Cordial Labeling for Star of Bistar Graph

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Abstract

Bistar graph $B_{m,n}$ is a graph derived by joining the center node by an edge of two-star graph $K_{1,m}$ and $K_{1,n}$ and Star of Bistar graph is formed by adding a bistar graph to each vertex at the $K_{1,m}$. In this work, we have looked into the cordial labeling pattern for star of bistar graph.

Keywords

Star graph, Bistar graph, Cordial labeling, Cordial graph.

1. Introduction

Graph labeling was first established by Alexander Rosa in 1967 in his first paper. For basic definition, we refer Harary [3]. We considered here Simple, finite, and undirected graphs. Graph labeling is assigning non-negative integers to vertices and edges. If the labeling is assigned to both vertices and edges then it is termed as total labeling. Cordial labeling was first instigate by Cahit [2]. Sudhakar et.al [5] demonstrated the cordiality for star graphs. Vaidya and Shah [6] demonstrated cordiality of bistar related graphs. In this paper, we will look into the labeling pattern for star of bistar graph for its cordiality.

1.1. Definition

Let G be a graph with vertex set V and edge set E, Assume f is a function from the vertex set $V(G) \rightarrow \{0,1\}$, then f is said to be cordial labeling if $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$, and G is cordial graph if it admits cordial labeling.

1.2. Definition

A star graph $K_{1,m}$ is a complete bipartite graph with one internal node and m – leaves. A star having three edges is called as claw

1.3. Definition

Bistar graph $B_{m,n}$ is a graph derived by joining the centre node by an edge of two-star graph $K_{1,m}$ and $K_{1,n}$. For reference, consider the below example $B_{2,2}$ which is joined by an edge of two-star graph $K_{1,2}$ and $K_{1,2}$.

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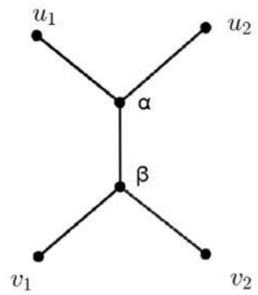

Figure 1:

1.4. Definition

Star of Bistar graph is formed by adding a bistar graph to each vertex of $K_{1,m}$. For reference, consider the below example – Star of Bistar graph $B_{2,2}$.

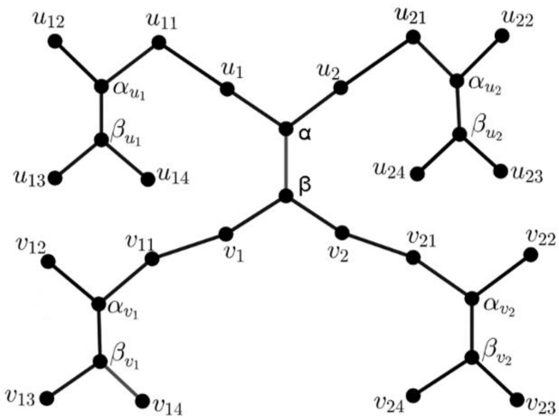

Figure 2:

2. Results

2.1. Theorem: Star of bistar graph $B_{m,n}$ is a cordial graph when $m = n$.

Proof: Let $K_{1,m}$ and $K_{1,n}$ are two-star graph forming a bistar graph $B_{m,n}$ by joining their centre vertex to an edge.

Let $u_1, u_2, u_3, \dots \dots \dots \dots, u_n$ and $v_1, v_2, v_3, \dots \dots \dots \dots, v_n$ are the nodes of $K_{1,m}$ and $K_{1,n}$ for internal node of star graph and α and β are the centre nodes of bistar graph by which $K_{1,m}$ and $K_{1,n}$ are joined.

Let $(u_{11}, u_{12}, \dots \dots u_{1n})$, $(u_{21}, u_{22}, \dots \dots .. u_{2n})$,……..,($u_{m1}, u_{m2}, \dots \dots \dots u_{mn})$ and $(v_{11}, v_{12}, \dots \dots v_{1n})$, $(v_{21}, v_{22}, \dots \dots .. v_{2n})$,……..,($v_{m1}, v_{m2}, \dots \dots \dots v_{mn})$ be the nodes of $K_{1,m}$ and $K_{1,n}$ of m – leaves of star of bistar graph.

The vertices of graph u_i are attached to u_{ij} and v_i are attached to v_{ij} and for the centre vertices of graph $K_{1,n}$ and $K_{1,m}$ of m – leaves are denoted as α_{u_i} and β_{v_i}.

Let G is a star graph of bistar graph. To define the function $f: V(G) \to \{0,1\}$, below cases will arise:

Case 1: When $m = n \equiv 0,2\ (mod 4)$ then we define labeling as:

$$f(\alpha) = 0$$
$$f(\beta) = 1$$
$$f(v_i) = f(u_i) = \begin{cases} 1\ ;\ i \equiv 1,3\ (mod\ 4) \\ 0;\ i \equiv 0,2\ (mod\ 4) \end{cases}$$
$$f(v_{ij}) = f(u_{ij}) = \begin{cases} 1\ ;\ i+j \equiv 1,3\ (mod\ 4) \\ 0;\ i+j \equiv 0,2\ (mod\ 4) \end{cases}$$
$$f(\beta_{v_i}) = f(\beta_{u_i}) = f(\alpha_{v_i}) = f(\alpha_{u_i}) = \begin{cases} 1\ ;\ i \equiv 1,3\ (mod\ 4) \\ 0;\ i \equiv 0,2\ (mod\ 4) \end{cases}$$

Case 2: When $m = n \equiv 1,3\ (mod\ 4)$ then we define labeling as:

$$f(\alpha) = 0$$
$$f(\beta) = 0$$
$$f(v_i) = f(u_i) = \begin{cases} 1\ ;\ i \equiv 1,3\ (mod\ 4) \\ 0;\ i \equiv 0,2\ (mod\ 4) \end{cases}$$
$$f(v_{ij}) = f(u_{ij}) = \begin{cases} 1\ ;\ i+j \equiv 1,3\ (mod\ 4) \\ 0;\ i+j \equiv 0,2\ (mod\ 4) \end{cases}$$
$$f(\beta_{u_i}) = f(\alpha_{u_i}) = \begin{cases} 0;\ i \equiv 1,3\ (mod\ 4) \\ 1;\ i \equiv 0,2\ (mod\ 4) \end{cases}$$
$$f(\beta_{v_i}) = f(\alpha_{v_i}) = \begin{cases} 1;\ i \equiv 1,3\ (mod\ 4) \\ 0;\ i \equiv 0,2\ (mod\ 4) \end{cases}$$

The above explained cases satisfy the condition for both vertices and edges, to be cordial which is as shown below in table 1.

Table 1: Labeling Pattern for Edges and Vertices

	Edge Labeling	**Vertices Labeling**
when $m = n = 0,2\ (mod\ 4)$	$e_f(0) + 1 = e_f(1)$	$v_f(0) = v_f(1)$
when $m = n = 1,3\ (mod\ 4)$	$e_f(1) = e_f(0) + 1$	$v_f(0) = v_f(1)$

The above cases justify that the graph G is cordial graph
Example 1.1. Star of Bistar graph $B_{3,3}$ is a cordial graph.

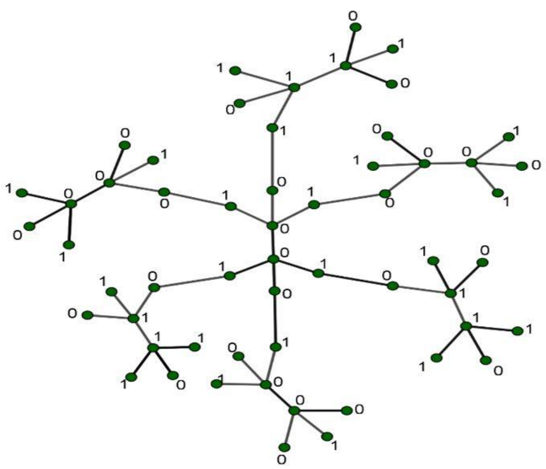

Figure 3:

2.2. Theorem: Star of Bistar graph $B_{m,n}$ is a cordial graph when $m \neq n$.

Proof: Let $K_{1,m}$ and $K_{1,n}$ are two-star graph forming a bistar graph $B_{m,n}$ by joining their centre vertex to an edge.

Let $u_1, u_2, u_3, \dots\dots\dots\dots, u_n$ and $v_1, v_2, v_3, \dots\dots\dots\dots, v_n$ are the nodes of $K_{1,m}$ and $K_{1,n}$ for internal node of star graph and α and β are the centre nodes of bistar graph by which $K_{1,m}$ and $K_{1,n}$ are joined.

Let $(u_{11}, u_{12}, \dots\dots u_{1n})$, $(u_{21}, u_{22}, \dots\dots.. u_{2n})$,……..,($u_{m1}, u_{m2}, \dots\dots\dots u_{mn})$ and $(v_{11}, v_{12}, \dots\dots v_{1n})$, $(v_{21}, v_{22}, \dots\dots.. v_{2n})$,……..,($v_{m1}, v_{m2}, \dots\dots\dots v_{mn})$ be the nodes of $K_{1,m}$ and $K_{1,n}$ of m – leaves of star of bistar graph.

The vertices of graph u_i are attached to u_{ij} and v_i are attached to v_{ij} and for the centre vertices of graph $K_{1,n}$ and $K_{1,m}$ of m – leaves are denoted as α_{u_i} and β_{v_i}.

Let G is a star graph of bistar graph. To define the function $f: V(G) \to \{0,1\}$, the below cases will arise:

Case 1: When $m \equiv 0,2\ (mod\ 4)$ and $n \equiv 0,2\ (mod\ 4)$ and $m \neq n$ then we define labeling as:

$$f(\alpha) = 0$$
$$f(\beta) = 1$$
$$f(u_i) = f(v_i) = \begin{cases} 0\ ;\ i \equiv 0,2\ (mod\ 4) \\ 1;\ i \equiv 1,3\ (mod\ 4) \end{cases}$$
$$f(v_{ij}) = f(u_{ij}) = \begin{cases} 1\ ;\ i+j \equiv 1,3\ (mod\ 4) \\ 0;\ i+j \equiv 0,2\ (mod\ 4) \end{cases}$$
$$f(\beta_{v_i}) = f(\beta_{u_i}) = f(\alpha_{v_i}) = f(\alpha_{u_i}) = \begin{cases} 1\ ;\ i \equiv 1,3\ (mod\ 4) \\ 0;\ i \equiv 0,2\ (mod\ 4) \end{cases}$$

Case 2: When $m \equiv 1,3\ (mod\ 4)$ and $n \equiv 1,3\ (mod\ 4)$ and $m \neq n$ then we define labeling as:

$$f(\alpha) = 0$$
$$f(\beta) = 0$$
$$f(u_i) = f(v_i) = \begin{cases} 1\ ;\ i \equiv 1,3\ (mod\ 4) \\ 0\ ;\ i \equiv 0,2\ (mod\ 4) \end{cases}$$
$$f(v_{ij}) = f(u_{ij}) = \begin{cases} 1\ ;\ i+j \equiv 1,3\ (mod\ 4) \\ 0;\ i+j \equiv 0,2\ (mod\ 4) \end{cases}$$
$$f(\beta_{v_i}) = f(\alpha_{v_i}) = \begin{cases} 1\ ;\ i \equiv 1,3\ (mod\ 4) \\ 0;\ i \equiv 0,2\ (mod\ 4) \end{cases}$$
$$f(\beta_{u_i}) = f(\alpha_{u_i}) = \begin{cases} 0\ ;\ i \equiv 1,3\ (mod\ 4) \\ 1;\ i \equiv 0,2\ (mod\ 4) \end{cases}$$

Case 3: When $m \equiv 1\ (mod\ 4)$ and $n \equiv 2\ (mod\ 4)$:

$$f(\alpha) = 0$$
$$f(\beta) = 0$$
$$f(u_i) = \begin{cases} 1\ ;\ i \equiv 0,1\ (mod\ 4) \\ 0\ ;\ i \equiv 2,3\ (mod\ 4) \end{cases}$$
$$f(v_i) = \begin{cases} 0\ ;\ i \equiv 0,1\ (mod\ 4) \\ 1\ ;\ i \equiv 2,3\ (mod\ 4) \end{cases}$$

$$f(v_{ij}) = f(u_{ij}) = \begin{cases} 1\ ;\ i+j \equiv 1,3\ (mod\ 4) \\ 0;\ i+j \equiv 0,2\ (mod\ 4) \end{cases}$$

$$f(\beta_{v_i}) = f(\beta_{u_i}) = f(\alpha_{v_i}) = f(\alpha_{u_i}) = \begin{cases} 1\ ;\ i \equiv 1,3\ (mod\ 4) \\ 0;\ i \equiv 0,2\ (mod\ 4) \end{cases}$$

Case 4: When $m \equiv 1\ (mod\ 4)$ and $n \equiv 0\ (mod\ 4)$:

$$f(\alpha) = 0$$

$$f(\beta) = 0$$

$$f(u_i) = \begin{cases} 1\ ;\ i \equiv 0,1\ (mod\ 4) \\ 0\ ;\ i \equiv 2,3\ (mod\ 4) \end{cases}$$

$$f(v_i) = \begin{cases} 0\ ;\ i \equiv 0,1\ (mod\ 4) \\ 1\ ;\ i \equiv 2,3\ (mod\ 4) \end{cases}$$

$$f(v_{ij}) = f(u_{ij}) = \begin{cases} 1\ ;\ i+j \equiv 1,3\ (mod\ 4) \\ 0;\ i+j \equiv 0,2\ (mod\ 4) \end{cases}$$

$$f(\beta_{v_i}) = f(\beta_{u_i}) = f(\alpha_{v_i}) = f(\alpha_{u_i}) = \begin{cases} 1\ ;\ i \equiv 1,3\ (mod\ 4) \\ 0;\ i \equiv 0,2\ (mod\ 4) \end{cases}$$

Case 5: When $m \equiv 3\ (mod\ 4)$ and $n \equiv 0\ (mod\ 4)$

$$f(\alpha) = 0$$

$$f(\beta) = 1$$

$$f(u_i) = \begin{cases} 1\ ;\ i \equiv 0,1\ (mod\ 4) \\ 0\ ;\ i \equiv 2,3\ (mod\ 4) \end{cases}$$

$$f(v_i) = \begin{cases} 0\ ;\ i \equiv 0,1\ (mod\ 4) \\ 1\ ;\ i \equiv 2,3\ (mod\ 4) \end{cases}$$

$$f(v_{ij}) = f(u_{ij}) = \begin{cases} 1\ ;\ i+j \equiv 1,3\ (mod\ 4) \\ 0;\ i+j \equiv 0,2\ (mod\ 4) \end{cases}$$

$$f(\beta_{v_i}) = f(\beta_{u_i}) = f(\alpha_{v_i}) = f(\alpha_{u_i}) = \begin{cases} 1\ ;\ i \equiv 1,3\ (mod\ 4) \\ 0;\ i \equiv 0,2\ (mod\ 4) \end{cases}$$

Case 6: When $m \equiv 3\ (mod\ 4)$ and $n \equiv 2\ (mod\ 4)$

$$f(\alpha) = 0$$

$$f(\beta) = 1$$

$$f(u_i) = \begin{cases} 1\ ;\ i \equiv 0,1\ (mod\ 4) \\ 0\ ;\ i \equiv 2,3\ (mod\ 4) \end{cases}$$

$$f(v_i) = \begin{cases} 0\ ;\ i \equiv 0,1\ (mod\ 4) \\ 1\ ;\ i \equiv 2,3\ (mod\ 4) \end{cases}$$

$$f(v_{ij}) = f(u_{ij}) = \begin{cases} 1\ ;\ i+j \equiv 1,3\ (mod\ 4) \\ 0;\ i+j \equiv 0,2\ (mod\ 4) \end{cases}$$

$$f(\beta_{v_i}) = f(\beta_{u_i}) = f(\alpha_{v_i}) = f(\alpha_{u_i}) = \begin{cases} 1\ ;\ i \equiv 1,3\ (mod\ 4) \\ 0;\ i \equiv 0,2\ (mod\ 4) \end{cases}$$

Case 7: When $m \equiv 2\ (mod\ 4)$ and $n \equiv 1\ (mod\ 4)$

$$f(\alpha) = 0$$

$$f(\beta) = 0$$

$$f(u_i) = \begin{cases} 0\ ;\ i \equiv 0,1\ (mod\ 4) \\ 1\ ;\ i \equiv 2,3\ (mod\ 4) \end{cases}$$

$$f(v_i) = \begin{cases} 1\ ;\ i \equiv 0,1\ (mod\ 4) \\ 0\ ;\ i \equiv 2,3\ (mod\ 4) \end{cases}$$

$$f(v_{ij}) = f(u_{ij}) = \begin{cases} 1\ ;\ i + j \equiv 1,3\ (mod\ 4) \\ 0;\ i + j \equiv 0,2\ (mod\ 4) \end{cases}$$

$$f(\beta_{v_i}) = f(\beta_{u_i}) = f(\alpha_{v_i}) = f(\alpha_{u_i}) = \begin{cases} 1\ ;\ i \equiv 1,3\ (mod\ 4) \\ 0;\ i \equiv 0,2\ (mod\ 4) \end{cases}$$

Case 8: When $m \equiv 2\ (mod\ 4)$ and $n \equiv 3\ (mod\ 4)$

$$f(\alpha) = 0$$

$$f(\beta) = 1$$

$$f(u_i) = \begin{cases} 0\ ;\ i \equiv 0,1\ (mod\ 4) \\ 1\ ;\ i \equiv 2,3\ (mod\ 4) \end{cases}$$

$$f(v_i) = \begin{cases} 1\ ;\ i \equiv 0,1\ (mod\ 4) \\ 0\ ;\ i \equiv 2,3\ (mod\ 4) \end{cases}$$

$$f(v_{ij}) = f(u_{ij}) = \begin{cases} 1\ ;\ i + j \equiv 1,3\ (mod\ 4) \\ 0;\ i + j \equiv 0,2\ (mod\ 4) \end{cases}$$

$$f(\beta_{v_i}) = f(\beta_{u_i}) = f(\alpha_{v_i}) = f(\alpha_{u_i}) = \begin{cases} 1\ ;\ i \equiv 1,3\ (mod\ 4) \\ 0;\ i \equiv 0,2\ (mod\ 4) \end{cases}$$

Case 9: When $m \equiv 0\ (mod\ 4)$ and $n \equiv 1\ (mod\ 4)$

$$f(\alpha) = 0$$

$$f(\beta) = 1$$

$$f(u_i) = f(v_i) = \begin{cases} 1\ ;\ i \equiv 2,3\ (mod\ 4) \\ 0\ ;\ i \equiv 0,1\ (mod\ 4) \end{cases}$$

$$f(v_{ij}) = f(u_{ij}) = \begin{cases} 1\ ;\ i + j \equiv 1,3\ (mod\ 4) \\ 0;\ i + j \equiv 0,2\ (mod\ 4) \end{cases}$$

$$f(\beta_{v_i}) = f(\beta_{u_i}) = f(\alpha_{v_i}) = f(\alpha_{u_i}) = \begin{cases} 1\ ;\ i \equiv 1,3\ (mod\ 4) \\ 0;\ i \equiv 0,2\ (mod\ 4) \end{cases}$$

Case 10: When $m \equiv 0\ (mod\ 4)$ and $n \equiv 3\ (mod\ 4)$

$$f(\alpha) = 0$$

$$f(\beta) = 0$$

$$f(u_i) = f(v_i) = \begin{cases} 1\ ;\ i \equiv 2,3\ (mod\ 4) \\ 0\ ;\ i \equiv 0,1\ (mod\ 4) \end{cases}$$

$$f(v_{ij}) = f(u_{ij}) = \begin{cases} 1\ ;\ i + j \equiv 1,3\ (mod\ 4) \\ 0;\ i + j \equiv 0,2\ (mod\ 4) \end{cases}$$

$$f(\beta_{v_i}) = f(\beta_{u_i}) = f(\alpha_{v_i}) = f(\alpha_{u_i}) = \begin{cases} 1\ ;\ i \equiv 1,3\ (mod\ 4) \\ 0;\ i \equiv 0,2\ (mod\ 4) \end{cases}$$

The above explained cases satisfy the condition for both vertices and edges, to be cordial which is as shown below in table 2.

Table 2: Labeling Pattern for Edges and vertices

	Edge Labeling	Vertices Labeling
when $m \equiv 0,2 \pmod 4$ & $n \equiv 0,2 \pmod 4$; $m \neq n$	$e_f(1) = e_f(0) + 1$	$v_f(0) = v_f(1)$
when $m \equiv 1,3 \pmod 4$ & $n \equiv 1,3 \pmod 4$; $m \neq n$	$e_f(1) = e_f(0) + 1$	$v_f(1) = v_f(0)$
when $m \equiv 1 \pmod 4$ & $n \equiv 2 \pmod 4$	$e_f(0) = e_f(1) + 1$	$v_f(1) = v_f(0)$
when $m \equiv 1 \pmod 4$ & $n \equiv 0 \pmod 4$	$e_f(1) = e_f(0) + 1$	$v_f(1) = v_f(0)$
when $m \equiv 3 \pmod 4$ & $n \equiv 0 \pmod 4$	$e_f(1) = e_f(0) + 1$	$v_f(1) = v_f(0)$
when $m \equiv 3 \pmod 4$ & $n \equiv 2 \pmod 4$	$e_f(0) = e_f(1) + 1$	$v_f(1) = v_f(0)$
When $m \equiv 2 \pmod 4$ & $n \equiv 1 \pmod 4$	$e_f(0) = e_f(1) + 1$	$v_f(1) = v_f(0)$
When $m \equiv 2 \pmod 4$ & $n \equiv 3 \pmod 4$	$e_f(1) = e_f(0) + 1$	$v_f(1) = v_f(0)$
When $m \equiv 0 \pmod 4$ & $n \equiv 1 \pmod 4$	$e_f(1) = e_f(0) + 1$	$v_f(1) = v_f(0)$
When $m \equiv 0 \pmod 4$ & $n \equiv 3 \pmod 4$	$e_f(0) = e_f(1) + 1$	$v_f(1) = v_f(0)$

The above cases justify that the graph G is cordial graph
Example 1.2: Star of Bistar graph $B_{4,5}$ is a cordial graph.

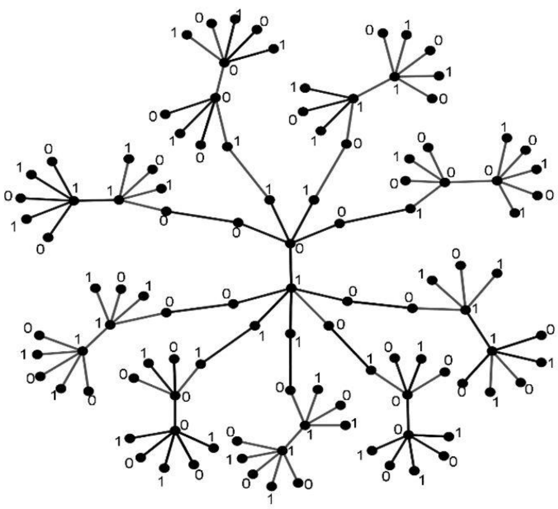

Figure 4:

Example 1.3: Star of Bistar graph $B_{3,4}$ is a cordial graph.

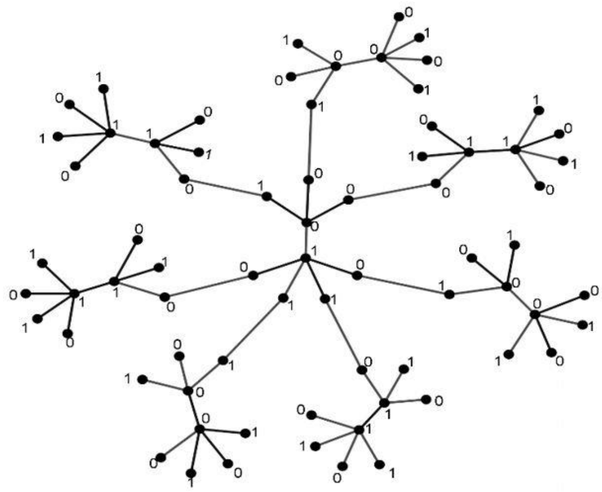

Figure 5:

3. Conclusion

Graph labeling in graphs is an important area of research. We have presented the cordiality for star of bistar graph. Also, we have seen some examples which justify the above theorem. Labeling for star of other graphs is an open research area.

4. Acknowledgements

Our thanks are due to those who motivated us and for being a good support to finish this work.

5. References

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