Computer Algebra and Automated Reasoning

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1 Computer Algebra and Definite Integration

My current work is in the area of using Automated Theorem Proving (ATP) to support Computer Algebra Systems (CAS). CAS, as the name suggests, are systems designed originally for performing algebraic calculations on computer. They generally began life as a collection of specific algorithms for algebra, with a common specification language. Gradually their domain of application was expanded and eventually they were packaged and marketed as general purpose mathematical environments. They retain two major legacies of computer algebra: firstly they are very good at calculation but poor at dealing with logical side conditions. Secondly they include many transformation routines which, while algebraically completely valid, are only analytically valid for a small subset of the real line.

As a pilot study in using ATP technology to support better CAS calculation (there are a number of different ways to approaching hytbrid CAS/ATP systems) we have focussed on the problem of definite integration. This is a specific problem area that highlights both of the problems mentioned above.

While indefinite integration is "simply" (it's not always so simple) a matter of solving the differential algebra question:

Given a function f find a function F such that:

$$F' = f,$$

definite integration involves calculating the area under the curve of f and as such requires much more attention to the domain of definition of f (the location and nature of discontinuities).

CAS systems deal badly with the analytical side conditions of continuity that are inlcuded in the fundamental theorem of calculus:

Given a function f and a function F such that:

$$F' = f,$$

and given two limits a, b such that

$[a, b] \subseteq \mathrm{Dom}(f)$ and f is continuous on $[a, b]$ then

$$\int_a^b f(x)\mathrm{dx} = F(b) - F(a)$$

While there are tricks to avoid problems with discontinuities computer algebra systems in general ignore many discontinuities and deal poorly with most others. It is usually left to the user (sometimes without informing them of this fact) that they must manually check the side conditions on using the fundamental theorem of calculus.

In particular, computer algebra systems deal very poorly with calcuations involving parameters. In cases where they will correctly identify (and sometimes correctly work around) problem points in completely concrete cases, they will often fail when presented with identical problems involving even one parameter.

As part of a larger scheme considering strong definite integration algorithms we have produced a prototype Definite Integral Table Look Up (DITLU) system.

Details of the system may be found in [AGLM99a, AGLM99b].

2 Real Number Theorem Proving

Harrison developed a theory of real numbers and a medium sized analysis library as part of his PhD [Har98]. Unfortunately, this development was performed within HOL-Light, an unreleased and unsupported version of HOL. Since then much of his library has been ported to HOL 98. In parallel with this, Gottliebsen has developed an analogue of much of Harrison's work within PVS [Got00]. Our ongoing work includes developments of this library to solve two major problems: automatic solution of sets of inequations involving real parameters and transcendetal functions; automatic checking of continuity of elementary functions.

The development of libraries and tactics for real number theorem proving is now an acheivable goal, and the possible benefits in software verification and automated support of other systems are huge. Until recently this realm seemed beyond the capabilities of theorem proving software, but we have reached a stage where that is no longer true, and the development of continuous mathematics within theorem proving is becoming more and more important.

References

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