Audit Of Mathematical Models For Software Specification Of The Workplace Decision Support System At The Logistics Management Point

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Abstract

The article analyzes the process of functioning of the system of technical support of combat operations in order to determine it`s capabilities and areas for improvement through for solving problems in modern local wars with the restriction of the use of heavy armored vehicles through application of the models of states and transitions.

In addition, the possibility of creating a mathematical basis for the management of maintenance and restoration of lightly armored vehicles for software implementation of the workplace of a logistics officer on evacuation management and lightly armored vehicles recovery, which will not only explore real support systems, but also solve complex problems of technical support of combat operations in real time - on the battlefield.

Keywords

technical support, armament and military equipment, lightly armored vehicles, intensity, probability, graph, Kolmogorov's equation

1. Introduction

Formulation of the problem.

Analysis of the models of the main states of the technical support system, which were used before beginning of hostilities in the anti-terrorist operation area, shows that the task of managing the evacuation and recovery of arming and military machinery (AMM) during hostilities is more difficult in terms of preconditions and initial data for planning than known it`s solutions. [1-4].

Based on the combat experience of servicemen of the Ukrainian Armed Forces and other military formations that directly participated in repelling the Russian armed aggression, it is well known that Ukraine clearly complies with the requirements of the Minsk agreements and does not use heavy armored vehicles such as tanks and artillery on the line of contact. Therefore, the main armored vehicles are light armored vehicles such as (BMP, armored personnel carrier, BBM).

It is known that many factors that affect the management of evacuation and recovery of arming and AMM hostilities are accompanied by uncertainties of random, natural and antagonistic nature.

An appropriate way out of this situation is to reduce the dimensionality of the analysis problem by comparing it in order to rank the types of technical support tasks according to some general indicator. As such an indicator can be used the probability stay of the evacuation management system and the restoration of AMM in each state of solving the tasks of combat operations. The choice of the solution of the problem is possible by averaging the optimal partial solutions over

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time on the highest level of probability. It is the correspondence of the probability models to the realities that requires further research. [5].

The purpose of the article.

Currently, Light Armored Vehicles (LAV) of all-military units are the most common type of military equipment in the armed forces. Also, such equipment is most often affected and fails, and therefore requires constant correction of planned activities of managing the evacuation and recovery of AMM in real time.

Systematic shelling of the positions of the Ukrainian Armed Forces in the area of anti-terrorist operation (ATO) leads to the decommissioning of those samples of armaments and military equipment that are located directly at the bases on the line of the collision of the parties. In the context of integration of logistics management as a mechanism for providing and managing evacuation and recovery and subsequent repair of lost samples of armaments and military equipment, it turned out that mathematical models of logistics management and operation, as well as software based on them do not meet the requirements of real-time decision support.

The purpose of the article is to provide a mathematical justification for the management of evacuation and recovery of LAV for software implementation of the workplace logistic officer for evacuation management and recovery of LAV at the logistics management point, which will not only explore real support systems, but also solve complex problems of technical support of combat operations in real time, including in the battlefield.

2. Analysis of the model of the main states of the technical system of combat operations (Conceptual Modeling)

To study the process of technical support, various types of technical support models are currently used. If the models adequately reflect all the states of the system, it is better to use model of states and transitions [5].

The adequacy of the model for processes without aftereffect is explained by the fact that it most accurately reflects the system, in the case when any of its current state does not depend on the state in which the system was before. It is the identity of the model to the real processes that explains the choice of the state model for the software specification of the decision support system of the logistics officer's workplace for evacuation management and light armored vehicles recovery at the logistics management point. This is the system of technical support of warfare. A variant of the graph of states and transitions of this system to different states is presented in **Figure 1.**

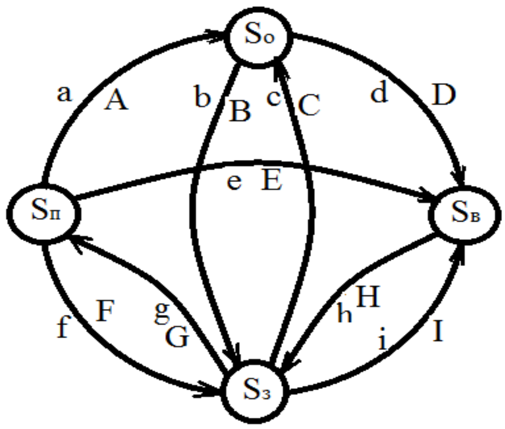

Figure 1. Graph of transitions of the technical system of combat operations in the states: $S_п$ – preparation of LAV for use; $S_з$ – combat use of LAV; $S_в$ – restoration of LAV after its damage; S_o – maintenance of LAV before or after combat actions.

The list of transition intensities and the corresponding probabilities of these transitions is as follows:

a, A – intensity and probability of transitions of the technical support system from the state of preparation of LAV for its maintenance;

b, B – intensity and probability of transitions from the state of LAV maintenance to the state of its combat use;

c, C – intensity and probability of transitions from the state of combat use of the LAV to the state of its maintenance;

d, D – intensity and probability of transitions from the state of maintenance of LAV to the state of recovery of LAV after damage;

e, E – intensity and probability of transitions from the state of preparation of LAV to the state of recovery of LAV after damage;

f, F - intensity and probability of transitions from the state of preparation of LAV to the state of its employment;

g, G – intensity and probability of transitions from the state of combat use of LAV to the state of preparation of LAV for the purpose of their employment;

h, H – intensity and probability of transitions from the state of recovery of LAV after damage to the state of its combat use;

i, I – intensity and probability of transitions from the state of combat use of LAV to the state of recovery of LAV after its damage.

It is also easy to imagine a situation where it is necessary to perform maintenance of LAV after its preparation for use, or after its combat application, as well as a situation when combat use of LAV has shown the need for new training for deployment, for example, taking into account unsatisfactory combat results due to insufficiently careful preliminary preparation.

In the process of functioning of the system of technical support of combat operations in time, it is in any state with probabilities:

$P_1(t)$ - probability that the system is in a state of preparation of weapons and ammunition for their use;

$P_2(t)$- the probability that the system is in a state of use of weapons for their intended purpose;

$P_3(t)$- the probability that the system is in a state of recovery after damage;

$P_4(t)$- the probability that the system is in a state of maintenance.

Find the probability $P_1(t)$. We provide t small increase Δt and find the probability that at the moment $t+\Delta t$ the system will be in a state S_n. This event can happen in two ways:

- at the moment t the system was already in condition S_n , but by the time Δt did not come out of this state, either

- at the moment t the system was in the state $S_в$, by the time Δt moved from it to the state S_n

The probability of the first variant is shown as the product of the probability $P_1(t)$ that at the moment t the system was in the state S_n, on the conditional probability that, being in a state S_n, system by the time Δt will not pass from it into a state $S_з$.

This conditional probability (up to infinitesimal higher orders of magnitude) is equal to: $1-\lambda_{12}\Delta t$

Similarly, the probability of the second option is equal to the probability of that at the moment t system was at the state $S_в$, which is multiplied by the conditional probability of transition over time Δt into the state S_n: $P_з(t)\lambda_{31}\Delta t$.

Applying the rule of adding probabilities, we obtain:

$$P_1(t+\Delta t)=P_1(t)(1-\lambda_{12}\Delta t)+P_3(t)\lambda_{31}\Delta t \quad (1)$$

Open the brackets on the right side, move $P_1(t)$ to the left and divide both parts of the equation by Δt; we will get:

$$\frac{P_1(t+\Delta t)-P_1(t)}{\Delta t}=\lambda_{12}P_1(t)+\lambda_{31}P_3(t) \quad (2)$$

Now direct Δt to zero and go to the limit:

$$\lim_{\Delta t\to 0}\frac{P_1(t+\Delta t)-P_1(t)}{\Delta t}=\lambda_{12}P_1(t)+\lambda_{31}P_3(t) \quad (3)$$

The left part is nothing but a derivative of the function $P_1(t)$:

$$\frac{\partial P_1(t)}{\partial t}=-\lambda_{12}P_1(t)+\lambda_{31}P_3(t) \quad (4)$$

Thus, the differential equation obtained by the function $P_1(t)$. Similar differential equations can be derived for other probabilities of states $P_2(t)$, $P_3(t)$, $P_4(t)$, which provides initial data for the search of computational methods for solving problems that replace theoretical models in the form of differential equations.

Consider the second state $S_з$. Find the probability that at the moment $t+\Delta t$ the system will be in a state $S_з$. This event can occur in two ways:

- at the moment t the system was already in condition $S_з$, by the time Δt did not come out of this state;

or

- at the moment t system was in condition S_n; by the time Δt moved from it to the state $S_з$;

or

- at the moment t system was in condition S_o, by the time Δt moved from it to the state $S_з$.

The probability of the first option is calculated as follows: $P_2(t)$ multiplied by the conditional probability that the system over time Δt will not pass either $S_в$, nor in S_o. Since the events that are the transition over time Δt into $S_в$ and from $S_з$ into S_o, are incompatible, the probability that one of these transitions will occur is equal to the sum of their probabilities, to wit $\lambda_{23}\Delta t+\lambda_{24}\Delta t$ (up to infinitesimal higher orders). The probability that none of these transitions will occur is equal $1-\left(\lambda_{23}\Delta t+\lambda_{24}\Delta t\right)$. Hence the probability of the first option: $P_2(t)\left(\lambda_{23}\Delta t+\lambda_{24}\Delta t\right)$.

Adding here the probabilities of the second and third options, we obtain:

$$\begin{aligned}&P_2(t+\Delta t)=P_2(t)\left(1-\lambda_{23}\Delta t+\lambda_{24}\Delta t\right)+\\&+P_1(t)\lambda_{12}\Delta t+P_4(t)\lambda_{42}\Delta t\end{aligned} \quad (5)$$

Moving $P_2(t)$ to the left side, dividing by Δt and crossing to the limit, we obtain a differential equation for $P_2(t)$:

$$\begin{aligned}&\frac{\partial P_2(t)}{\partial t}=-\lambda_{23}P_2(t)-\lambda_{24}P_2(t)+\\&+\lambda_{12}P_1(t)+\lambda_{42}P_4(t)\end{aligned} \quad (6)$$

Reasoning similarly for states $S_в$ and S_o, we obtain as a result a system of differential equations composed by type **(5)**, **(6)**. Rejecting them for the sake of convenience argument t in functions P_1, P_2, P_3, P_4 rewrite the system in the form:

$$\begin{aligned}&\frac{\partial P_1}{\partial t}=-\lambda_{12}P_1+\lambda_{31}P_3,\\&\frac{\partial P_2}{\partial t}=-\lambda_{23}P_2-\lambda_{24}P_2+\lambda_{12}P_1-\lambda_{42}P_4,\\&\frac{\partial P_3}{\partial t}=-\lambda_{31}P_3-\lambda_{34}P_3+\lambda_{23}P_2,\\&\frac{\partial P_4}{\partial t}=-\lambda_{42}P_4+\lambda_{24}P_2+\lambda_{34}P_3.\end{aligned} \quad (7)$$

These equations for the probabilities of states are Kolmogorov's equations.

The integration of this system of equations will give the desired probabilities of states as a function of time. The initial conditions are taken depending on what was the initial state of the system. For example, if at the initial time (at $t=0$) the system was in a state S_n, then the initial conditions must be accepted: $t=0$, $P_1=1$, $P_2=P_3=P_4=0$, which gives an understanding of the universality of the model under study, in terms of its further use as an element of the software specification of the workplace logistic officer for evacuation management and recovery of light armored vehicles decision support system at the logistics management point.

Note that all four equations for P_1,P_2,P_3,P_4 one could not write because $P_2+P_2+P_3+P_4=1$ for all t, and any of the probabilities P_1,P_2,P_3,P_4 can be expressed through the other three. For example, $P_4=1-P_1,P_2,P_3$.

Then a special equation for P_4 not necessary to write. In the future, this fact will reduce the requirements for productivity and speed of the hardware components of the decision support system.

Let's pay attention to the structure of equations **(7)**. They are all built on a general rule that can be formulated as follows. In the left part of each equation there is a derivative of the probability of the state, and the right part contains as many terms as there are gaps connected with the given state.. If the gap leaves the state, the corresponding member has a sign "minus", and if the gap enters the state - the sign "plus". Each term is equal to the product of the intensity of the transition corresponding to a given gap and the probability of the state from which the arc emerges.

If the matrix of transition intensities or the state graph is known, the state probability vector can be determined $P_9(t)=\left(P_1(t),...,P_n(t)\right)$, through the matrix equation $P(t)=P(t)\bullet\Delta$.

From a practical point of view, to ensure the combat effectiveness of the unit is important to reduce the intensity and probability (*g*, *G*) its transition to the state (S_n) preparation of LAV for the purpose of their application, and also increase in intensity and probability (*f*, *F*) transition of the system to the state ($S_з$) use of LAV for its intended purpose. This requires keeping the LAV at a high level of its readiness factor, accelerated and sufficient level of preparation of the LAV for the start of combat actions.

It is necessary to significantly reduce the intensity and probability (*i*, *I*) transition of the technical support system to the state ($S_в$) recovery of LAV after damages, reduce the intensity and probability (*e*, *E*) transition of the system from the state (S_n) preparation of LAV for the purpose of their application in a condition ($S_в$) recovery from damage, ie before the start of the use of LAV for its intended purpose.

It is necessary to increase the intensity and probability (*h*, *H*) transition of the system from the state ($S_в$) recovery of LAV after damage to the condition ($S_з$) application for intended use.

The greatest attention is paid to the study of the condition ($S_з$) use of LAV by purpose and condition ($S_в$) recovery of LBT after damages is not accidental. This is due to the fact that these states of the technical system of combat operations are the most important in terms of the importance of the functions of the technical support system, and the structure of unconditional relations in this system.

It is safe to say in advance that, given the uncertainties of a random nature, namely, equally intense and equally probable transitions of the technical system of combat operations from any state to any other state, the total probability ($P_{зв} = P_з + P_в$) stay of this system in a condition ($S_з$) use of LAV on purpose and in condition ($S_в$) recovery of LAV after damage is always the highest in comparison with other general probability, equal to the sum of the probability of the system in the state of preparation of LAV for their application and the probability of the system in a state of maintenance, that is, with the total probability $P_{оп} = P_о + P_п$.

Indeed, it is easy to see this in some arbitrary but concrete example.

2.1. Verification and specification of the obtained models for their further implementation in the software of decision support systems (Modeling verification & validation)

The first test example.

The initial prerequisites for modeling are equally intense and equally likely transitions of the system of technical support of hostilities from any state to any state, namely (check. **Figure 1**): *a = b = c = d = e = f = g = i = h = 1/2* hours; *A = B = C = D = E = F = G = I = H = 1/9*; *t = (6…48)* hours.

Identify the general probabilities that need to be quantified, namely: $P_{зв}(t) = P_з(t) + P_в(t)$; $P_{оп}(t) = P_о(t) + P_п(t)$,

where $P_п(t)$ - the probability that the system is in a state of preparation of weapons and ammunition for their use;

$P_з(t)$ - the probability that the system is in a state of use of weapons for their intended purpose;

$P_в(t)$ - the probability that the system is in a state of recovery after damage;

$P_о(t)$ - the probability that the system is in a state of maintenance of weapons.

The solution is carried out on machines for data packaging, provided that for the representation of numerical values that the decimal system of systematization of calculations are in the range from 0 to 1, respectively in the binary calculation system the number of characters for the mantissa is 8 bits with the corresponding mantissa.

According to the formulas **(1-7)** we will get:

$P_п(t = 6...48) = 0,13...0,09$;

$P_о(t = 6...48) = 0,17...0,18$; $P_п + P_о = 0,30...0,27$.

$P_з(t = 6...48) = 0,59...0,26$;

$P_в(t = 6...48) = 0,11...0,47$;

$P_з + P_в = 0,70...0,73$.

Graphs of general probabilities in the form of time functions during the process of technical support of hostilities, obtained according to the initial data **example 1** and emphasize the validity of the statement which was made earlier.

Thus, in the system of technical support of combat actions there is a pattern, namely: under conditions of equally probable transitions of the system from state to state, it is in a state of application or recovery more often (approximately three times) than in a state of maintenance or training.

It is clear that this result is not a new discovery. The problem is solved by a known method for the new initial conditions and the new content of the problem of evacuation and recovery of armaments and military equipment. It only confirms the peculiarity of the structure and the essence of the functioning of a complex system of technical support of combat actions. This is what is needed carefully and always consider.

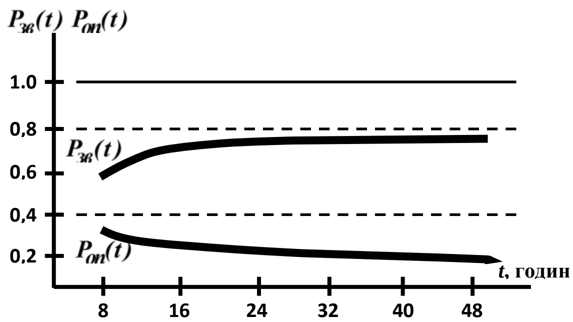

Figure 2. General probabilities of the technical support system being in combat during states: application or recovery, $P_{зв}(t)$ LAV; maintenance or preparing, $P_{оп}(t)$ LAV for combat actions.

Next, it is necessary to investigate (for conditions similar to the data according to **Example 1**) the dependence of the time of technical support of combat operations of each of the probabilities, namely: $P_n(t)$ - the probability of the system being in a state of preparation of LAV for the purpose of their application; $P_з(t)$ - the probability that the system is in a state of use of LAV for its intended purpose; $P_в(t)$ - the probability that the system is in a state of recovery LAV after its damage; $P_o(t)$ - the probability that the system is in a state of maintenance LAV.

The second test example.

Output data. We have equally intense and equally probable transitions of the system of technical support of combat actions from any state to any of its states, namely (check. Ошибка! Источник ссылки не найден.): *a = b = c = d = e = f = g = i = h= 1/2 hours*; *A = B = C = D = E = F = G = I = H = 1/9*; *t = (6…48) hours*.

Identify and plot graph of probabilities: $P_n(t)$, $P_з(t)$, $P_в(t)$, $P_o(t)$, *t = (6…48) hours.*

Regarding the representation of numerical values in the binary calculation system, the assumption introduced in the first test example.

The results obtained from the simulation results of determining and comparing the probabilities of the technical support system in each of the main states are typical for combat operations. These results characterize the full group of phenomena, under conditions of commensurate intensities and commensurate probabilities of transitions of this system to different states. They show the following.

First, with the start of combat actions, the technical support system is: in a state of preparation of weapons and ammunition for the fight with a probability **13%**; in the state of use of weapons for their intended purpose - with probability **60%**; in a state of restoration of armament after damage - with probability **10%**; in a state of service - with probability **17%**.

Secondly, after two days of combat actions, the technical support system is in a state of preparation of weapons and ammunition - with a probability **9%**; in the state of use of weapons for their intended purpose - with probability **26%**; in a state of restoration of armament after damage - with probability **47%**; in the state of service of armaments - with probability **17%**.

This shows that the data obtained (under conditions of equally intense and equally probable transitions of the system to different states) using the model, in the presence of random and antagonistic uncertainties, do not contradict the known experimental results of real events of typical support, according to local fight. Third, the weakest point of a typical unit's technical support system is its ability to recover weapons and military equipment (AMM) damaged during combat.

This situation necessitates further research on the technical system of combat operations, in order to identify measures to increase opportunities for the restoration of AAM damaged during combat.

The solution of the problem of evacuation and recovery using the model of states and transitions confirms the adequacy of the model with real measures of evacuation and recovery of armaments and military equipment.

Therefore, it seems appropriate measures aimed at increasing the survivability of AAM. According to the classical definition, the survivability of AAM is its ability to maintain its functions during the action of the enemy's means of destruction and the ability to quickly recover from damage and return to service.

It is clear that to increase the survivability of weapons part is necessary and sufficient: first, to organize and implement a set of measures to reduce its radio and optical visibility by air and ground reconnaissance by the enemy and before and during its intended use; secondly, to organize and carry out measures and means for artillery and technical reconnaissance: thirdly, to organize and carry out the use of a set of repair forces, the use of replacement units, blocks, devices and materials, to organize the evacuation and rapid recovery of damaged AAM.

According to the graph of states and transitions of the technical system of combat operations, the above measures and means should clearly: first, reduce the intensity and probability of transition of the system from the state of use of LAV for its intended purpose to recovery after damage; secondly, these measures and means will increase the intensity and probability of the transition of the system from the state of recovery of LAV after damage to the state of use for its intended purpose..

We will further determine the direction of change in the operation of the technical system of combat operations for some specific conditions that differ (from the conditions of **Example 2**) by reducing the intensity and probability of transition of the system from the intended use to the recovery state after damage, for example, in two times, in addition, differ in the increase in the intensity and probability of the transition of the system from the state of recovery of LAV after damage to the state of use for its intended purpose also in two times.

The system of technical support of combat operations is: in the state of preparation of the LAV for combat with a probability of (13… 14)%; in the state of application of LAV for the purpose - with a probability of (62… 47)%; in a state of recovery after damage - with probability; in the state of service - with a probability of (18… 29)%. The implementation of measures aimed at increasing the survivability of LAV, compared with measures, showed that the probability of recovery of LAV after damage and its return to service increases more than four times; the probability of the maintenance system in the state of use for its intended purpose (as of two days) is doubled, the probability of being in the state of maintenance is also increased by one and a half times.

3. Conclusions

1. The weakest point of the standard system of technical support of combat operations of the unit is its ability to restore weapons damaged during combat. The solution of the problem of evacuation and recovery using the model of states and transitions confirms the adequacy of the model with real measures of evacuation and recovery of weapons. This situation necessitates further research on the technical system of combat operations, in order to identify measures to increase the ability to restore weapons damaged during combat.

2. Analysis of the functioning of the technical system of combat operations in order to determine its capabilities and areas for improvement in conditions of random and antagonistic uncertainties - all this necessitates the search for and application of effective models and appropriate quantitative analysis and synthesis for adequate scientific management of technical problems.

3. The use of models of states and transitions allows by building an adequate model and appropriate simple calculations, even in conditions of random and antagonistic uncertainties to obtain sufficiently reliable quantitative estimates of the capabilities of the technical system of combat operations, and to determine appropriate directions and ways to improve it and increase important parameters. its functioning.

3. Under the conditions of creating a software product and implementing a dialog-information model of the operation of the technical system of combat operations using a personal computer, it is possible not only to explore real support systems, but also to solve complex problems of technical support of combat operations in real time and in including on the battlefield.

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