Propositional Temporal Resolution Revised

Anatoli Degtiarev and Michael Fisher
Department of Computing and Mathematics, Manchester Metropolitan University
Manchester M1 5GD, U.K.
(e-mails: {A.Degtiarev,M.Fisher}@doc.mmu.ac.uk)

We propose a refinement of clausal proposional temporal resolution introduced in [Fis91] and considered thoroughly in [FDP00]. Knowledge of all basic notions related to this version of propositional temporal resolution is assumed.

Recall that the general form of the temporal resolution rule from [Fis91] is the following

$$\frac{A \Rightarrow \bigcirc \Box \neg l \quad C \Rightarrow \Diamond l}{C \Rightarrow (\neg A)\,\mathcal{W}\, l}\ (gtr)$$

Now, in some cases instead of involving the connective $\mathcal{W}$ ("unless"), we can manage with an "ordinary" version of resolution[1], which will be referred to as *(general) weak temporal resolution*:

$$\frac{A \Rightarrow \bigcirc \Box \neg l \quad C \Rightarrow \Diamond l}{A \wedge C \Rightarrow l}\ (gwtr)$$

This rule gives a weaker conclusion, however, it is more convenient. The convenience and relative sufficiency of this rule can be justified in particular by checking all examples in [FDP00], where only this rule is required. Moreover, in some cases, we retain completeness replacing temporal resolution by weak temporal resolution .

To give a standard form of weak temporal resolution for PLTL-clauses we follow [Fis91, FDP00] and join the derivation of $A \Rightarrow \bigcirc \Box \neg l$ and temporal resolution in a single combined rule:

$$\frac{A_1 \Rightarrow \bigcirc B_1 \quad \ldots \quad A_n \Rightarrow \bigcirc B_n \quad C \Rightarrow \Diamond l}{(\bigvee_{i=1}^{n} A_i) \wedge C \Rightarrow l}\ (wtr)$$

where the *loop* side conditions

$$\vdash B_i \supset \neg l \quad \text{and} \quad \vdash B_i \supset \bigvee_{j=1}^{n} A_j \quad \text{for all} \quad i \in \{1, \ldots, n\}.$$

have to be satisfied[2].

Proposition 1 A propositional temporal resolution system consisting of *initial resolution, step resolution and weak temporal resolution* rules is complete with respect to sets of PLTL-clauses containing at most one eventuality literal[3].

Unfortunately if an initial set of PLTL clauses contains more than one eventuality literal weak resolution is not sufficient. It is demonstrated by the following example.

[1]taking into account that the clause $C \Rightarrow \Diamond l$ is equivalent to $C \Rightarrow l \vee \bigcirc \Diamond l$.

[2]Let $I = \bigvee_{j=1}^{n} A_j$. Under the side conditions given above I can be considered as an *invariant formula* that gives derivability of $\Box \bigcirc \neg l$. Indeed, $I = \bigvee A_j \Rightarrow \bigvee \bigcirc B_j = \bigcirc \bigvee B_j \Rightarrow \bigcirc \neg l$ and $I = \bigvee A_j \Rightarrow \bigvee \bigcirc B_j = \bigcirc \bigvee B_j \Rightarrow \bigcirc \bigvee A_j = \bigcirc I$.

[3]A formula $\Diamond l$ (a literal l) from the right-hand side of a sometime PLTL-clause is called *eventuality* (*eventuality literal*).

Example Consider the following set of (merged) PLTL-clauses containing two eventuality literals:

1. $\textbf{start} \Rightarrow a \wedge \neg l_1 \wedge \neg l_2$;
2. $a \Rightarrow \bigcirc(\neg a \wedge (l_1 \vee l_2) \wedge (\neg l_1 \vee \neg l_2))$;
3. $(\neg a \wedge l_1 \wedge \neg l_2) \Rightarrow \bigcirc(\neg a \wedge l_1 \wedge \neg l_2)$;
4. $(\neg a \wedge \neg l_1 \wedge l_2) \Rightarrow \bigcirc(\neg a \wedge \neg l_1 \wedge l_2)$;
5. $a \Rightarrow \Diamond l_1$;
6. $a \Rightarrow \Diamond l_2$.

This set is unsatisfiable, however we cannot derive a contradiction by weak temporal resolution (wtr).

To get completeness in general we use the augmentation approach of [FDP00] in a slightly different way. For each eventuality literal l occurring in an initial set S we introduce a new proposition w_l as a name of the formula $\bigcirc\Diamond l$. After that we apply, to every *sometime clause* $C \Rightarrow \Diamond l$ containing this eventuality literal, a fix point definition of the connective $\Diamond$ renaming the subformulas $\bigcirc\Diamond l$ by w_l. Namely,

$$
\begin{array}{ll}
C \Rightarrow \Diamond l & \text{is replaced by} \\
\{C \Rightarrow l \vee w_l, w_l \Rightarrow \bigcirc\Diamond l\} & \text{is replaced by} \\
\{C \wedge \neg l \Rightarrow w_l, w_l \Rightarrow \bigcirc\Diamond l, w_l \Rightarrow \bigcirc(l \vee \bigcirc\Diamond l)\} & \text{is replaced by} \\
\{C \wedge \neg l \Rightarrow w_l, w_l \Rightarrow \bigcirc\Diamond l, w_l \Rightarrow \bigcirc(l \vee w_l)\} & \text{is replaced by} \\
\{\textbf{start} \Rightarrow \neg C \vee l \vee w_l, \textbf{true} \Rightarrow \neg C \vee l \vee w_l, w_l \Rightarrow \bigcirc\Diamond l, w_l \Rightarrow \bigcirc(l \vee w_l)\}. &
\end{array}
$$

It results in the same augmented set of clauses as in [FDP00]. Let us denote it by S^{aug}. This set is satisfiable if, and only if, the initial set S is satisfiable because renaming does not affect satisfiability.

Now, we define an analogue of the weak temporal resolution rule for S^{aug} as follows:

$$
\frac{A_1 \Rightarrow \bigcirc B_1 \quad \ldots \quad A_n \Rightarrow \bigcirc B_n \quad w_l \Rightarrow \bigcirc\Diamond l}{(\bigvee_{i=1}^{n} A_i) \Rightarrow \neg w_l} \ (atr)
$$

with the same loop side conditions for the step clauses as above.

Let us refer to this rule as *augmented temporal resolution* (atr) . It is not difficult to see that augmented resolution is sound. Indeed, the loop side conditions imply $(\bigvee_{i=1}^{n} A_i) \supset \bigcirc\Box\neg l$, and in addition the last premise of the rule is equivalent to $\bigcirc\Box\neg l \supset \neg w_l$. Completeness of augmented resolution is obtained by simple adaptation of the completeness proof given in [FDP00].

Proposition 2 A propositional temporal resolution system consisting of *initial resolution, step resolution and augmented temporal resolution* rules is complete with respect to augmented sets of PLTL-clauses.

Example (continuation) We can obtain a contradiction from 1-4 as follows.

At first let us produce augmentation clauses corresponding to eventuality literals l_1 and l_2.

7. $w_1 \Rightarrow \bigcirc\Diamond l_1$;
8. $w_2 \Rightarrow \bigcirc\Diamond l_2$;
9. $w_1 \Rightarrow \bigcirc(l_1 \vee w_1)$;
10. $w_2 \Rightarrow \bigcirc(l_2 \vee w_2)$;
11. $\textbf{start} \Rightarrow (\neg a \vee l_1 \vee w_1)$;
12. $\textbf{start} \Rightarrow (\neg a \vee l_2 \vee w_2)$;
13. $\textbf{true} \Rightarrow \bigcirc(\neg a \vee l_1 \vee w_1)$;
14. $\textbf{true} \Rightarrow \bigcirc(\neg a \vee l_2 \vee w_2)$.

Now, we can apply augmented temporal resolution (atr) to pairs $7, 4$ and $8, 3$ obtaining clauses 15 and 16:

15. $\textbf{true} \Rightarrow \bigcirc(a \vee l_1 \vee \neg l_2 \vee \neg w_1)$;
16. $\textbf{true} \Rightarrow \bigcirc(a \vee \neg l_1 \vee l_2 \vee \neg w_2)$.

Further we can derive by step resolution from 2 and $9, 10, 15, 16$

17. $a \wedge w_1 \wedge w_2 \Rightarrow \bigcirc\textbf{false}$.

At last we can get a contradiction just by initial resolution from $17, 1, 11, 12$.

The refinements of clausal propositional temporal resolution described in this abstract have appeared in the process of our work on developing resolution decision procedures for restricted fragments of the the first-order temporal logic. This work is in progress now.

References

[FDP00] M. Fisher, C. Dixon, and M. Peim. Clausal temporal resolution. *ACM Transactions on Computation Logic*, 2000. To appear.

[Fis91] M. Fisher. A resolution method for temporal logic. In *Proceedings of the Twelfth International Joint Conference on Artificial Intelligence (IJCAI)*. Morgan Kaufman, 1991.