Rational Defeasible Subsumption in DLs with Nested Quantifiers: the Case of $\mathcal{ELI}_\perp$

(Extended Abstract)

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Abstract

Defeasible description logics (DDLs) support nonmonotonic reasoning by admitting defeasible concept inclusions in the knowledge base. Early reasoning methods for subsumption did not always use defeasible information for objects in the scope of nested quantifiers and thus neglected un-defeated information. The reasoning approach employing typicality models for the DDL $\mathcal{EL}_\perp$ overcomes this effect for existentially quantified objects.

In this extended abstract we report on how to lift typicality model-based reasoning to the DDL $\mathcal{ELI}_\perp$, which extends $\mathcal{EL}_\perp$ with inverse roles. These can capture a form of universal quantification and extend expressivity of the DDL substantially. Reasoning in DDLs often employs rational closure according to the propositional KLM postulates. We can show that the proposed subsumption algorithm yields more entailments than rational propositional entailment.

Keywords

Description Logics, defeasible reasoning, Non-monotonic reasoning, Typicality

Description logics (DLs) are knowledge representation formalisms that are designed to model terminological knowledge. Important notions from an application domain are modeled by concepts, which are essentially unary first-order logic predicates. Each DL offers a set of concept constructors which can be used to build complex concepts. The so-called roles correspond to binary relations and can be used in concept constructors to relate members of one concept to members of another. The use of roles and the quantification over the role-successors is what sets DLs apart from propositional logic. Concepts can be related to each other by so-called *general concept inclusions* (GCIs), which state material implications for a pair of (complex) concepts. A finite set of GCIS is called a *TBox* $\mathcal{T}$.

Reasoning in description logics is usually the classical, monotone first-order reasoning. Two prominent reasoning problems are satisfiability of a concept w.r.t. an ontology and to decide subsumption for two given concepts w.r.t. an ontology. The latter is to test whether membership to the first concept implies membership to the second w.r.t. to the GCIs in $\mathcal{T}$ and is a classical entailment relation. For certain applications monotone reasoning can be a short-coming and variants of DLs with non-monotonic reasoning have been investigated by the research community. A popular nonmonotonic variant are defeasible description logics (DDLs) which can express knowledge that holds until it is defeated by contradictory information. DDLs can express what properties typical members of a concept fulfill by the use of *defeasible concept inclusions* (DCIs) . A finite set of GCIS is a *DBox* $\mathcal{D}$. A *defeasible knowledge base* (DKB) is a pair of a TBox and a DBox: $\mathcal{K} = (\mathcal{T}, \mathcal{D})$.

There are several proposals for semantics of defeasible DLs in the literature, such as [1, 2, 3, 4, 5, 6]. Many of them use a kind of preferential semantics that often relies on a preference relation on the interpretation domain. Another well-investigated approach is supply the semantics by materialization-based reasoning, where essentially the information from the DCIs is used in conjunction with the (potential) subsumee. This approach has the severe short-coming of *quantification neglect* which means that defeasible information is not used for all the elements in the relational neighborhood of the subsumee. Thus even un-defeated defeasible information can be omitted when performing reasoning over existentially quantified objects—as it was observed in [5] and later and independently in [7, 6].

One approach that alleviates quantification neglect and does not rely on a preference relation over the domain is defeasible reasoning by typicality models. These models were introduced for the DL $\mathcal{EL}_\perp$ and provide a supraclassical inference relation. The classical DL $\mathcal{EL}_\perp$ provides conjunction and a form of existential quantifica-

NMR'22: 20th International Workshop on Non-Monotonic Reasoning, August 07-09, 2022, Haifa, Israel

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tion called *existential restrictions* as concept constructors. It can also state disjointness of concepts by the use of $\bot$. $\mathcal{EL}_\bot$ has the canonical model property, i.e. there always exists a model that can be embedded into all other models. Thus testing whether α is entailed (i.e. holds in all models) can be done by computing the canonical model and test whether α is satisfied in it. Reasoning in $\mathcal{EL}_\bot$ can be done in polynomial time [8].

To compute the canonical model in classical $\mathcal{EL}_\bot$, the ontology is normalized such that complex concepts get assigned a name. The domain of the canonical model consists of a representative for each of the named concepts. The canonical models are a main building block of the typicality model used for to characterize the semantics of defeasible reasoning in $\mathcal{EL}_\bot$.

Typicality interpretations used for defeasible reasoning have 2-dimensional domains. One dimension is the *representative domain*, which coincides with the domain of the classical canonical model. The other dimension is determined by which subsets of the DBox $\mathcal{D}$ are "applied" to the elements. Each element of the typicality domain for $\mathcal{EL}_\bot$ is a pair of a concept name and a subset of $\mathcal{D}$.

Now, the use of different collections of subsets from $\mathcal{D}$ induces different strengths of reasoning. The use of a chain of subsets given by the exceptionality chain computed according to [9] gives reasoning of rational strength. This chain would always include the empty set indicating that no defeasible information needs to be satisfied. (To achieve reasoning of relevant strength the whole lattice of subsets of $\mathcal{D}$ is used.) Besides the parameter for strength of reasoning, the semantics is also determined by the parameter of coverage. *Coverage* of reasoning, which determines whether defeasible information is only "applied" to the root object of a concept (and not necessarily to the objects in its relational neighborhood) or to all objects in the relational neighborhood of a concept. The first is called propositional strength and the latter is called nested strength. Different forms of coverage of reasoning are induce by the relational structure on the domain, i.e. by forcing successors to be as typical as possible or not forcing this.

Reasoning of propositional strength is mainly of interest to us to be able to compare the resulting inference relation to materialization-based reasoning. Reasoning under nested coverage results in an inference relation that does not cause quantification neglect.

The results presented in this extended abstract are initial steps on a longer research path. We want to investigate defeasible reasoning by means of typicality models for defeasible Horn-$\mathcal{ALC}$. This DDL is fairly expressive, as (non-Horn) $\mathcal{ALC}$ is propositionally complete and admits the use of both quantifiers. For all quantification can be captured by the concept constructor called value restriction. Horn-$\mathcal{ALC}$ restricts $\mathcal{ALC}$ to GCIs that are Horn rules and Horn-$\mathcal{ALC}$ enjoys the canonical model property. To lift the method for $\mathcal{EL}_\bot$ to Horn-$\mathcal{ALC}$ two extensions of the logic need to be addressed:

- the more general form of negation and
- forall quantification

We address the latter by investigating $\mathcal{ELI}_\bot$, which extends $\mathcal{EL}_\bot$ by inverse roles which, in turn, can express value restrictions as consequences.

The goal of this paper is to develop a characterization of defeasible entailment (and thus of defeasible subsumption) that alleviates quantification neglect and provides reasoning of rational strength. To that end we proceed as it was done in [6] for $\mathcal{EL}_\bot$. First we develop a characterization of entailment under rational strength and propositional coverage. From this we develop a characterization of entailment under rational strength and nested coverage.

Entailment under rational strength and propositional coverage in $\mathcal{ELI}_\bot$. The first step to lift the technique from [6] to $\mathcal{ELI}_\bot$ is to adapt the typicality domain. We use for the first dimension the representative domain for $\mathcal{ELI}_\bot$. The classical canonical model for $\mathcal{ELI}_\bot$ uses sets of concept names as domain elements, since the combination of existential restrictions and value restrictions can cause conjunctions for which no name exists in the DKB. For instance, when $\exists r.E$ and $\forall r.F$ get combined, there need not be a name for the concept $E \sqcap F$ that the r-successor belongs to. This extended representative domain makes several of the technical constructions for $\mathcal{EL}_\bot$ more involved for $\mathcal{ELI}_\bot$. It also incurs an increase of computational complexity from polynomial time to ExpTime for reasoning already in for classical reasoning [8].

The second dimension of the typicality domain for $\mathcal{ELI}_\bot$ is—as before—the exceptionality chain computed according to [9]. This gives the domain for rational strength reasoning in $\mathcal{ELI}_\bot$ in general. It is the relational structure on the typicality domain that determines the coverage of reasoning. In case of propositional coverage, we extend the *minimal typicality models* for $\mathcal{EL}_\bot$ to the use of inverse roles.

In *minimal typicality models*, the root element belonging to a named concept can have any degree of typicality admitted by the domain, i.e. it can satisfy any subset of DCIs available in the (second dimension of the) typicality domain. The elements that are in the relational neighborhood of this root element, however, do not need to satisfy any of the defeasible information. Therefore every role successor necessitated by existential restrictions for roles or their inverse, are elements from the typicality domain, where the second component is empty, i.e. where no DCI needs to be satisfied.

We show that defeasible subsumption w.r.t. a $\mathcal{ELI}_\bot$ DKB and under rational strength and propositional coverage can be decided by testing satisfaction of it in the rational minimal typicality model alone.

Entailment under rational strength and nested coverage in $\mathcal{ELI}_\bot$. The characterization of nested rational reasoning for $\mathcal{EL}_\bot$ is achieved by means of maximal typicality models. The idea for this kind of models is that not only the root element of a concept is *as typical as it gets*, but that also all the elements that the root element is connected to via (inverse) roles are. Maximal typicality models use the same typicality domain as their minimal counterparts. The generation of maximal typicality models is done by a fixed-point construction starting from the minimal typicality model. It successively makes elements that a role edge starts from or ends in more typical, i.e. reconnects at an element in the typicality domain that represents the same set of named concepts, but is coupled with a bigger subset of $\mathcal{D}$. The fixed-point construction proceeds in two steps in every round:

1. identify an edge in the active set of models that can be upgraded to a more typical successor or predecessor and upgrade that edge
2. for the obtained interpretation, restore it to be a model of the DKB again

This construction operates on a set of models, where only the ones with elements that are maximally typical are kept. From this set the maximal typicality model is obtained as the one model capturing the information common to all models in the set.

The major technical challenge for defining an upgrade method for $\mathcal{ELI}_\bot$ is that here the element that causes an edge in the model and the role predecessor of that edge need no longer coincide as it is the case in $\mathcal{EL}_\bot$. This required a much more elaborate technique for upgrading the models.

Our main result is that defeasible subsumption w.r.t. a $\mathcal{ELI}_\bot$ DKB and under rational strength and nested coverage can be decided by testing satisfaction of it in the rational maximal typicality model alone. This establishes the computation of rational maximal typicality models as the main step in the decision procedure for entailment (and subsumption) for $\mathcal{ELI}_\bot$ that does not commit quantification neglect.

We also investigate the relationship between the materialization-based semantics and the one given by nested rational reasoning realized by maximal typicality models. We show that the latter semantics indeed yields consequences that are a superset of the consequences obtained by the materialization-based semantics.

By these results we have provided a method to decide entailment in DDLs that admit the use of both quantifiers and that does not omit defeasible information unless a contradiction is encountered.

Currently we are working on typicality models that can achieve reasoning of relevant strength. We need to see whether a mere change of the underlying typicality domain—the full lattice $\mathcal{P}(\mathcal{D})$ instead of the exceptionality chain—is enough to for this or whether new techniques in comparison to [6] are required. Also, a comparison between the resulting inference relations for $\mathcal{ELI}_\bot$ would be a asset to understand defeasible reasoning in DDLs better. In the long run, it is interesting to extend these results to the DDL Horn-$\mathcal{ALC}$.

Acknowledgments

This study financed in part by the Coordenação de Aperfeiçoamento de Pessoal de Nível Superior – Brasil (CAPES) – Finance Code 001 and also by the Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq) and by the AI competence center ScaDS.AI Dresden/Leipzig.

References

[1] L. Giordano, V. Gliozzi, N. Olivetti, G. L. Pozzato, Preferential description logics, in: N. Dershowitz, A. Voronkov (Eds.), Logic for Programming, Artificial Intelligence, and Reasoning, 14th International Conference, LPAR, 2007, Proceedings, volume 4790 of *LNCS*, Springer, 2007, pp. 257–272.

[2] L. Giordano, V. Gliozzi, N. Olivetti, G. L. Pozzato, Reasoning about typicality in preferential description logics, in: European Workshop on Logics in Artificial Intelligence, Springer, 2008, pp. 192–205.

[3] K. Britz, J. Heidema, T. Meyer, Modelling object typicality in description logics, in: Australasian Joint Conference on Artificial Intelligence, Springer, 2009, pp. 506–516.

[4] K. Britz, T. Meyer, I. Varzinczak, Semantic foundation for preferential description logics, in: Australasian Joint Conference on Artificial Intelligence, Springer, 2011, pp. 491–500.

[5] P. A. Bonatti, M. Faella, I. M. Petrova, L. Sauro, A new semantics for overriding in description logics, Artificial Intelligence 222 (2015) 1–48.

[6] M. Pensel, A.-Y. Turhan, Reasoning in the defeasible description logic $\mathcal{EL}_\bot$—computing standard inferences under rational and relevant semantics, International Journal of Approximate Reasoning (IJAR) 103 (2018) 28–70. doi:`https://doi.org/10.1016/j.ijar.2018.08.005`.

[7] M. Pensel, A.-Y. Turhan, Including quantification in defeasible reasoning for the description logic $\mathcal{EL}_{\perp}$, in: M. Balduccini, T. Janhunen (Eds.), Proceedings of the 14th International Conference on Logic Programming and Nonmonotonic Reasoning - LPNMR, Springer, 2017, pp. 78–84. doi:https://doi.org/10.1007/978-3-319-61660-5_9.

[8] F. Baader, S. Brandt, C. Lutz, Pushing the $\mathcal{EL}$ envelope, in: L. P. Kaelbling, A. Saffiotti (Eds.), IJCAI-05, Proceedings of the Nineteenth International Joint Conference on Artificial Intelligence, Professional Book Center, 2005, pp. 364–369. URL: http://ijcai.org/Proceedings/05/Papers/0372.pdf.

[9] G. Casini, T. Meyer, K. Moodley, R. Nortjé, Relevant closure: A new form of defeasible reasoning for description logics, in: Proceedings of the 14th European Conference on Logics in Artificial Intelligence (JELIA'14), 2014, pp. 92–106.