Application Of Greedy Algorithms On Classes (ψ, β) – differentiable Periodic Functions In Lebesgue Spaces For Optimization Problems

Iryna Zamrii [1], Viktoriya Shkapa [1], Valentyn Sobchuk [1] and Hanna Vlasyk [1]

[1] *State University of Telecommunications, 7, Solomyanska str., Kyiv, 03110, Ukraine*

Abstract

Many problems in economics, industry, science, as well as the problems of managing complex technical objects lead to the need to solve optimization problems. The problem of constructing algorithms for the approximate solution of optimization problems is of considerable interest. To do this, the properties of the space of variables are investigated and the regularities of the behavior of functions in this space are revealed. The paper describes the application of greedy algorithms to obtain estimates of functions in special classes. Sparse representations of a function are not only a powerful analytical tool, but they are used in many areas, such as image processing, signal processing, numerical computing, directly in optimization problems, as they significantly increase the ability to process large data sets. The key to the search for sparse representations is the concept of m-term approximation of the objective function by the elements of this system of functions. A universal method that allows this is the greedy algorithm, the principle of which is to use the greedy step in search of a new element to be added to this m-term approximation. In this work, using approximations by greedy algorithms (ψ, β)-differentiable functions in Lebesgue spaces, the exact order estimates under conditions $1 < p < q \leq 2$, $1 < p \leq 2 \leq q < \infty$ and $2 \leq p \leq q < \infty$ were found. The estimates obtained allow us to effectively use mathematical models that describe the routes between atomic nodes of the system, which require the use of (ψ, β)-differentiable functions in the space L_q, in optimization problems.

Keywords

(ψ, β) −derivative, greedy approximation, greedy algorithms, best approximations, optimization problem

1. Introduction

The solution of most real problems in the field of decision-making requires the formalization of the situation when the choice should be made, in the form of an optimization problem of a certain class. The optimal choice of one of several valid alternatives according to a certain criterion corresponds to the assignment of variables to specific values from the range of acceptable values. Often variables can take only one of two values - zero or one. The corresponding problems are called optimization problems with Boolean variables or pseudo-Boolean optimization problems. This issue has been actively studied in recent years in the works of many scientists [1-11]. This approach allows to obtain good results for adaptive algorithms for optimal prefix coding of the alphabet with minimal redundancy, algorithms for finding a minimum weight

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EMAIL: irinafraktal@gmail.com (A. 1); vshkapa@ukr.net (A. 2); v.v.sobchuk@gmail.com (A. 3); annawlasik@gmail.com (A. 4)
ORCID: 0000-0001-5681-1871 (A. 1); 0000-0003-3591-7583 (A. 2); 0000-0002-4002-8206 (A. 3); 0000-0002-0680-4128 (A. 4)

spanning tree in a graph and finding a minimum weight spanning tree in a connected graph, and so on.

In essence, these are greedy algorithms, which implement the following principles: at each step of the algorithm we abstract from the previous and next steps and think only about the optimal solution at this stage. The approach does not provide for the cancellation of the choice already made (return to previous steps) and does not predict anything for the future; the speed of program execution is easy to predict, because the complexity of the algorithm is linear. However, it is necessary to understand when this approach can be used and when not. Even if the greedy algorithm gives the optimal solution in certain cases, it is difficult to prove that the approach will work in all other possible cases.

Most known optimization methods involve specifying objective functions and constraints in the form of algebraic expressions, while in many real problems some or all functions are given, algorithmically, which makes it impossible to apply standard algorithms to them; and requires the development of search engine optimization procedures and their evaluation. At the same time, the analysis of many practical problems, to the solution of which greedy algorithms can be applied, allows to reveal in them some features in the form of constructive properties, inherent both in objective functions, and the restriction imposed by the conditions of the problem. It should also be noted that when solving a specific problem, it is useful to have information about the effectiveness of algorithms, which allows you to get the result with the appropriate accuracy.

Often, when solving practical problems, the researcher deals with a specific problem statement. This paper aims to evaluate the solutions of a class of problems described by certain classes of (ψ,β) −differentiable functions. The study of these functions is of particular practical interest, for example, for problems in the description of which models are used, which to describe the routes between the atomic nodes of the system require the use of classes of $(\psi,\beta)-$ differentiable functions.

Before proceeding to the presentation of the main results, we present necessary notations and we will give definitions of the approximate characteristic which will be investigated.

Let L_q be the space of 2π −periodic functions f summable to a power q, $1\le q<\infty$ (resp., essentially bounded for $q=\infty$), on the segment $[-\pi,\pi]$. The norm in this space is defined as follows:

$$\| f \|_{L_q} = \| f \|_q = \\ = \begin{cases} \left(\frac{1}{2\pi}\int\limits_{-\pi}^{\pi} |f(x)|^q dx\right)^{\frac{1}{q}}, 1\le q<\infty, \\ \operatorname*{ess\,sup}\limits_{x\in[-\pi,\pi]} |f(x)|, \qquad q=\infty. \end{cases}$$

For a function $f\in L_1$, we consider its Fourier series

$$\sum_{k\in\mathbb{Z}} \hat{f}(k)e^{ikx},$$

where

$$\hat{f}(k)=\frac{1}{2\pi}\int\limits_{-\pi}^{\pi} f(x)e^{-ikx}dx$$

are the Fourier coefficients of the function f. In what follows, we always assume that the function $f\in L_1$ satisfies the condition

$$\int\limits_{-\pi}^{\pi} f(x)dx=0.$$

Further, let $\psi\ne 0$, be an arbitrary function of natural argument and let β be an arbitrary fixed real number. If a series

$$\sum_{k\in\mathbb{Z}\setminus\{0\}} \frac{\hat{f}(k)}{\psi(|k|)} e^{i(kx+\beta\frac{\pi}{2}\operatorname{sign}k)}$$

is the Fourier series of a summable function, then, following Stepanets [12] we can introduce the (ψ,β)–derivative of the function f and denote it by f_β^ψ. By L_β^ψ we denote the set of functions f satisfying this condition. In what follows we assume that the function f belongs to the class $L_{\beta,p}^\psi$ if $f\in L_{\beta,p}^\psi$ and

$$f_\beta^\psi \in U_p=\{\varphi:\varphi\in L_p, \| \varphi \|_p\le 1\}, \\ 1\le p\le\infty.$$

If

$$\psi(|k|)=|k|^{-r},\ r>0,\ k\in\mathbb{Z}\setminus\{0\},$$

then the (ψ,β)–derivative of the function f coincides with its (r,β)–derivative (denoted by f_β^r) in the Weyl–Nagy sense.

We give definition of the greedy approximation under investigation. Let $\{\hat{f}(k(l))\}_{l=1}^\infty$ — the Fourier coefficients $\{\hat{f}(k)\}_{k\in\mathbb{Z}}$ of the function $f\in L_1$, that are arranged in non-increasing order of their absolute value, i.e

$$|\hat{f}(k(1))|\ge|\hat{f}(k(2))|\ge\ldots$$

Denote for $f\in L_q$

$$G_m(f,x)=\sum_{l=1}^{m}\hat{f}(k(l))e^{ik(l)x}$$

and if $F\subset L_q$ is a certain function class, then we set

$$G_m(F)_q:=\sup_{f\in F}\parallel f(\cdot)-G_m(f,\cdot)\parallel_q. \quad (1)$$

At present, there are many works devoted to the investigation of quantity (1) for important classes of functions. For details and the corresponding references, see, e.g., [13, 14].

By B we denote the set of functions ψ, satisfying the following conditions:

1) ψ — are positive and nonincreasing;

2) there exists a constant $C>0$ such that

$$\frac{\psi(\tau)}{\psi(2\tau)}\leq C,\quad \tau\in\mathbb{N}.$$

Thus, the functions $\frac{1}{\tau^r}, r>0$; $\frac{\ln^{\gamma}(\tau+1)}{\tau^r},\gamma\in\mathbb{R}$, $r>0,\tau\in\mathbb{N}$, and some other functions belong to the set B.

For the quantities A and B, the notation $A\asymp B$ means that there exist positive constants C_1 and C_2 such that $C_1A\leq B\leq C_2A$. If $B\leq C_2A$ $(B\geq$ $\geq C_1A)$, than we can write $B\ll A$ $(B\gg A)$. All C_i, $i=1,2,\ldots$, encountered in our paper may depend only on the parameters appearing in the definitions of the class and metric in which we determine the error of approximation.

2. Main results

The following assertion is true:

Theorem 1. Let $\boldsymbol{1<p<q\leq 2}$, $\boldsymbol{\psi\in B}$, $\boldsymbol{\beta}\in\mathbb{R}$ and let, in addition, there exist $\boldsymbol{\varepsilon>0}$ such that the sequence $\boldsymbol{\psi(t)t^{\frac{1}{p}-\frac{1}{q}+\varepsilon}}$, $\boldsymbol{t}\in\mathbb{N}$, does not increase. Then the following order estimate is true:

$$G_m(L^{\psi}_{\beta,p})_q\asymp\psi(m)m^{\frac{1}{p}-\frac{1}{2}}.$$

Proof. The upper bounds follow from the estimate for the approximation of functions from the classes $L^{\psi}_{\beta,p}$ by their Fourier sums [15, p. 215]:

$$\mathcal{E}_m(L^{\psi}_{\beta,p})_2=$$

$$=\sup_{f\in L^{\psi}_{\beta,p}}\parallel f(x)-\sum_{k=-m}^{m}\hat{f}(k)e^{ikx}\parallel_2\asymp$$

$$\asymp\psi(m)m^{\frac{1}{p}-\frac{1}{2}}.$$

We now determine the lower bounds. We will use the Rudin-Shapiro polinomials $\mathcal{R}_l(x)$:

$$\mathcal{R}_l(x)=\sum_{j=2^{l-1}}^{2^l-1}\varepsilon_je^{ijx},$$

$$\varepsilon_j=\pm1,\quad x\in\mathbb{R},$$

satisfying the order estimate (see, e.g., [16, p. 155])

$$\parallel\mathcal{R}_l\parallel_\infty\ll 2^{\frac{l}{2}}.$$

We also need the well-known de la Vallee-Poussin kernels

$$V_m(x)=\frac{1}{m}\sum_{l=m}^{2m-1}D_l(x),$$

$x\in\mathbb{R},\quad m\in\mathbb{N}$, where

$$D_l(x)=\sum_{|k|\leq l}e^{ikx}$$

is the Dirichlet kernel.

Further, we set for

$$\varepsilon=\pm1\ \Lambda_{\pm1}:=\{k:\hat{\mathcal{R}}_l(k)=\pm1\},$$

and let $\varepsilon=\pm1$ will be such that

$$|\Lambda_\varepsilon|>|\Lambda_{-\varepsilon}|.$$

Then for given m, we take $l\in\mathbb{N}$ from the relation $2^{l-2}\leq m<2^{l-1}$, take a small positive parameter δ and consider a function

$$f(x)=C_3\psi(2^l)2^{l\left(\frac{1}{p}-1\right)}f_1(x),\quad C_3>0,$$

where

$$f_1(x)=V_m(x)+\varepsilon\delta\mathcal{R}_m(x),$$

$$0<\delta\leq m^{\frac{1}{2}-\frac{1}{p}}.$$

We now show that, for a certain choice of the constant $C_3>0$, the function f belongs to the class $L^{\psi}_{\beta,p}$. To this end, it suffices to verify that

$$\parallel f^{\psi}_{\beta}\parallel_p\ll 1.$$

For this purpose, we use the estimate [17]

$$\parallel t^{\psi}_{\beta}\parallel_p\ll\psi^{-1}(n)\parallel t\parallel_p \quad (2)$$

(for any polynomial $t\in T_n$, $1<p<\infty$), and the well-known relation (see, e.g., [18, p. 66])

$$\parallel V_{2^l}\parallel_p\asymp 2^{l\left(1-\frac{1}{p}\right)},\ 1\leq p\leq\infty. \quad (3)$$

Hence, we can write

$$\parallel f^{\psi}_{\beta}\parallel_p\ll\psi^{-1}(m)\parallel f\parallel_p\leq$$

$$\leq\psi^{-1}(m)\psi(2^l)2^{l\left(\frac{1}{p}-1\right)}\cdot$$

$$\cdot\left(\parallel V_m\parallel_p+\delta\parallel\mathcal{R}_m\parallel_p\right)\leq$$

$$\leq\psi^{-1}(m)\psi(2^l)2^{l\left(\frac{1}{p}-1\right)}\cdot$$

$$\cdot\left(\parallel V_m\parallel_p+\delta\parallel\mathcal{R}_m\parallel_\infty\right)\ll$$

$$\ll\psi^{-1}(m)\psi(2^l)2^{l\left(\frac{1}{p}-1\right)}\cdot$$

$$\cdot\left(2^{l\left(1-\frac{1}{p}\right)}+2^{l\left(\frac{1}{2}-\frac{1}{p}\right)}2^{\frac{l}{2}}\right)\ll 1.$$

This implies that, for a proper choice of the constant $C_3>0$, function $f\in L^{\psi}_{\beta,p}$.

By using the estimate (see, e.g., [14], p. 581) for $1\leq q\leq 2$ and $1<p\leq 2$

$$\parallel f_1-G_m(f_1)\parallel_q\gg m^{\frac{1}{2}},$$

we obtain

$$\sup_{f\in L^{\psi}_{\beta,p}} \| f - G_m(f) \|_q \gg \psi(2^l) 2^{l\left(\frac{1}{p}-1\right)}.$$

$$\| f_1 - G_m(f_1) \|_q \gg$$

$$\gg \psi(m) m^{\frac{1}{p}-1} m^{\frac{1}{2}} = \psi(m) m^{\frac{1}{p}-\frac{1}{2}}.$$

The required lower bound is established, which proves the theorem.

Theorem 2. Let $\mathbf{1} < \boldsymbol{p} \le \mathbf{2} \le \boldsymbol{q} < \infty$, $\boldsymbol{\psi} \in \boldsymbol{B}$, $\boldsymbol{\beta} \in \mathbb{R}$ and let, in addition, there exist $\boldsymbol{\varepsilon} > \mathbf{0}$ such that the sequence $\boldsymbol{\psi(t) t^{\frac{1}{p}+\varepsilon}}$, $\boldsymbol{t} \in \mathbb{N}$, does not increase. Then the following order estimate is true:

$$G_m(L^{\psi}_{\beta,p})_q \asymp \psi(m) m^{\frac{1}{p}-\frac{1}{q}}.$$

Proof. The upper bound follows from the following inequality

$$\| f - G_m(f) \|_q \le$$

$$\le \left(1 + 3m^{\left|\frac{1}{2}-\frac{1}{q}\right|}\right) e_m(f)_q,$$

$1 \le q \le \infty$, (see [14]), and the estimate

$$e_m(L^{\psi}_{\beta,p})_q =$$

$$= \sup_{f\in L^{\psi}_{\beta,p}} \inf_{\Theta_m} \inf_{T(\Theta_m,\cdot)} \| f(\cdot) - T(\Theta_m,\cdot) \|_q \asymp$$

$$\asymp \psi(m) m^{\frac{1}{p}-\frac{1}{2}},$$

$$1 < p \le 2 \le q < \infty,$$

where

$$T(\Theta_m, x) = \sum_{k=1}^{m} c_k e^{i n_k x},$$

Θ_m is a set of m integers $n_1, \dots, n_m$ and c_k are arbitrary complex numbers (see [19]).

$$\| f - G_m(f) \|_q \le$$

$$\le (1 + 3m^{\frac{1}{2}-\frac{1}{q}}) e_m(f)_q \ll$$

$$\ll m^{\frac{1}{2}-\frac{1}{q}} \psi(m) m^{\frac{1}{p}-\frac{1}{2}} = \psi(m) m^{\frac{1}{p}-\frac{1}{q}}.$$

Therefore

$$G_m(L^{\psi}_{\beta,p})_q \ll \psi(m) m^{\frac{1}{p}-\frac{1}{q}}. \qquad (4)$$

We now determine the lower bounds. For given m we take $l \in \mathbb{N}$ from the relation $2^{l-1} \le m < 2^l$ and consider a function

$$f_2(x) = C_4 \psi(2^l) 2^{l\left(\frac{1}{p}-1\right)} V_{2^l}(x), \quad C_4 > 0.$$

It is easy to see that the function f_2 belongs to $L^{\psi}_{\beta,p}$. Indeed, according to relations (2) and (3), we can write

$$\| (f_2)^{\psi}_{\beta} \|_p \ll \psi^{-1}(m) \| f_2 \|_p \ll$$

$$\ll \psi^{-1}(m) \psi(2^l) 2^{l\left(\frac{1}{p}-1\right)} 2^{l\left(1-\frac{1}{p}\right)} = 1.$$

This implies that, for the proper choice of the constant $C_4 > 0$, the function f_2 belongs to $L^{\psi}_{\beta,p}$.

Using the inequality of different metrics, we obtain the ratio

$$\| T_n \|_p \ll n^{\frac{1}{q}-\frac{1}{p}} \| T_n \|_q,$$

$$1 \le q \le p \le \infty,$$

we obtain the ratio

$$\| V_{2^l} - G_m(V_{2^l}) \|_q \gg$$

$$\gg m^{-\frac{1}{q}} \| V_{2^l} - G_m(V_{2^l}) \|_{\infty} \gg$$

$$\gg m^{1-\frac{1}{q}}. \qquad (5)$$

Therefore, given (5), we will have

$$\sup_{f_2\in L^{\psi}_{\beta,p}} \| f_2 - G_m(f_2) \|_q \gg$$

$$\gg \psi(2^l) 2^{l\left(\frac{1}{p}-1\right)} \| f_2 - G_m(f_2) \|_q \gg$$

$$\gg \psi(m) m^{\frac{1}{p}-1} m^{1-\frac{1}{q}} =$$

$$= \psi(m) m^{\frac{1}{p}-\frac{1}{q}}.$$

Thus for

$$1 < p \le 2 \le q < \infty$$

we obtain

$$G_m(L^{\psi}_{\beta,p})_q \asymp \psi(m) m^{\frac{1}{p}-\frac{1}{q}}.$$

The estimate from below, and with it Theorem 2, is proved.

Theorem 3. Let $2 \le p \le q < \infty$, $\psi \in B$, $\beta \in \mathbb{R}$ and let, in addition, there exist $\varepsilon > 0$ such that the sequence $\psi(t) t^{\frac{1}{2}+\varepsilon}$, $t \in \mathbb{N}$, does not increase. Then the following order estimate is true:

$$G_m(L^{\psi}_{\beta,p})_q \asymp \psi(m) m^{\frac{1}{2}-\frac{1}{q}}.$$

Proof. We first establish the upper bound. Since

$$L^{\psi}_{\beta,p} \subset L^{\psi}_{\beta,2},$$

then

$$G_m(L^{\psi}_{\beta,p})_q \le G_m(L^{\psi}_{\beta,2})_q$$

and therefore, taking into account the ratio (4) for $p = 2$, we obtain the upper bounds

$$G_m(L^{\psi}_{\beta,p})_q \ll \psi(m) m^{\frac{1}{2}-\frac{1}{q}}.$$

To set the upper bound for given m we take $l \in \mathbb{N}$ so that the relation is fulfilled $m \asymp 2^l$, we will take small positive parameter δ and consider a function

$$f_3(x) = C_5 \psi(2^l) 2^{-\frac{l}{2}} f_4(x),$$

where

$$f_4(x) := \mathcal{R}_m(x) + \varepsilon\delta D_m(x),$$

$$0 < \delta \le m^{\frac{1}{p}-\frac{1}{2}}.$$

We now show that, for a certain choice of the constant $C_5 > 0$, the function f_3 belongs to the class $L^{\psi}_{\beta,p}$.
For this purpose, it suffices to check that

$$\| (f_3)^{\psi}_{\beta} \|_p \ll 1.$$

To this end, we use (2) and the known relation (see, e.g., [20, p. 155])

$$\| D_{2^l} \|_p \asymp 2^{l\left(1-\frac{1}{p}\right)}, \qquad 1 < p < \infty.$$

So we will have

$$\| (f_3)^{\psi}_{\beta} \|_p \ll \psi^{-1}(m) \| f_3 \|_p \leq$$
$$\leq \psi^{-1}(m)\psi(2^l)2^{-\frac{l}{2}} \cdot$$
$$\cdot \left(\| \mathcal{R}_m \|_p + \delta \| D_m \|_p\right) \leq$$
$$\leq \psi^{-1}(m)\psi(2^l)2^{-\frac{l}{2}} \cdot$$
$$\cdot \left(\| \mathcal{R}_m \|_\infty + \delta \| D_m \|_p\right) \ll$$
$$\ll \psi^{-1}(m)\psi(2^l)2^{-\frac{l}{2}} \cdot$$
$$\cdot \left(2^{\frac{l}{2}} + 2^{l\left(\frac{1}{p}-\frac{1}{2}\right)}2^{l\left(1-\frac{1}{p}\right)}\right) \ll 1.$$

This implies that, for a proper choice of the constant $C_5 > 0$, the function f_3 belongs to the class $L^{\psi}_{\beta,p}$.
Further, use the estimate set in [14, p. 582]:

$$\| f_4 - G_m(f_4) \|_q \gg m^{1-\frac{1}{q}}, \quad 2 \leq q \leq \infty.$$

Taking into account this ratio, we will have

$$\sup_{f_3 \in L^{\psi}_{\beta,p}} \| f_3 - G_m(f_3) \|_q \gg$$
$$\gg \psi(2^l)2^{-\frac{l}{2}} \| f_4 - G_m(f_4) \|_q \gg$$
$$\gg \psi(m)m^{-\frac{1}{2}}m^{1-\frac{1}{q}} = \psi(m)m^{\frac{1}{2}-\frac{1}{q}}.$$

Thus for

$$2 \leq p \leq q < \infty$$

we obtain

$$G_m(L^{\psi}_{\beta,p})_q \asymp \psi(m)m^{\frac{1}{2}-\frac{1}{q}}.$$

The lower bound is established. This completes the proof of the theorem.

Remark. The assertion of Theorems for a special case of the classes $W^r_{p,\beta}$ were established by Temlyakov [21].

3. Conclusions

The possibilities of precise methods are very limited, especially when solving large-scale problems. For many classes of discrete optimization problems that occur in practice, no effective (polynomial) exact algorithms have been developed. In addition, the use of regular algorithms is possible only in the presence of a priori information about the properties of the target functional. This leads to the need to develop and study approximate algorithms to obtain the necessary solution. Because if the dimension is close to the hundredth step, then the exact algorithm is no longer able to find a solution in real time. This paper proposes the use of a greedy algorithm, the essence of which is to select the next element at each step in an optimal way, to effectively solve problems of optimization of functions in the presence of constraints. In particular, we obtain the exact order estimates of approximations by greedy algorithms of the classes $L^{\psi}_{\beta,p}$ of periodic functions in the space L_q for some relations between parameters p and q. Using approximation by greedy algorithms (ψ, β) - differentiable functions in Lebesgue spaces, the exact order estimates under conditions $1 < p < q \leq 2$, $1 < p \leq 2 \leq q < \infty$ and $2 \leq p \leq q < \infty$ were found. The estimates obtained allow us to effectively use mathematical models that describe the routes between atomic nodes of the system, which require the use of (ψ,β)-differentiable functions in the space L_q, in optimization problems.

4. References

[1] S. Yevseiev, R. Korolyov, A. Tkachov,O. Laptiev, I. Opirskyy, O. Soloviova, Modification of the algorithm (OFM) S-box, which provides increasing crypto resistance in the post-quantum period, International Journal of Advanced Trends in Computer Science and Engineering (IJATCSE) 9 (2020) 8725-8729. doi:10.30534/ijatcse/2020/261952020

[2] O. Barabash, O. Laptiev, O. Kovtun, O. Leshchenko, K. Dukhnovska, A. Biehun, The Method dynavic TF-IDF, International Journal of Emerging Trends in Engineering Research (IJETER) 8 (2020) 5713-5718. doi:10.30534/ijeter/2020/130892020

[3] O. Barabash, O. Laptiev, V. Tkachev, O. Maystrov, O. Krasikov, I. Polovinkin, The Indirect method of obtaining Estimates of the Parameters of Radio Signals of covert means of obtaining Information, International Journal of Emerging Trends in Engineering Research (IJETER) 82020, Indexed- ISSN: 2278 – 3075, 4133 – 4139. doi:10.30534/ijeter/2020/17882020

[4] V. Savchenko, O. Ilin, N. Hnidenko, O.Tkachenko, O. Laptiev, S. Lehominova, Detection of Slow DDoS Attacks based on

User's Behavior Forecasting, International Journal of Emerging Trends in Engineering Research (IJETER) 8 (2020) 2019 – 2025. doi:10.30534/ijeter/2020/90852020

[5] O. Laptiev, O. Stefurak, I. Polovinkin, O. Barabash, S. Vitalii, O. Zelikovska, The method of improving the signal detection quality by accounting for interference, 2020 IEEE 2nd International Conference on Advanced Trends in Information Theory (IEEE ATIT 2020) Conference Proceedings Kyiv, Ukraine, 2020, pp. 172 –176. doi:10.1109/ATIT50783.2020.9349259

[6] O. Laptiev, V. Savchenko, S. Yevseiev, H. Haidur, S. Gakhov, S. Hohoniants, The new method for detecting signals of means of covert obtaining information, 2020 IEEE 2nd International Conference on Advanced Trends in Information Theory (IEEE ATIT 2020) Conference Proceedings Kyiv, Ukraine, 2020, pp. 176 –181. doi:10.1109/ATIT50783.2020.9349322

[7] V. Sobchuk, V. Pichkur, O. Barabash, O. Laptiev, I. Kovalchuk, A. Zidan, Algorithm of control of functionally stable manufacturing processes of enterprises, 2020 IEEE 2nd International Conference on Advanced Trends in Information Theory (IEEE ATIT 2020) Conference Proceedings Kyiv, Ukraine, 2020, pp. 206 –211. doi:10.1109/ATIT50783.2020.9349332

[8] V. Savchenko, O. Laptiev, O. Kolos, R. Lisnevskyi, V. Ivannikova, I. Ablazov, Hidden Transmitter Localization Accuracy Model Based on Multi-Position Range Measurement, 2020 IEEE 2nd International Conference on Advanced Trends in Information Theory (IEEE ATIT 2020) Conference Proceedings Kyiv, Ukraine, 2020, pp. 246 –251. doi:10.1109/ATIT50783.2020.9349304

[9] I. Zamrii, M. Prats'ovytyi, Singularity of the Digit Inversor for the Q_3-Representation of the Fractional Part of a Real Number, Its Fractal and Integral Prorerties, Journal of Mathematical Sciences 215 (2016) 323–340. doi:10.1007/s10958-016-2841-y

[10] O. Barabash, O. Kopiika, I. Zamrii, V. Sobchuk, A. Musienko, Fraktal and Differential Properties of the Inversor of Digits of $\mathrm{Q_s}$-Representation of Real Number, Modern Mathematics and Mechanics. Fundamentals, Problems and Challenges. Springer International Publishing AG (2019) 79 – 95. doi: 10.1007/978-3-319-96755-4_5

[11] H. M. Vlasyk, V. V. Shkapa, I. V. Zamrii, Estimates of the best orthogonal trigonometric approximations and orthoprojective widths of the classes of periodic functions of many variables in a uniform metric, Journal of Mathematical Sciences 246 (2020) 110-119. doi:10.1007/s10958-020-04725-0

[12] A. I. Stepanets', Methods of Approximation Theory. Vols. 1,2, Institute of Mathematics, Ukrainian National Academy of Sciences, Kyiv, 2002.

[13] V. N. Temlyakov, Greedy approximation, Cambridge, Cambridge University Press, 2011. doi: 10.1017/CBO9780511762291

[14] V. N. Temlyakov, Greedy algorithm and m-term trigonometric approximation, Constr. Approx 14 (1998) 569-587.

[15] A. I. Stepanets, Classification and Approximation of Periodic Functions [in Russian], Naukova Dumka, Kiev (1987).

[16] B. S. Kashin, A. A. Sahakyan, Orthogonal series [in Russian], M .: Science, 1984.

[17] A. S. Romanyuk, Inequalities for the L_p -norms of (ψ,β)-derivatives and Kolmogorov widths of the classes of functions of many variables $L^{\psi}_{\beta,p}$, in: Investigations in the Approximation Theory of Functions [in Russian], Proc. of the Institute of Mathematics, Academy of Sciences of Ukr. SSR, Kiev (1987), 92–105.

[18] V. N. Temlyakov, Approximation of functions with bounded mixed derivative, Tr. Mat. Inst. Akad. Nauk SSSR, 178 (1986).

[19] Barabash Oleg, Laptiev Oleksandr, Tkachev Volodymyr, Maystrov Oleksii, Krasikov Oleksandr, Polovinkin Igor. The Indirect method of obtaining Estimates of the Parameters of Radio Signals of covert means of obtaining Information. International Journal of Emerging Trends in Engineering Research (IJETER), Volume 8. No. 8, August 2020. Indexed- ISSN: 2278 – 3075. pp4133 – 4139. DOI:10.30534/ijeter/2020/17882020.

[20] Serhii Yevseiev, Roman Korolyov, Andrii Tkachov, Oleksandr Laptiev, Ivan Opirskyy, Olha Soloviova. Modification of the algorithm (OFM) S-box, which provides increasing crypto resistance in the post-quantum period. International Journal of Advanced Trends in Computer Science and Engineering (IJATCSE) Volume 9. No. 5,

September-Oktober 2020, pp 8725-8729. DOI: 10.30534/ijatcse/2020/261952020.

[21] Vitalii Savchenko, Oleksandr Laptiev, Oleksandr Kolos, Rostyslav Lisnevskyi, Viktoriia Ivannikova, Ivan Ablazov. Hidden Transmitter Localization Accuracy Model Based on Multi-Position Range Measurement. 2020 IEEE 2nd International Conference on Advanced Trends in Information Theory (IEEE ATIT 2020) Conference Proceedings Kyiv, Ukraine, November 25-27. pp.246 –251