 = u_1$, $t_2 = u_2$, $t_3 = u_4$, $s_1 = s_2 = s_3 = u_3$. If $u_4 = 3$, then $\rho_n = 0$ at $n \neq 1 + u_2$, $\rho_{1+u_2} = \sqrt{6}$, $\alpha_{1+u_2} = \pi/6 + u_3\pi/3 + u_4 2\pi/3$. The resulting set of signals can be matched to the hierarchy and then use HS.

6. Results

Generalized results of estimating the signal distance reduction effect with improving the operational efficiency of code structure of multidimensional signal are given in Table 1.

Table 1 lists the parameters of signal code constructs built using the results of solutions to the examples of the specified types of code structures [13].

The first column shows the dimensionality of a signal system. The second – the dimensionality of elementary or, in the form of the sum of dimensionalities, a composite elementary polytope. The third – the minimum energy (Euclid square) distance (with a single average energy per coordinate). The fourth column shows the speed R in bits per coordinate. Then the R_1 speed of the best quaternary CPM (or other PM specified in the footnote) with the same N_v and D upper Shannon speed limits R_{Sh} [1] and Kabatyansky-Levenstein speed R_{KL} [21]. The last column shows the example number and the structure (HeS stands for a heterogeneous structure).

Table 1

Results of computing the signal distances of code constructs of multidimensional signals

N_v	v	D	R	R_1	R_{Sh}	R_{KL}	Примітка
30	2	6	1,161	0,791 ¹	2,212	1,591	1, HS
64	2	12	1,098	0,894	2,241	1,638	1, HS
40	4	16	0,965	0,824	2,191	1,081	1, HS
6	2	6	1,027	0,791 ¹	1,224	1,064 ⁵	2, HeS
6	3	2,07	1,513	0,291 ²	1,886	–	2, HeS

Note: ¹ – non-redundant 3-dimensional PM; ² – non-redundant 6-dimensional PM; ³ – at $D=8$; ⁴ – at $D=16$; ⁵ – from [20, 21].

7. Discussion

Our analysis of the results, summarized in Table 1, reveals the following. The win of HS is the more noticeable the larger the dimensionality. Note, for comparison, that a single (binary) PM corresponds to the speed $R=0.5$ and $D=8$, and the two-time (quaternary) – $R=1.0$, $D=4$ [13].

This is explained by the fact that the HS, due to the peculiarities of formation and the possibility of varying the changes in the components of the code construct, can provide all nonzero distances with the same value. First of all, this is an opportunity to form the correct simplex from two signals of a simple HS. Or a set of orthogonal signals with the same norms in more complex HS constructs. An additional advantage of the HS, which is confirmed by the data in Table 1, is that it has the ability to replace all nonzero signal distances with the smallest of them. That is, the possibility of binary quantization of the signal distance. This gives significant advantages in the speed of information transmission. As shown by the data in Table 1 [8, 13].

We shall define the features of the proposed method of forming code structures of multidimensional signals. Implementing modulation procedures, as can be seen from their description, does not encounter fundamental difficulties even in a general case, unless, of course, an acceptable encoding procedure is known for each code. Difficulties may rather arise due to the fact that HS usually require non-binary codes and even optionally codes over the prime number. Not much is known about specific codes of this type. Recently, significant progress has been made in the theory of non-binary codes. The new ternary and quaternary codes [20, 22, 23] are described. Attractive is the direction associated with the

codes above the rings of deductions [24, 25]. Sometimes, the necessary codes can be built using HS. The possibilities of reversible structures are also limited primarily by a small number of known structures of reversible codes, especially non-binary ones. Obviously, the most promising HS is with a reversible code in the form of a composition of equilibrium ones.

Thus, the connection between the signal distance of a certain type of code constructions and the speed of a continuous multidimensional signal transmission channel is established and substantiated against the background of maintaining a certain level of noise immunity in educational and training telecommunication systems.

8. Conclusions

1. It is established that code constructions, which are based on the construction of a hierarchical structure, specify a system of signals with minimum signal distances, which can be determined by the values of the minimum Heming distances in the code structure of the code structure.

Varying the values of signal distances to the minimum, comparable to Heming distances, for hierarchical code constructions of multidimensional signals can significantly increase the volume and speed of transmitted information through a continuous communication channel of the training telecommunication system.

2. As a result of simulation it is established that changes in the signal distance can significantly increase the speed of information transfer in bits per coordinate.

For a hierarchical code structure, this can be up to 20 percent when the signal distance is halved.

3. The implementation of the modulation procedure has no fundamental difficulties, provided that for each code of the code structure known coding procedure using binary codes.

The simplest approximate demodulation procedure that implements the signal distance can be constructed as a sequence of descending procedures of the reception procedures as a whole for individual codes that determine the design of the code system as a whole.

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