Repairing Ontologies via Kernel Pseudo-Contraction

Vinícius Bitencourt Matos[1,*], Renata Wassermann[1,*], PhD

[1]Universidade de São Paulo (USP), São Paulo, Brazil

Abstract

Rational agents must have some internal representation of their knowledge or belief system. Belief Revision is a research area that aims at understanding how they should change their representations when they are faced with new information. In a contraction operation, a sentence is removed from a knowledge base and must not be logically entailed by the resulting set. Pseudo-contraction was proposed by Hansson as an alternative to base contraction where some degree of syntax independence is allowed. In this work, we analyse kernel constructions for pseudo-contraction operations and their formal properties. Also, we show the close relationship between concepts and definitions of Belief Revision and Ontology Repair (such as pseudo-contractions and gentle repairs, respectively).

Keywords

Belief revision, Description logics, Ontology repair, Pseudo-contraction

1. Introduction

Belief Revision is a research area that deals with problems related to changing knowledge bases or logical theories, especially in the face of new, possibly conflicting, information. The work of Alchourrón, Gärdenfors, and Makinson [1] is widely recognised as the initial hallmark of this area, and gave rise to what is known as the AGM paradigm. Originally, it required the underlying logic to satisfy some assumptions, such as compactness, monotonicity and the deduction theorem, and most work following AGM was developed with propositional logic in mind. In the AGM paradigm, the beliefs of an agent are represented by sets closed under logical consequences, the *belief sets*. Over the past decades, the AGM theory has been adapted to *belief bases* (sets of sentences that are not necessarily closed) represented in different logical formalisms, such as Horn or Description Logics [2, 3, 4].

The AGM paradigm defines three change operations on belief sets: expansion, which incorporates a new belief; contraction, which removes a belief; and revision, which incorporates a new belief retaining consistency. In this paper, we will only address the problem of retracting beliefs, thus contraction operations and their variations. In a contraction operation, a sentence must be removed from a set and must not be entailed by the contracted set. Some of the minimal requirements for an operation to be a belief contraction are that it satisfies *success* — the removed set is not entailed anymore — and *inclusion* — no new beliefs are added [5]. There are two main constructions associated to contraction operations: *partial meet contraction* [1] with respect to a sentence is defined as the intersection of some inclusion-maximal subsets that do not entail it, and *kernel contraction* [6] is obtained by removing at least one sentence from each inclusion-minimal subset entailing the sentence to be removed.

The area of Ontology Repair groups together tools and formal definitions related to the task of debugging ontologies and getting rid of unwanted inferences. Different approaches have been proposed, depending on which parts of the knowledge base one wants to change [7, 8, 9, 10, 11].

Both in Belief Revision and in Ontology Repair, classical approaches assume that no information can be added to a knowledge base when we perform the task of removing some unwanted consequence. Whilst this assumption may be reasonable, it is usually formalised as a syntactic requirement of inclusion, in a way that forces the removal of too much information. The assumption can be formalised as a less restrictive constraint which only states that we cannot add new *consequences* to the knowledge base, thus allowing to add sentences that were logically entailed by the original set. Note that this formalisation still captures the intuition that no new information should be added, but "information" is now seen as independent from the syntax. This idea has been proposed and developed in both areas in the last decades, with different terminologies and notations. In Belief Revision, pseudo-contractions are generalisations of contractions that allow the addition of sentences as long as they were already entailed by the initial set. Similarly, in Ontology Repair, a gentle repair of an ontology is built by removing sentences or replacing them with weaker versions so that the resulting set does not imply the unwanted sentence, and new consequences are not allowed [12].

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*Corresponding author.
vinicius.matos@alumni.usp.br (V. B. Matos); renata@ime.usp.br (R. Wassermann)
0000-0001-7342-6668 (V. B. Matos); 0000-0001-8065-1433 (R. Wassermann)

Recently, a pseudo-contraction construction based on partial meet contraction was proposed and characterised [13, 14]. It uses a weak consequence operator (i.e. a consequence operator that may not include all the consequences of a classical Cn) to expand the initial set of sentences before applying the classical partial meet contraction.

In this text, we analyse a pseudo-contraction construction that is based on a kernel contraction and expands the set with some of its consequences before applying the classical kernel contraction. Furthermore, we show that some concepts and definitions of Belief Revision and Ontology Repair are closely related, extending some previous work and showing that the new kernel pseudo-contraction is also connected to gentle repairs. In order to facilitate the integration between the areas, we will adopt a functional notation for Belief Revision, which we have proposed in a previous work (e.g. the contraction of B by α will be denoted by $\mathrm{c}(B, \alpha)$ rather than $B - \alpha$). We expect it to be clearer and less ambiguous than the classical infix notation.

The results of this paper appeared in the first author's thesis [15, sections 3.2, 3.3 and 4.2], which contains the proofs that have been omitted here due to space constraints. The proofs are also available at https://www.ime.usp.br/~renata/papers/NMR2022_supplement.pdf.

This text is structured as follows. Section 2 introduces pseudo-contractions and presents some definitions that will be used throughout the paper. In Section 3, we define our new operation (Cn^* kernel pseudo-contraction), explain its properties and characterise it by means of a set of postulates. Section 4 shows our prototype of a tool that computes some pseudo-contractions in ontologies. Ontology Repair is introduced in Section 5, and its connections with Belief Revision are presented in Section 6. Section 7 finishes the text with the conclusions.

2. Pseudo-contraction Operations

Contractions over belief bases can lead to unnecessary waste of information, largely due to the inclusion postulate [16]:

(inclusion) $\mathrm{c}(B, \alpha) \subseteq B$.

The postulate requires that the result of contracting a belief base B by a sentence α is included in the original belief base. This postulate prevents the weakening of formulae, which can be seen as an argument against its use for belief bases.

Example 1. Consider a knowledge base that contains the following three sentences:
- All Swedish things are European;
- European Swans are white;
- s is a Swedish Swan.

This can be formalised, for example, in a Description Logic:

$$Swedish \sqsubseteq European;$$

$$European \sqcap Swan \sqsubseteq \exists hasColour.\{white\};$$

$$s : Swan \sqcap Swedish.$$

If we want to contract by s *hasColour white*, one of the sentences must be removed; thus, for example, if we choose to remove the third sentence, the fact that s is a swan is lost.

Intuitively, in Example 1, we should consider replacing the sentence $s : Swan \sqcap Swedish$ with a weaker version $s : Swan$, which is forbidden by the inclusion postulate. Another intuitive idea prohibited by that postulate is to weaken $European \sqcap Swan \sqsubseteq \exists hasColour.\{white\}$ by adding an intersection to the left-hand side, in order to convey the idea that all European swans that satisfy a certain property (e.g. "normal" or "typical") are white.

Hansson has proposed a weakening of inclusion, *logical inclusion* [5], which is satisfied by operations he has called *pseudo-contractions* [16]:

(logical inclusion) $\mathrm{Cn}(\mathrm{c}(B, \alpha)) \subseteq \mathrm{Cn}(B)$.

(success) If $\alpha \notin \mathrm{Cn}(\emptyset)$, then $\alpha \notin \mathrm{c}(B, \alpha)$.

Definition 2 (Pseudo-contraction [16])**.** An operation c is a *pseudo-contraction* if c satisfies success and logical inclusion.

With logical inclusion, whilst we still do not allow the addition of arbitrary sentences, the resulting set no longer has to be a subset of the original set, thus making it possible to insert the sentence $s : Swan$ in Example 1.

From now on, we will consider a generic consequence operator Cn that is Tarskian and compact, such as $\mathrm{Cn}_{\mathrm{FOL}}$ and the consequence operators that correspond to some fragments of first-order logic. Thus, the operations we will present do not assume any other syntactic or semantic features of the logic, which makes them applicable to logics that do not satisfy the AGM requirements (such as Description Logics, which usually do not have a sentence $\neg\varphi$ for every sentence φ). Results that require extra properties will explicitly mention them. The set of all sentences in the language will be denoted by $\mathfrak{L}$. Subclassicality will be defined with respect to Cn.

In the following sections, we will present some pseudo-contraction constructions that depend on the kind of formulae that we are allowed to add when contracting by a formula. Before computing the kernel set, our operations will "close" the set under a new consequence operator, Cn^*, which will make possible the insertion of new sentences. This is a generic operator whose definition is deliberately unspecified, and we will explicitly state the conditions that are required by each theorem.

The properties of pseudo-contraction constructions depend on the properties that are satisfied by Cn^*, especially inclusion and subclassicality: if both are satisfied, then Cn^* is in an intermediate level between the original base (which would be used in a base contraction) and its closure (as in classical AGM contraction), i.e. $B \subseteq \mathrm{Cn}^*(B) \subseteq \mathrm{Cn}(B)$. For practical applications, $\mathrm{Cn}^*(B)$ should always be finite if B is finite, but we will not assume this restriction.

Automatic reasoners for ontologies — such as HermiT[1] and Fact++[2] — allow the user to choose the types of consequences that will be generated. Each configuration can be seen as a Cn^*, which satisfies inclusion[3] and is usually subclassical[4]. Since they are syntactically restricted, they are good examples of weak consequence operators. Those reasoners can be embedded in ontology editors, such as Protégé[5], as shown in Figure 1.

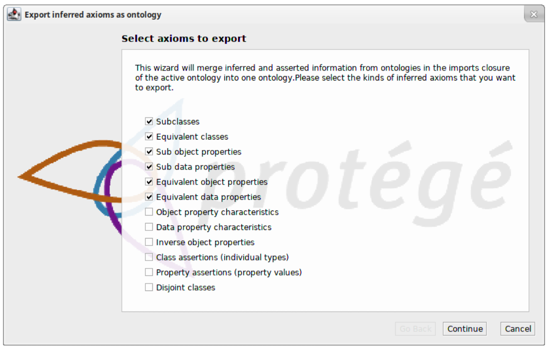

Figure 1: Screenshot of the Protégé window where the user can choose which types of consequences will be exported.

Another example of Cn^* is the conversion of sentences into some standard format, such as conjunctive and disjunctive normal forms, and normal forms tailored to specific logics, as long as no new symbols are introduced. Since inclusion is not satisfied by those consequence operators, some desirable properties may not be satisfied by constructions that use them; on the other hand, they may be of use if good inference algorithms exist for them.

3. Cn* kernel pseudo-contraction

In this section, we will present a pseudo-contraction operation that is defined as a kernel contraction starting from the expanded set $\mathrm{Cn}^*(B)$.

[1]http://www.hermit-reasoner.com/
[2]http://owl.cs.manchester.ac.uk/tools/fact/
[3]One step after the window shown in Figure 1, users can choose whether the original sentences should be kept.
[4]Except, maybe, for some highly-complex ontologies that cannot be represented in classical logic.
[5]http://protege.stanford.edu/

Definition 3 (Kernel and kernel set [6])**.** Let $B \subseteq \mathfrak{L}$ and $\alpha \in \mathfrak{L}$. The *kernel set* of B with respect to α, denoted by $\mathrm{Ker}[B, \alpha]$, is such that a set X is in $\mathrm{Ker}[B, \alpha]$ if and only if $X \subseteq B$, $\alpha \in \mathrm{Cn}(X)$, and there is no $Y \subset X$ such that $\alpha \in \mathrm{Cn}(Y)$. Each such X is an *α-kernel.*

We will show later (Proposition 28) that the definition above is related to that of justification (Definition 18), which is well-known in Ontology Repair.

Definition 4 (Incision function [6])**.** Let $B \subseteq \mathfrak{L}$. A function f is an *incision function*[6] for B if, for every $\alpha \in \mathfrak{L}$, it is the case that $f(\mathrm{Ker}[B, \alpha]) \subseteq \bigcup \mathrm{Ker}[B, \alpha]$ and $f(\mathrm{Ker}[B, \alpha]) \cap X \neq \emptyset$ for every non-empty $X \in \mathrm{Ker}[B, \alpha]$.

Definition 5 (Cn^* kernel pseudo-contraction)**.** Let B be a set of sentences, Cn^* a consequence relation and f an incision function for $\mathrm{Cn}^*(B)$. The *Cn^* kernel pseudo-contraction* of B by a sentence α, denoted by $\mathrm{kc}_f^{\mathrm{Cn}^*}(B, \alpha)$, is such that, for all sentences α:

$$\mathrm{kc}_f^{\mathrm{Cn}^*}(B, \alpha) = \mathrm{Cn}^*(B) \setminus f(\mathrm{Ker}[\mathrm{Cn}^*(B), \alpha]).$$

The following examples illustrate this construction:

Example 6. Let $\mathrm{Cn}^*_{\mathrm{break}}$ be a consequence operator that preserves the existing sentences and adds $a : C_i$ (for $i = 1, \ldots, n$) for every sentence $a : C_1 \sqcap \cdots \sqcap C_n$ in the original set. This is analogous to the consequence operator that "breaks conjunctions into conjuncts", originally presented in [17]. If B is the knowledge base defined in Example 1, then $\mathrm{Cn}^*_{\mathrm{break}}(B) = B \cup \{s : \mathit{Swan}, s : \mathit{Swedish}\}$. Let α be the sentence s *hasColour white*. We have $\mathrm{Ker}[\mathrm{Cn}^*_{\mathrm{break}}(B), \alpha] = \{\{\beta_1, \beta_2, \beta_3\}, \{\beta_1, \beta_2, \beta_3', \beta_3''\}\}$, where

$$\begin{aligned} \beta_1 &= \mathit{Swedish} \sqsubseteq \mathit{European}, \\ \beta_2 &= \mathit{European} \sqcap \mathit{Swan} \sqsubseteq \exists \mathit{hasColour}.\{\mathit{white}\}, \\ \beta_3 &= s : \mathit{Swan} \sqcap \mathit{Swedish}, \\ \beta_3' &= s : \mathit{Swan} \text{ and} \\ \beta_3'' &= s : \mathit{Swedish}. \end{aligned}$$

If the definition of the incision function f is such that $f(\mathrm{Ker}[\mathrm{Cn}^*_{\mathrm{break}}(B), \alpha]) = \{\beta_3, \beta_3''\}$, then the result of the operation is $\mathrm{kc}_f^{\mathrm{Cn}^*_{\mathrm{break}}}(B, \alpha) = (B \setminus \beta_3) \cup \{\beta_3'\}$, i.e., the pseudo-contraction replaces β_3 with its weaker version β_3'.

In order to characterise this operation, we will need a starred version of some postulates:

(inclusion*) $\mathrm{c}(B, \alpha) \subseteq \mathrm{Cn}^*(B)$.

(uniformity*) If for all $B' \subseteq \mathrm{Cn}^*(B)$, $\alpha \in \mathrm{Cn}(B')$ if and only if $\beta \in \mathrm{Cn}(B')$, then $\mathrm{c}(B, \alpha) = \mathrm{c}(B, \beta)$.

[6]It should be noted that incision functions depend only on the kernel set, which may be the same for distinct pairs of B and α.

(core-retainment*) If $\beta \in \mathrm{Cn}^*(B) \setminus \mathrm{c}(B, \alpha)$, then there is some $B' \subseteq \mathrm{Cn}^*(B)$ such that $\alpha \in \mathrm{Cn}(B' \cup \{\beta\}) \setminus \mathrm{Cn}(B')$.

The representation theorem follows.

Theorem 7 (Cn* kernel pseudo-contraction: representation theorem)**.** *If Cn* satisfies monotonicity, then an operation is a Cn* kernel pseudo-contraction if and only if it satisfies success, inclusion*, core-retainment* and uniformity*.*

Proof sketch. Construction-to-postulates: Success can be shown by contradiction: if $\alpha \in \mathrm{Cn}(\mathrm{kc}_{\boldsymbol{f}}^{\mathrm{Cn}^*}(B, \alpha))$, then the fact that Cn is Tarskian implies that there is some non-empty $X \in \mathrm{Ker}[\mathrm{Cn}^*(B) \setminus \boldsymbol{f}(\mathrm{Ker}[\mathrm{Cn}^*(B), \alpha]), \alpha]$, and such X must be in $\mathrm{Ker}[\mathrm{Cn}^*(B), \alpha]$, but this implies that $\boldsymbol{f}(\mathrm{Ker}[\mathrm{Cn}^*(B), \alpha]) \cap X = \emptyset$, violating the definition of incision function. Inclusion* and core-retainment follow directly from the definitions. For uniformity*, if α and β satisfy the antecedent but not the consequent, then there must be some $X \in \mathrm{Ker}[\mathrm{Cn}^*(B), \alpha] \setminus \mathrm{Ker}[\mathrm{Cn}^*(B), \beta]$ (w.l.o.g., swapping α and β for the other case); thus, either $\beta \notin \mathrm{Cn}(X)$ or there is some $X' \subsetneq X$ such that $\beta \in \mathrm{Cn}(X')$, and neither can hold because $X \in \mathrm{Ker}[\mathrm{Cn}^*(B), \alpha]$. *Postulates-to-construction:* This part of the proof is analogous to the proof of the corresponding theorem in [6]. If $\mathrm{c}^{\mathrm{Cn}^*}$ satisfies the postulates, then the function $\boldsymbol{f}$ defined as $\boldsymbol{f}(\mathrm{Ker}[\mathrm{Cn}^*(B), \alpha]) := \mathrm{Cn}^*(B) \setminus \mathrm{c}^{\mathrm{Cn}^*}(B, \alpha)$ is a well-defined incision function for $\mathrm{Cn}^*(B)$ and the operations $\mathrm{kc}_{\boldsymbol{f}}^{\mathrm{Cn}^*}$ and $\mathrm{c}^{\mathrm{Cn}^*}$ are equivalent. ☐

Cn* partial meet pseudo-contraction, a pseudo-contraction construction that "closes" the set under Cn* before applying a classical partial meet contraction, was proposed by [13]. The result of the operation, denoted by $\mathrm{pmc}_{\boldsymbol{g}}^{\mathrm{Cn}^*}(B, \alpha)$, is obtained by taking the intersection of the output of a *selection function* $\boldsymbol{g}$ that chooses some elements (at least one) of $\mathrm{Rem}[\mathrm{Cn}^*(B), \alpha]$, which is the set of all inclusion-maximal subsets of $\mathrm{Cn}^*(B)$ that do not entail α.

Cn* partial meet pseudo-contractions satisfy relevance*, and Cn* kernel pseudo-contractions satisfy core-retainment*; moreover, the other three postulates are identical (success, inclusion* and uniformity*). Hence, every Cn* partial meet pseudo-contraction is also a Cn* kernel pseudo-contraction. We will now show how to obtain the explicit construction of a Cn* kernel pseudo-contraction from a Cn* partial meet pseudo-contraction. This will use the definition of an incision function derived from a selection function:

Definition 8 (Incision function associated to a selection function [18])**.** Let $\boldsymbol{g}$ be a selection function for X. The function $\boldsymbol{f_g}$ defined as

$$\boldsymbol{f_g}(\mathrm{Ker}[X, \alpha]) = X \setminus \bigcap \boldsymbol{g}(\mathrm{Rem}[X, \alpha])$$

is the $\boldsymbol{g}$*-associated incision function* for X.

Theorem 9. [18] *The function $\boldsymbol{f_g}$ (as in Definition 8) is an incision function for X.*

As mentioned earlier, Cn* kernel pseudo-contraction subsumes Cn* partial meet pseudo-contraction. The following proposition shows the explicit construction:

Proposition 10. *If $\mathrm{pmc}_{\boldsymbol{g}}^{\mathrm{Cn}^*}$ is a Cn* partial meet pseudo-contraction, then it is equivalent to the Cn* kernel pseudo-contraction $\mathrm{kc}_{\boldsymbol{f_g}}^{\mathrm{Cn}^*}$.*

Proof. Let $\mathrm{pmc}_{\boldsymbol{g}}^{\mathrm{Cn}^*}$ be a Cn* partial meet pseudo-contraction. We can rewrite it as follows:

$$\begin{aligned}
&\mathrm{pmc}_{\boldsymbol{g}}^{\mathrm{Cn}^*}(B, \alpha) \\
&= \bigcap \boldsymbol{g}(\mathrm{Rem}[\mathrm{Cn}^*(B), \alpha]) \\
&= \mathrm{Cn}^*(B) \setminus \left[\mathrm{Cn}^*(B) \setminus \bigcap \boldsymbol{g}(\mathrm{Rem}[\mathrm{Cn}^*(B), \alpha])\right] \\
&= \mathrm{Cn}^*(B) \setminus \boldsymbol{f_g}(\mathrm{Ker}[X, \alpha]) \\
&= \mathrm{kc}_{\boldsymbol{f_g}}^{\mathrm{Cn}^*}(B, \alpha),
\end{aligned}$$

where $\boldsymbol{f_g}$ is the incision function defined as in Definition 8 (for $X = \mathrm{Cn}^*(B)$). ☐

In general, not every kernel contraction is equivalent to a partial meet contraction. By taking Cn* as the identity function, Cn* partial meet and kernel pseudo-contractions become partial meet and kernel contractions for belief bases, which means that they are not equivalent. Therefore, Cn* kernel pseudo-contractions may not have the same properties as Cn* partial meet pseudo-contractions.

Since inclusion* implies logical inclusion for every subclassical Cn* [14], we can see that a Cn* kernel pseudo-contraction is indeed a pseudo-contraction as long as Cn* satisfies subclassicality. If Cn* also satisfies inclusion, then the following property holds:

(logical core-retainment) If $\beta \in B \setminus \mathrm{c}(B, \alpha)$, then there is a B' such that $B' \subseteq \mathrm{Cn}(B)$ and $\alpha \in \mathrm{Cn}(B' \cup \{\beta\}) \setminus \mathrm{Cn}(B')$.

Observation 11. *If Cn* satisfies subclassicality and inclusion, then any operation that satisfies core-retainment* also satisfies logical core-retainment.*

A desirable property that is not necessarily satisfied by kernel contractions (hence, not always satisfied by Cn* kernel pseudo-contractions) is *relative closure* [19]:

(relative closure) $B \cap \mathrm{Cn}(\mathrm{c}(B, \alpha)) \subseteq \mathrm{c}(B, \alpha)$.

Nonetheless, kernel contractions and Cn* kernel pseudo-contractions satisfy relative closure if they are *smooth*:

Definition 12 (Smooth incision function and smooth kernel contraction [6])**.** An incision function f for X is *smooth* if $X' \cap f(\text{Ker}[X, \alpha]) \neq \emptyset$ for all $X' \subseteq X$ such that $\text{Cn}(X') \cap f(\text{Ker}[X, \alpha]) \neq \emptyset$. A kernel (pseudo-)contraction is *smooth* if its incision function is smooth.

Proposition 13. *If f is smooth and Cn* satisfies inclusion, then the Cn* kernel pseudo-contraction $\text{c}_f^{\text{Cn}^*}$ satisfies relative closure.*

Proof sketch. If the proposition does not hold, then there must be a sentence $\beta \in (B \cap \text{Cn}(\text{c}_f^{\text{Cn}^*}(B, \alpha)))$ such that $\beta \notin \text{c}_f^{\text{Cn}^*}(B, \alpha)$, and β must be in $f(\text{Ker}[\text{Cn}^*(B), \alpha])$ and in $\text{Cn}^*(B)$ due to inclusion of Cn^*. The set $B' := B \setminus f(\text{Ker}[\text{Cn}^*(B), \alpha])$ is such that $\beta \in \text{Cn}(B')$ from its definition, thus $\text{Cn}(B') \cap f(\text{Ker}[\text{Cn}^*(B), \alpha])$ is non-empty, and smoothness of f implies that $B' \cap f(\text{Ker}[\text{Cn}^*(B), \alpha]) \neq \emptyset$, which contradicts the definition of B'. □

The vacuity postulate states that the set should remain unchanged if it does not entail the sentence to be contracted:

(vacuity) If $\alpha \notin B$, then $\text{c}(B, \alpha) = B$.

Cn^* kernel pseudo-contraction does not satisfy vacuity: as shown by [13], it is not satisfied by Cn^* partial meet pseudo-contraction. However, a weaker version of this property is satisfied:

(vacuity*) If $\alpha \notin \text{Cn}(B)$, then $\text{c}(B, \alpha) = \text{Cn}^*(B)$.

Proposition 14. *If Cn* satisfies subclassicality, an operation that satisfies inclusion* and core-retainment* also satisfies vacuity*.*

Proof sketch. Inclusion* implies that $\text{c}(B, \alpha) \subseteq \text{Cn}^*(B)$, and $\supseteq$ is a consequence of core-retainment* and the assumptions that Cn^* is subclassical and Cn is Tarskian. □

4. Pseudo-contraction plug-in for Protégé: a prototype

We have implemented a prototype of a tool that computes Cn^* (partial meet and kernel) pseudo-contractions. The algorithms for obtaining remainder and kernel sets were adapted from Guimarães' repository [20][7]. It is a Protégé plug-in that provides a tab which can be added to the program window. The user can type the sentence to be contracted using Manchester syntax [21] and the traditional notation of Description Logics is displayed below the input field, as shown in Figure 2.

Besides the sentence α to be removed, the user chooses:

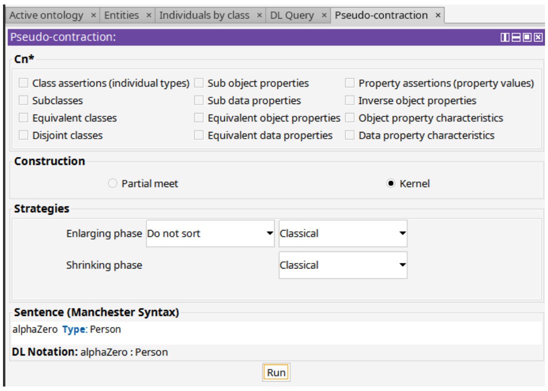

Figure 2: Screenshot of Protégé window with the pseudo-contraction tab.

- Cn^*: the types of consequences that will be generated by the reasoner before the operation;
- construction: partial meet or kernel;
- strategies: methods used to compute the remainder and kernel sets (see: [22, 23, 24, 25, 20]).

After the user clicks on *Run*, the program obtains the set $\text{Cn}^*(\mathcal{O})$ from the original ontology $\mathcal{O}$ and computes $\text{Rem}[\text{Cn}^*(\mathcal{O}), \alpha]$ or $\text{Ker}[\text{Cn}^*(\mathcal{O}), \alpha]$, according to the chosen construction. The remainders or kernels are then shown on a dialogue window, where they can be selected by the user. The kernel set and the incision function discussed in Example 6 are depicted in Figure 3 (it contains more kernels than in the example because the consequence generator *Class assertions* is more general than $\text{Cn}^*_{\text{break}}$ and generates the additional sentence $s : European$).

In all cases, the sentences that will be removed are displayed in red. To avoid cluttering the window, *Declaration* sentences and obvious tautologies (such as $a : \top$ and $C \sqsubseteq \top$) are omitted. After a click on the button *Execute operation*, the plug-in transforms the active ontology $\mathcal{O}$ into $\text{pmc}_g^{\text{Cn}^*}(\mathcal{O}, \alpha)$ or $\text{kc}_f^{\text{Cn}^*}(\mathcal{O}, \alpha)$, where Cn^* is determined by the selected sentence generators and the chosen remainders or kernel elements define the function g or f.

The plug-in is written in Java 8 and supports Protégé 5.5.0, which is the latest version at the time of writing. It uses OWL API[8] 4.2.5 to manipulate OWL objects[9]. The source code is available on GitLab[10].

[7] https://gitlab.com/rfguimaraes/owl-change

[8] http://owlcs.github.io/owlapi/

[9] Version 5.1.17 of OWL API is already available, but it is not supported by Protégé yet, which is why we had to use a previous version.

[10] The source code is publicly available in a GitLab repository: https://gitlab.com/viniciusbm/pseudo-contraction-protege-plugin.

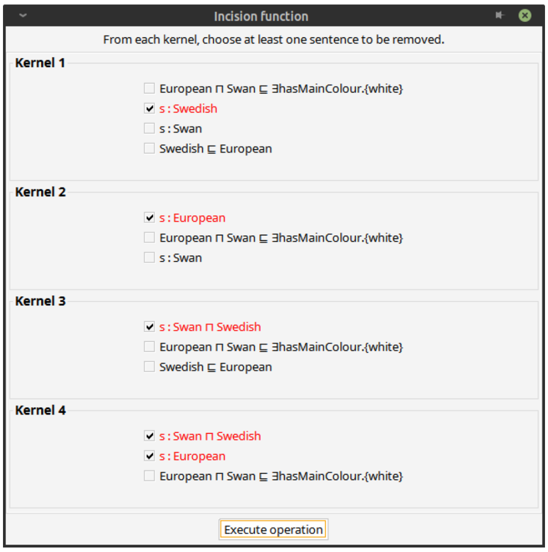

Figure 3: Screenshot of Protégé window with the kernel set of Example 6.

5. Ontology Repair

Ontology Repair consists in transforming an ontology so that it does not imply a certain formula. In what follows, we define the main concepts based on the presentation given by [12]. Consider that $\mathcal{O} = \langle \mathcal{O}_s, \mathcal{O}_r \rangle$ is an ontology consisting of a static and a refutable part ($\mathcal{O}_s$ and $\mathcal{O}_r$, respectively), which are assumed to be disjoint.[11] The static part contains the axioms which we want to preserve when we repair the ontology, while the refutable part contains those which we are willing to give up if needed. We assume that the separation into static and refutable is given.

Definition 15 (Repair). Let $\mathcal{O} = \langle \mathcal{O}_s, \mathcal{O}_r \rangle$ be an ontology and let α be a sentence entailed by $\mathcal{O}$ but not by $\mathcal{O}_s$. An ontology $\mathcal{O}'$ is a *repair* of $\mathcal{O}$ with respect to α if $\mathrm{Cn}(\mathcal{O}_s \cup \mathcal{O}') \subseteq \mathrm{Cn}(\mathcal{O}) \setminus \{\alpha\}$.

Classically, a repair consists of a subset of the refutable part of the ontology:

Definition 16 (Classical repair). A repair $\mathcal{O}'$ of the ontology $\mathcal{O}$ with respect to the sentence α is a *classical repair* if it is contained in $\mathcal{O}_r$.

In order to preserve as much knowledge as possible, we look for an optimal repair (which in general does not exist [12]):

[11] The notation $\langle \mathcal{O}_s, \mathcal{O}_r \rangle$ is meant to represent the set $\mathcal{O}_s \cup \mathcal{O}_r$ in a way that makes it possible to tell if a sentence is in the static part or in the refutable part.

Definition 17 (Optimal repair). A repair $\mathcal{O}'$ of the ontology $\mathcal{O}$ with respect to the sentence α is an *optimal repair* if no other repair $\mathcal{O}''$ (of $\mathcal{O}$ w.r.t. α) is such that $\mathrm{Cn}(\mathcal{O}_s \cup \mathcal{O}') \subset \mathrm{Cn}(\mathcal{O}_s \cup \mathcal{O}'')$.

An *optimal classical repair* is a classical repair which is optimal in the sense that no classical repair contains it. Unlike optimal repairs, optimal classical repairs always exist.

In order to find classical repairs, a construction based on justifications and hitting sets can be used. Justifications are minimal subsets of a base that imply the unwanted sentence:

Definition 18 (Justification [9]). Let $\mathcal{O} = \langle \mathcal{O}_s, \mathcal{O}_r \rangle$ be an ontology and α a sentence entailed by $\mathcal{O}$ but not by $\mathcal{O}_s$. A *justification* for α in $\mathcal{O}$ is an inclusion-minimal subset J of $\mathcal{O}_r$ such that $\alpha \in \mathrm{Cn}(\mathcal{O}_s \cup J)$. We will denote the set of all justifications for α in $\mathcal{O}$ as $\mathrm{Just}(\mathcal{O}, \alpha)$.

The definition above is often presented without partitioning the ontology, which corresponds to a particular case where $\mathcal{O}_r = \emptyset$.

[26] has proposed an algorithm to debug incoherent ontologies inspired by Reiter's hitting set tree [22]. Other authors [9, 24, 11] extended and generalised this algorithm to find all justifications for any given entailment.

Definition 19 (Hitting set [22]). Given a set $\mathcal{J}$ of justifications for a sentence in an ontology, a *hitting set* of $\mathcal{J}$ is a set H of sentences contained in $\bigcup \mathcal{J}$ such that $H \cap J \neq \emptyset$ for every $J \in \mathcal{J}$.

A repair $\mathcal{O}'$ of $\mathcal{O} = \langle \mathcal{O}_s, \mathcal{O}_r \rangle$ with respect to α is obtained by computing an inclusion-minimal hitting set H of $Just(\mathcal{O}, \alpha)$ and defining $\mathcal{O}'$ as the set $\mathcal{O}_r$ after the removal of each sentence in H.

Example 20. Let α be the sentence $alphaZero : Person$. Consider the knowledge base $\mathcal{O} = \langle \mathcal{O}_s, \mathcal{O}_r \rangle$, where $\mathcal{O}_s = \{alphaZero : ComputerProgram\}$ and

$$\begin{aligned}\mathcal{O}_r = \{&alphaZero : ChessPlayer,\\ &alphaZero : GoPlayer,\\ &ChessPlayer \sqcup GoPlayer \sqsubseteq Person\}.\end{aligned}$$

In order to obtain a repair of the ontology $\mathcal{O}$ with respect to α, we start by computing the set of justifications $\mathcal{J}$, which in this case is $\{\{alphaZero : ChessPlayer, ChessPlayer \sqcup GoPlayer \sqsubseteq Person\}, \{alphaZero : GoPlayer, ChessPlayer \sqcup GoPlayer \sqsubseteq Person\}\}$. Then, it obtains a minimal hitting set of $\mathcal{J}$, which may be the set $H := \{\{ChessPlayer \sqcup GoPlayer \sqsubseteq Person\}\}$. Lastly, it returns the set obtained by removing from $\mathcal{O}_r$ the elements of H, i.e. the set $\mathcal{O}' := \{alphaZero : ChessPlayer, alphaZero : GoPlayer\}$.

A special case of Ontology Repair is *ABox Repair*, where the TBox is fixed, i.e., the TBox is contained in $\mathcal{O}_s$. It is easy to see that when $\mathcal{T} = \mathcal{O}_s$ and $\mathcal{A} = \mathcal{O}_r$, an ABox repair is an optimal repair according to Definition 17.

Previously, we have shown that contraction operations in classical Belief Revision are too restrictive for belief bases because of the inclusion postulate, and we analysed pseudo-contraction operations — a generalisation of contraction that satisfies logical inclusion rather than inclusion. Similarly, in Ontology Repair, classical repairs do not allow the inclusion of new sentences, and the same issue is present: sentences are either kept or removed altogether. In our Example 20, the sentence *ChessPlayer* $\sqcup$ *GoPlayer* $\sqsubseteq$ *Person* was discarded, but we might want to replace it with a less constraining sentence that preserves some of the original information. A very similar idea was introduced by [12] in Ontology Repair: in a *gentle repair*, one can either remove a sentence or substitute it with a weaker version, retaining part of the information it represented.

Definition 21 (Weakening [12])**.** A sentence α_1 is *weaker* than a sentence α_2 if $\mathrm{Cn}(\{\alpha_1\}) \subset \mathrm{Cn}(\{\alpha_2\})$.

Definition 22 (Gentle Repair [12][12])**.** Let $\mathcal{O} = \langle \mathcal{O}_s, \mathcal{O}_r \rangle$ be an ontology and let α be a sentence entailed by $\mathcal{O}$ but not by $\mathcal{O}_s$. An ontology $\mathcal{O}'$ is a *gentle repair* of $\mathcal{O}$ with respect to α if $\mathrm{Cn}(\mathcal{O}_s \cup \mathcal{O}') \subseteq \mathrm{Cn}(\mathcal{O}) \setminus \{\alpha\}$ and, for every $\varphi \in \mathcal{O}'$, either $\varphi \in \mathcal{O}_r$ or φ is weaker than ψ for some $\psi \in \mathcal{O}_r \setminus \mathcal{O}'$.

The algorithm that computes a gentle repair is very similar to the procedure described earlier. The only difference is that $\mathcal{O}'$ is defined by replacing sentences that are in H with weaker versions rather than removing them. More specifically, for each β that would be removed by the original algorithm, we replace it with a β' weaker than β such that $\alpha \notin \mathrm{Cn}\left(\mathcal{O}_s \cup (J \setminus \{\beta\}) \cup \{\beta'\}\right)$ for every $J \in \mathcal{J}$ such that $\beta \in J$. Such a β' always exists: a tautology satisfies the requirements (note, though, that replacing a sentence with a tautology is logically equivalent to removing it, which means that a classical repair is obtained if we only use tautologies). In order to illustrate what is different in the outcome of this algorithm, we will use the same example:

Example 23. Consider again the problem discussed in Example 20. Starting with $\mathcal{O}' = \mathcal{O}_r$, we compute $\mathcal{J}$ and H as before. Then, instead of removing the sentence $\beta :=$ *ChessPlayer* $\sqcup$ *GoPlayer* $\sqsubseteq$ *Person*, it is replaced with a weaker version, such as $\beta' := (\mathit{ChessPlayer} \sqcup \mathit{GoPlayer}) \sqcap \neg \mathit{ComputerProgram} \sqsubseteq \mathit{Person}$. This procedure is repeated until the set $\mathcal{O}_s \cup \mathcal{O}'$ fails to entail α. In our example, replacing β with β' is enough to prevent such entailment, and the algorithm stops, returning the repair $\{\mathcal{O}_r \setminus \{\beta\}) \cup \{\beta'\}$.

A modified version of the procedure above was proposed by [12] where, instead of weakening each element of the minimal hitting set, only a single formula in each justification needs to be changed. Starting with $\mathcal{O}_r = \mathcal{O}'$, if $\alpha \in \mathrm{Cn}(\mathcal{O}_s \cup \mathcal{O}')$, a single justification J for α in $\mathcal{O}_s \cup \mathcal{O}'$ is computed, and for some arbitrary sentence β in J, we replace it with a weaker β' such that $\alpha \notin \mathrm{Cn}\left(\mathcal{O}_s \cup (J \setminus \{\beta\}) \cup \{\beta'\}\right)$ (as discussed earlier, such as β' always exists). [12] remark that as the unmodified version requires the computation of minimal hitting sets, which is expensive, the modified version has an important advantage, even though both may consume exponential time [12]. They are guaranteed to stop after a number of steps that grows at most exponentially in the size of the refutable part [12].

[12]In [12], the concept of gentle repair has not been formally defined, only explained in intuitive terms. We will use this definition, which we proposed in [27].

6. Correspondence between Belief Revision and Repairs in Description Logics

In this section, we will analyse the close relationship between the concepts and constructions presented for Ontology Repair and Belief Revision.

We start by giving two definitions that generalise several concepts in the literature.

Definition 24 (Maximal Non-Implying Subsets [27])**.** Let B be a knowledge base, α a sentence, and Φ a set of static sentences (i.e. which should be preserved in any operation). The set of *maximal α-non-implying subsets of B with respect to Φ*, denoted by $\mathrm{MaxNon}(B, \alpha, \Phi)$, is such that $X \in \mathrm{MaxNon}(B, \alpha, \Phi)$ if and only if $X \subseteq B$, $\alpha \notin \mathrm{Cn}(\Phi \cup X)$, and there is no Y such that $X \subset Y \subseteq B$ and $\alpha \notin \mathrm{Cn}(\Phi \cup Y)$.

For brevity, we shall omit the last argument of MaxNon whenever it is empty: $\mathrm{MaxNon}(B, \alpha, \emptyset)$ is the same as $\mathrm{MaxNon}(B, \alpha)$.

Remark 25 ([27])**.** *If $\Phi \subseteq B$, then the maximal α-non-implying subsets of B with respect to Φ contain all of the elements of Φ, i.e., $X \supseteq \Phi$ for every $X \in \mathrm{MaxNon}(B, \alpha, \Phi)$.*

Definition 24 corresponds to a remainder if $\Phi = \emptyset$, i.e., $\mathrm{MaxNon}(B, \alpha) = \mathrm{Rem}[B, \alpha]$.

Definition 26 (Minimal Implying Subsets [27])**.** Let B be a knowledge base, α a sentence, and Φ a set of static sentences. The set of *minimal α-implying subsets of B with respect to Φ*, denoted by $\mathrm{MinImp}(B, \alpha, \Phi)$, is such that $X \in \mathrm{MinImp}(B, \alpha, \Phi)$ if and only if $X \subseteq B$, $\alpha \in \mathrm{Cn}(\Phi \cup X)$, and there is no $Y \subset X$ such that $\alpha \in \mathrm{Cn}(\Phi \cup Y)$.

As in the previous definition, the last argument will be omitted if empty: $\text{MinImp}(B, \alpha) = \text{MinImp}(B, \alpha, \emptyset)$.

Remark 27 ([27]). *The minimal α-implying subsets of B with respect to Φ do not contain elements of Φ, i.e., $X \cap \Phi = \emptyset$ for every $X \in \text{MinImp}(B, \alpha, \Phi)$.*

If $\Phi = \emptyset$, Definition 26 corresponds to Definition 3, i.e., $\text{MinImp}(B, \alpha) = \text{Ker}[B, \alpha]$. Moreover, Definition 26 is closely related to Definition 18: $\text{MinImp}(B, \alpha, \Phi) = \text{Just}(\langle \Phi, B \setminus \Phi \rangle, \alpha)$, or conversely, $\text{Just}(\langle \mathcal{O}_s, \mathcal{O}_r \rangle, \alpha) = \text{MinImp}(\mathcal{O}_s \cup \mathcal{O}_r, \alpha, \mathcal{O}_s)$. As shown in [27], the definitions of MaxNon and MinImp also encompass concepts such as MaNAs (maximal non-axiom sets), MinAs (minimal axiom sets), MISs (minimal inconsistent sets) and arguments [28, 29, 30].

We can now analyse the relation between the definitions and operations of Belief Revision and Ontology Repair.

Let $B \subseteq \mathfrak{L}$ and $\alpha \in \mathfrak{L}$. The following two properties follow straightly from Definition 3 and Definition 18.

Proposition 28 (Kernel $\sim$ Justification [27]). *If $\alpha \in \text{Cn}(B)$, then a set X is an α-kernel of B with respect to α if and only if X is a justification for α in $\langle \emptyset, B \rangle$.*

The set of those sets, which we denote by $\text{MinImp}(B, \alpha)$, unifies the concepts of the following proposition:

Remark 29 (Kernel set $\sim$ Set of all justifications [27]). *If a sentence α and a set B are such that $\alpha \in \text{Cn}(B)$, then* $\text{Ker}[B, \alpha] = \text{Just}(\langle \emptyset, B \rangle, \alpha)$.

A classical repair (Definition 16) can be seen as a contraction operation that satisfies two of Hansson's postulates for base contraction (success and inclusion) [27].

The following proposition, which is an immediate consequence of the upper bound property [31], will be useful to show the connection between partial meet base contraction and classical repairs.

Proposition 30 (Existence of α-remainder preserving $\mathcal{O}_s$ [27]). *Let $\mathcal{O} = \langle \mathcal{O}_s, \mathcal{O}_r \rangle$ be an ontology and α be a sentence entailed by $\mathcal{O}$ but not by $\mathcal{O}_s$. Then, there is at least one α-remainder X of $\mathcal{O}_s \cup \mathcal{O}_r$ such that $\mathcal{O}_s \subseteq X$.*

Now we can show that partial meet base contractions that include the static part of the ontology yield classical repairs.

Theorem 31 (Partial meet base contraction $\Longrightarrow$ Classical repair [27]). *Under the conditions of Proposition 30, if $\boldsymbol{g}$ is such that $\mathcal{O}_s \subseteq X$ for every $X \in \boldsymbol{g}(\text{Rem}[\mathcal{O}, \alpha])$, then the operation $\text{Rep}_{\boldsymbol{g}}$ defined as $\text{Rep}_{\boldsymbol{g}}(\mathcal{O}, \alpha) = \text{pmc}_{\boldsymbol{g}}(\mathcal{O}, \alpha) \setminus \mathcal{O}_s$ yields a classical repair.*

We can now show the relationship between pseudo-contractions and gentle repairs.

Proposition 32 (Gentle Repair $\Longrightarrow$ Pseudo-contraction). *Let* GRep *be an operation that yields a gentle repair. Define the operation $\text{c}_{(\text{GRep})}$ as*

$$\text{c}_{(\text{GRep})}(B, \alpha) = \begin{cases} \text{GRep}(\langle \emptyset, B \rangle, \alpha), & \textit{if } B \models \alpha; \\ B, & \textit{otherwise.} \end{cases}$$

Then, $\text{c}_{(\text{GRep})}$ is a pseudo-contraction operation.

The result above follows from Definition 22, which guarantees that $\text{c}_{(\text{GRep})}$ satisfies success and logical inclusion.

For the other direction (pseudo-contractions as gentle repairs), we will introduce general partial meet pseudo-contractions and general kernel pseudo-contractions. Pseudo-contractions allow the result to contain some weakened versions of formulae that were originally in the belief base. This can be achieved by applying a partial meet or kernel operation on a "weak closure" of the belief base [14]. However, as this closure does not depend on the sentence that is being contracted, we cannot add only weakenings of formulae that would be removed. General (partial meet and kernel) pseudo-contractions employ a consequence operator (Cn^{**}) that depends on both the set of beliefs and the input sentence. Before defining them, we need the following concepts:

Definition 33 (Extension of a selection function [32, adapted]). Let $\boldsymbol{g}$ be a selection function for B, and let $B \subseteq B^* \subseteq \mathfrak{L}$. We say that a selection function $\boldsymbol{g'}$ for B^* is an *extension* of $\boldsymbol{g}$ to B^* if $\boldsymbol{g'}$ is such that for every $\alpha \in \mathfrak{L}$ and $X \in \boldsymbol{g}(\text{MaxNon}(B, \alpha))$ there is a $Y \in \boldsymbol{g'}(\text{MaxNon}(B^*, \alpha))$ such that $X \subseteq Y$.

Definition 34 (Extension of an incision function). Let $\boldsymbol{f}$ be an incision function for a set of sentences B, and let $B \subseteq B^* \subseteq \mathfrak{L}$. The incision function $\boldsymbol{f'}$ for B^* is an *extension* of $\boldsymbol{f}$ for B^* if $\boldsymbol{f'}(\text{MinImp}(B^*, \alpha)) \supseteq \boldsymbol{f}(\text{MinImp}(B^*, \alpha))$ for all sentences α.

The general partial meet pseudo-contraction[13] was proposed by [32] and generalised by [14] as a way to weaken sentences in belief base pseudo-contractions, instead of removing them:

Definition 35 (General partial meet pseudo-contraction). [32, 14] Let $\alpha \in \mathfrak{L}$, $B \subseteq \mathfrak{L}$, Cn' be a consequence relation, and $\boldsymbol{g}$ be a selection function for B. Let us define $\text{Cn}^{**}(B, \alpha) := \text{Cn'}(B \setminus \bigcap \boldsymbol{g}(\text{MaxNon}(B, \alpha))) \cup B$, and let $\boldsymbol{g'}$ be an extension of $\boldsymbol{g}$ to $\text{Cn}^{**}(B, \alpha)$. The *general partial meet pseudo-contraction* of B by α, denoted by $\text{gpmc}^{\text{Cn}^{**}}_{\boldsymbol{g},\boldsymbol{g'}}(B, \alpha)$, is defined as the set $\bigcap \boldsymbol{g'}(\text{MaxNon}(\text{Cn}^{**}(B, \alpha), \alpha))$.

[13] In [27], this operation was referred to as "two-place partial meet pseudo-contraction", which in [14] refers to a more general type of operations.

We will use a similar idea to define the general kernel pseudo-contraction:

Definition 36 (General kernel pseudo-contraction)**.** Let $\alpha \in \mathfrak{L}$, $B \subseteq \mathfrak{L}$, Cn' be a consequence relation, and $\boldsymbol{f}$ be an incision function for B. Let us define $\mathrm{Cn}^{**}(B, \alpha) := B \cup \mathrm{Cn'}(\boldsymbol{f}(\mathrm{MinImp}(B, \alpha)))$, and let $\boldsymbol{f}'$ be an extension of $\boldsymbol{f}$ to $\mathrm{Cn}^{**}(B, \alpha)$. The *general kernel pseudo-contraction* of B by α, denoted by $\mathrm{gkc}^{\mathrm{Cn}^{**}}_{\boldsymbol{f},\boldsymbol{f}'}(B, \alpha)$, is defined as the set $\mathrm{Cn}^{**}(B, \alpha) \setminus \boldsymbol{f}'(\mathrm{MinImp}(\mathrm{Cn}^{**}(B, \alpha), \alpha))$.

Consider the following properties for selection and incision functions:

Definition 37 (A-inclusion [27])**.** Let $B \subseteq \mathfrak{L}$, and let $A \subseteq B$. A selection function $\boldsymbol{g}$ for B satisfies *A-inclusion* if, for all $\alpha \notin \mathrm{Cn}(A)$, $A \subseteq X$ for every $X \in \boldsymbol{g}(\mathrm{MaxNon}(B, \alpha))$.

Definition 38 (A-exclusion)**.** Let $B \subseteq \mathfrak{L}$, and let $A \subseteq B$. An incision function $\boldsymbol{f}$ for B satisfies *A-exclusion* if $A \cap \boldsymbol{f}(\mathrm{MinImp}(B, \alpha)) = \emptyset$ for all $\alpha \notin \mathrm{Cn}(A)$.

Intuitively, a selection function (for B) that satisfies A-inclusion only selects α-remainders that preserve A, unless α itself is entailed by A, in which case A cannot be a subset of any α-remainder; similarly, an incision function (for B) that satisfies A-exclusion only selects sentences that are not in A, preserving A in the operation, unless α is entailed by A, in which case it is impossible to have an incision function that does not contain elements of A.

Lemma 39. *Consider a general partial meet pseudo-contraction defined as in Definition 35. For every sentence φ in $B \setminus \bigcap \boldsymbol{g}(\mathrm{MaxNon}(B, \alpha))$, there is a set X such that $X \in \boldsymbol{g}'(\mathrm{MaxNon}(\mathrm{Cn}^{**}(B, \alpha), \alpha))$ and $\varphi \notin X$.*

Proof sketch. The conditions imply the existence of a set $Y \in \boldsymbol{g}(\mathrm{MaxNon}(B, \alpha))$ such that $\varphi \notin Y$. Since $\boldsymbol{g}'$ is an extension of $\boldsymbol{g}$ to $\mathrm{Cn}^{**}(B, \alpha)$, there is an $X \in \boldsymbol{g}'(\mathrm{MaxNon}(\mathrm{Cn}^{**}(B, \alpha), \alpha))$ such that $Y \subseteq X$, and such X cannot contain φ due to the definition of remainder. □

If a consequence operator returns only sentences that are in the given set or are weaker than some of its sentences, then we say it is *strictly weakening*:

Definition 40 (Strictly weakening operator [27])**.** A consequence operator Con is *strictly weakening* if, for every $\varphi \in \mathfrak{L}$ and every $B \subseteq \mathfrak{L}$, $\varphi \in \mathrm{Con}(B)$ if and only if $\varphi \in B$ or $\mathrm{Con}(\{\varphi\}) \subset \mathrm{Con}(\{\psi\})$ for some $\psi \in B$.

Now we can show under which conditions a general (partial meet or kernel) pseudo-contraction yields a gentle repair.

Lemma 41. *Let* c *be a contraction operation for a set of sentences B. Let $\mathrm{c}^{\mathrm{Cn}^{**}}$ be a pseudo-contraction operation such that $\mathrm{c}^{\mathrm{Cn}^{**}}(B, \beta) \subseteq \mathrm{Cn}^{**}(B)$, where $(B \setminus \mathrm{c}(B, \beta)) \cap \mathrm{c}^{\mathrm{Cn}^{**}}(B, \beta) = \emptyset$ and $\mathrm{Cn}^{**}(B) := B \cup \mathrm{Cn'}(B \setminus \mathrm{c}(B, \beta))$ for all sentences β and the consequence relation* Cn' *is monotonic, subclassical and strictly weakening. If the ontology $\mathcal{O} := \langle \mathcal{O}_s, \mathcal{O}_r \rangle$ is such that $\mathcal{O}_s \subseteq \mathrm{c}(\mathcal{O}, \beta) \cap \mathrm{c}^{\mathrm{Cn}^{**}}(\mathcal{O}, \beta)$ for all sentences β and α is a sentence such that $\alpha \notin \mathrm{Cn}(\mathcal{O}_s)$, then the set $\mathcal{O}' := \mathrm{c}^{\mathrm{Cn}^{**}}(\mathcal{O}, \alpha) \setminus \mathcal{O}_s$ is a gentle repair of $\mathcal{O}$ with respect to α.*

Proof sketch. Using monotonicity, inclusion and idempotence of Cn, and also subclassicality of Cn' and success of $\mathrm{c}^{\mathrm{Cn}^{**}}$, we can show that $\mathcal{O}'$ is a repair, and the extra condition required for it to be a gentle repair is derived from the assumption that Cn' is strictly weakening. □

Theorem 42 (When a general partial meet pseudo-contraction is a gentle repair [27, adapted])**.** *Let $\mathrm{gpmc}^{\mathrm{Cn}^{**}}_{\boldsymbol{g},\boldsymbol{g}'}$ and Cn^{**} be as in Definition 35, Cn^{**} based on a consequence relation* Cn' *that satisfies subclassicality, $\boldsymbol{g}$ and $\boldsymbol{g}'$ satisfy $\mathcal{O}_s$-inclusion,* Cn' *be monotonic and strictly weakening, and $\mathcal{O} = \langle \mathcal{O}_s, \mathcal{O}_r \rangle$. If $\alpha \notin \mathrm{Cn}(\mathcal{O}_s)$, then $\mathcal{O}' := \mathrm{gpmc}^{\mathrm{Cn}^{**}}_{\boldsymbol{g},\boldsymbol{g}'}(\mathcal{O}, \alpha) \setminus \mathcal{O}_s$ is a gentle repair of $\mathcal{O}$ w.r.t. α.*

Proof sketch. The result follows from Lemma 41 by taking $\mathrm{pmc}_{\boldsymbol{g}}$ as c and $\mathrm{gpmc}^{\mathrm{Cn}^{**}}_{\boldsymbol{g},\boldsymbol{g}'}$ as $\mathrm{c}^{\mathrm{Cn}^{**}}$. □

Theorem 43 (When a general kernel pseudo-contraction is a gentle repair)**.** *Let $\mathrm{gkc}^{\mathrm{Cn}^{**}}_{\boldsymbol{f},\boldsymbol{f}'}$ and Cn^{**} be as in Definition 36, Cn^{**} based on a consequence relation* Cn' *that satisfies subclassicality, $\boldsymbol{f}$ and $\boldsymbol{f}'$ satisfy $\mathcal{O}_s$-exclusion,* Cn' *be monotonic and strictly weakening, and $\mathcal{O} = \langle \mathcal{O}_s, \mathcal{O}_r \rangle$. If $\alpha \notin \mathrm{Cn}(\mathcal{O}_s)$, then $\mathcal{O}' := \mathrm{gkc}^{\mathrm{Cn}^{**}}_{\boldsymbol{f},\boldsymbol{f}'}(\mathcal{O}, \alpha) \setminus \mathcal{O}_s$ is a gentle repair of $\mathcal{O}$ w.r.t. α.*

Proof sketch. The result follows from Lemma 41 by taking $\mathrm{kc}_{\boldsymbol{f}}$ as c and $\mathrm{gkc}^{\mathrm{Cn}^{**}}_{\boldsymbol{f},\boldsymbol{f}'}$ as $\mathrm{c}^{\mathrm{Cn}^{**}}$. □

7. Conclusions

In this paper, we have introduced a construction for pseudo-contraction based on kernel contraction, and we have characterised it by means of a representation theorem. Furthermore, we have implemented a prototype of a tool that computes Cn* partial meet and kernel pseudo-contractions, built upon existing software that computes remainder and kernel sets. Lastly, we have analysed the similarities between some concepts and definitions of Belief Revision and Ontology Repair (more specifically, pseudo-contractions and gentle repairs, respectively), extending previous work [27] and showing that the new operation that we have introduced is also connected to gentle repairs. The last two theorems show that gentle repairs can be constructed by restricted forms of pseudo-contraction. One question that remains is whether we need these restrictions from the point of view of Ontology Repair, i.e., whether it makes sense to define more general forms of repair that lie in between gentle and classical repairs.

In the future, we would like to evaluate the performance of pseudo-contractions in both artificial and real-world ontologies in order to compare the practical efficiency of the constructions. In particular, it would be useful to apply the operations to benchmarks designed for ontology repair problems such as [33].

Also, we think it would be relevant to explore families of Cn^* consequence operators that are interesting for theoretical or practical purposes. As an example, we can think of approximations as defined in [34] as generating consequences in less expressive logics.

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