Theorem Proving for Temporal Logics of Knowledge or Belief

Cláudia Nalon

Centre for Agent Research and Development
Department of Computing and Mathematics
Manchester Metropolitan University
Manchester M1 5GD, UK

Email: C.Nalon@doc.mmu.ac.uk

June, 2000

Temporal logics have been investigated in computer science for over twenty years. They were introduced in [12] as a tool for the specification of reactive systems and, since then, they have been shown to be useful in a variety of applications (e.g. temporal databases, model checking). Nevertheless, for some applications, a temporal component is not enough to describe the properties of the system. To deal with properties of distributed and multi-agent systems, temporal logic is often augmented with modal operators of either knowledge or belief. For description of particular systems, it is also necessary to restrict attention to a class of models. For instance, by adding the axiom $K \bigcirc \phi \Rightarrow \bigcirc K \phi$, synchronous systems with perfect recall can be described. Axioms involving operators from both logics are known as *interaction* axioms.

Within the CARD, a proof method, based on the principle of resolution [11], for temporal logics of knowledge and belief has been proposed [3, 6] and a prototype implementation has been developed. Interactions between knowledge and time, their properties, and issues of complexity have been discussed in [10, 4, 7]. A resolution-based method for a temporal logic of knowledge for synchronous systems with perfect recall has been described in [1]. It has been shown that by adding new clauses to the normal form, no new resolution rules are required and the theorem prover for temporal logics of knowledge (without interactions) can be used with a small number of changes. Current work involves the investigation of other useful interactions, such as perfect recall (alone) [10] and no learning with synchrony ($\bigcirc K \phi \Rightarrow K \bigcirc \phi$) [7]. Interactions between belief and time, as those described in [5] and [9], will also be investigated.

Adding interaction axioms increases (sometimes dramatically) the complexity of the logic. For instance, the complexity of validity for the single agent case for temporal logic of knowledge (without interactions) is PSPACE. If the synchrony and perfect recall axiom is added, complexity is double-exponential time [8]. So, the development of strategies to guide the search for a proof is essential. Successful strategies, such as set of support, which is applied in both classical [13] and temporal [2] logics, will be considered when developing strategies for interacting logics of time and knowledge or belief.

References

[1] C.Dixon and M.Fisher. *Clausal Resolution for Logics of Time and Knowledge with Synchrony and Perfect Recall.* Submitted, 2000.

[2] C.Dixon and M.Fisher. *The Set of Support Strategy in Temporal Resolution.* In Proceedings of TIME-98 the Fifth International Workshop on Temporal Representation and Reasoning, Sanibel Island, Florida, IEEE Computer Society Press, May, 1998.

[3] C.Dixon, M.Fisher, and M. Wooldridge. *Resolution for Temporal Logics of Knowledge.* Journal of Logic and Computation, volume 8, number 3, 1998.

[4] R.Fagin, J.Y.Halpern, Y.Moses, and M.Y.Vardi. *Reasoning About Knowledge.* MIT Press, 1995.

[5] M. Fisher and M. Wooldridge. *On the Formal Specifications and Verification of Multi-Agent Systems.* International Journal of Cooperative Information Systems, 6(1), January, 1997.

[6] M. Fisher, M. Wooldridge and C. Dixon. *A Resolution-Based Proof Method for Temporal Logics of Knowledge and Belief.* In Proceedings of the International Conference on Formal and Applied Practical Reasoning (FAPR-96), Bonn, Germany, June, 1996.

[7] J.Y.Halpern, R. van der Meyden, and M.Y. Vardi. *Complete Axiomatizations for Reasoning About Knowledge and Time* Submitted for publication, 1997.

[8] J.Y.Halpern and M.Y. Vardi. *The Complexity of Reasoning about Knowledge and Time. I Lower Bounds.* Journal of Computer and Systems Sciences, 38:195-237, 1989.

[9] J.J.C.Meyer and W. van der Hoek. *Epistemic Logic for Computer Science and Artificial Intelligence*, vol. 41 Cambridge Tracts in Theoretical Computer Science. Cambridge University Press, 1995.

[10] R. van der Meyden. *Axioms for Knowledge and Time in Distributed Systems with Perfect Recall.* In Proceedings of the Ninth IEEE Symposium on Logic in Computer Science, pages 448-457, 1994.

[11] J.A.Robinson. *A Machine-Oriented Logic Bases on The Resolution Principle.* ACM Journal, 12(1):23-41, January, 1965.

[12] A. Pnueli *The Temporal Logic of Programs.* In *Proceedings of the 18th Symposium on the Foundations of Computer Science*, Providence, November, 1977.

[13] L.Wos, D.Carson, and G. Robinson. *Efficiency and Completeness of the Set of Support Strategy in Theorem Proving.* ACM Journal, 12:536-541, October, 1965.