PSO-SVR Aided Robust EKF Algorithm for GNSS/INS Integrated Navigation

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Abstract

The traditional robust extended Kalman filter (EKF) usually fails to resist small and continuous outliers effectively when the abnormality occurs in GNSS data. To solve this issue, we propose a PSO-SVR aided robust EKF for GNSS/INS integrated navigation. First, this paper uses the particle swarm optimization (PSO) algorithm to optimize the SVR kernel function and penalty coefficient as well as establishes the PSO-SVR intelligent prediction model. Next, on the basis of robust EKF, a PSO-SVR model is employed to predict the navigation algorithm during abnormal periods of GNSS. Finally, this algorithm is verified by the vehicle field test, and the robustness of this algorithm to different outliers is further tested by artificially adding small and continuous outliers of different sizes. According to the experimental results, compared with the REKF algorithm, this PSO-SVR-REKF algorithm can resist outliers effectively whether the outlier of a single point is significant or not. A 10s-abnormality of GNSS data can bring out an improvement of accuracy by 64%.

Keywords

GNSS/INS integrated navigation, extended Kalman filter, support vector regression, particle swarm optimization

1. Introduction

Recently, the GNSS prediction combined with machine learning algorithms has become a popular approach to solve the problem of poor accuracy of GNSS/INS integrated navigation systems in abnormal periods of GNSS [1]. The common machine learning algorithms to assist GNSS/INS integrated navigation include Support Vector Machine (SVM) [2, 3], Recurrent Neural Network (RNN) [4, 5], Residual Attention Network (RAN) [6], Long Short-Term Memory (LSTM) [7, 8], Gated Controlled Recurrent Unit (GRU) [9], etc. The above algorithms can be divided into intelligent prediction based on neural network and regression prediction based on statistics, both of which are of shallow network layers to conduct time-series prediction easily. However, the prediction algorithm based on neural networks may be easily puzzled by local optimization, uncertain network topology, the "curse of dimensionality", and other issues [10, 11]. SVR, based on the statistical learning theory, is a method proposed to study statistical regression and prediction problems that can help solve small samples, nonlinearity, high dimension, and local minimum issues. Hence, it is very suitable for predicting GNSS abnormal information, and it is verified to be more efficient than the neural-network prediction algorithm.

However, to construct the SVR model, the parameter optimization must be carried out since choosing training parameters by experience will affect the regression accuracy and generalization ability. Though being widely applied in the navigation system, the genetic algorithm-support vector regression (GA-SVR) model is of low-level efficiency, as it requires complex processes such as selection, crossover, and mutation during the operation process [12]. Particle Swarm Optimization (PSO) is a population-based parallel optimization technique with few adjustable parameters, fast convergence speed, simple and easy operation, which is suitable for processing real-time navigation data [13].

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Based on this, this paper first constructs an intelligent prediction model based on PSO-SVR optimized kernel function parameters and penalty coefficients, taking the angular rate and acceleration velocity of the inertial navigation system (INS), the velocity and attitude estimations by mechanization equations as the input, and the increment of GNSS/INS integrated navigation solution as the output; Next, this paper investigates that how the PSO-SVR aided robust EKF can diagnose and repair the abnormality when single-point and continuous abnormality of GNSS signals to predict the navigation solution during the abnormal period of GNSS and improve the EKF performance to suppress outliers.

2. GNSS/INS robust EKF model

2.1. Extented Kalman filter

The standard Kalman filtering algorithm generally assumes that the system and observation equations are linear. However, GNSS/INS tight coupled navigation system usually cannot satisfy this assumption. EKF algorithm can realize the linear approximation of nonlinear systems to further improve the solution accuracy. The nonlinear system is assumed by the following expression:

$$x_{k} = f_{k-1}(x_{k-1}) + w_{k}, w_{k} \sim N(0, Q_{k})$$
(1)

$$z_k = h_k(x_k) + v_k, v_k \sim N(0, R_k)$$
(2)

Wherein x_k and x_{k-1} are state vectors at the time k and k-1 respectively; w_k and v_k are random noise; $f_{k-1}(\cdot)$ is the state transition function; $h_k(\cdot)$ is the transfer function between the state and observation vectors; noise variance matrix in dynamic systems Q_k and observation noise variance matrix R_k can be preset. The first-order discrete-time extended Kalman filter is predicted as follows:

$$\hat{x}_{k}(-) = f_{k-1}(\hat{x}_{k-1}(+)) \tag{3}$$

$$\hat{z}_k = h_k(\hat{x}_k(-)) \tag{4}$$

Filter valuation and its corresponding covariance matrix is: $\hat{x}_k(+) = \hat{x}_k(-) + \overline{K}_k(z_k - \hat{z}_k) = \hat{x}_k(-) + \overline{K}_k V_k$

$$\hat{x}_{k}(+) = \hat{x}_{k}(-) + \overline{K}_{k}(z_{k} - \hat{z}_{k}) = \hat{x}_{k}(-) + \overline{K}_{k}V_{k}$$

$$P_{k}(+) = [I - \overline{K}_{k}(H_{k})]P_{k}(-)$$
(6)

$$P_k(+) = [I - K_k(H_k)]P_k(-)$$

Wherein the prediction covariance matrix is:

$$P_{k}(-) = \Phi_{k-1}P_{k-1}(+)\Phi_{k-1}^{T} + Q_{k-1}$$
(7)
KF is:

The gain matrix of EKF is:

$$\overline{K}_{k} = P_{k}(-)H_{k}^{T}[H_{k}P_{k}(-)H_{k}^{T}+R_{k}]^{-1}$$
(8)

The prediction residual is $V_k = z_k - \hat{z}_k$, The GNSS/INS navigation system can be considered nearly linear but not absolutely linear, and EKF can effectively solve nonlinear problems to deliver better state estimation. In addition, as the first-order estimation is taken into the Taylor series expansion, the prediction residual is enough to describe dynamic characteristics even though failing to represent the real value.

2.2. Robust extented Kalman filter

The equivalent EKF gain matrix is constructed, a similar expression to the IGG III weight function:

$$\widetilde{K}_{ij} = \begin{cases} \overline{K}_{ij}, s_j \leq k_0 \\ \overline{K}_{ij} \times \frac{k_0}{s_j} \times [\frac{k_1 - s_j}{k_1 - k_0}]^2, k_0 \leq s_j \leq k_1 \\ 0, s_j > k_1 \end{cases}$$
(9)

Wherein k_0 and k_1 are threshold parameters with k_0 as 2.5-3.5 and k_0 as 3.5-4.5; , wherein *i* and *j* denote the dimensions of state and observation vectors respectively, and σ_j refers to the prediction residual, redundant observation component, and measurement standard deviation of the observation vector separately, and the iteration will be carried out after each update.

Given the number of iterations t, the state prediction value and the prediction residual are as follows:

$$x_{k,t}(-) = x_{k,t-1}(+) \tag{10}$$

$$V_{k,t}(-) = z_k - H_k x_{k,t}(-)$$
(11)

Wherein the state prediction value $z_{k,t}(-)$ at the t iteration is confirmed by the state filter value and its prediction residual after the T-1 iteration. After calculating the equivalent gain matrix according to Equation (9), we can obtain the robust filter value as

$$x_{k,t}(+) = x_{k,t-1}(-) + \tilde{K}V_{k,t}$$
(12)

If the difference between $x_{k,t}(+)$ and $x_{k,t-1}(-)$ is the estimated value of standard EKF at the time k, and the posterior covariance matrix will be:

$$P_{k}(+) = [I - \tilde{K}_{k,t}]P_{k}(-)$$
(13)

Wherein $\widetilde{K}_{k,t}$ means the final equivalent Kalman filter gain matrix at the end of an iteration.

3. PSO-SVR training regression model

3.1. Support vector regression (SVR)

SVR, one of machine learning methods, is commonly employed to study statistical regression and data prediction to solve the problems of small samples, nonlinearity, high dimensions, and local minimum. The basic concept of SVR is to map the dataset x to high-dimensional feature space for regression analysis and prediction by defining a kind of nonlinear transformation. Given the training samples $(x_i, y_i), x_i \in R, y_i \in R, i = 1 \cdots n$, we will set the objective regression function:

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$$\langle w_1, y_1, y_2, y_1, z_1, y_1, z_2, z$$

To obtain the optimal parameters w and b, we establish the optimization function as follows: $\min \frac{1}{2} \langle w \cdot w \rangle + C \sum_{i=1}^{n} (\xi_i + \xi_i^*)$ (15)

with constraint condition

 $y_i - \langle w \cdot x_i \rangle + b \le \varepsilon + \xi_i, \langle w \cdot x_i \rangle - y_i + b \le \varepsilon + \xi_i^*.$

Wherein C is the penalty factor to weigh the overall model and sample errors. The larger the value of C, the higher level of data fitting. \mathcal{E} refers to the loss function used in reflecting the generalization ability of models. The larger of \mathcal{E} , the lower level of data fitting. The above functions are transformed into the following optimal problem through the kernel method and corresponding duality theory.

$$(\alpha, \alpha^*) = \sum_{i=1}^n y_i (\alpha_i - \alpha *_i) - \varepsilon \sum_{i=1}^n (\alpha_i + \alpha_i^*) - \frac{1}{2} \sum_{i=1,j=1}^n (\alpha_i - \alpha_i^*) (\alpha_j - \alpha_j^*) K(x_i, x_j) \quad (16)$$
with constraint condition

$$\sum_{i=1}^{n} (\alpha_i - \alpha_i^*) = 0, 0 \le \alpha_i \le C, i = 1, 2, \dots, j = 1, 2, \dots, 0 \le \alpha_i^* \le C$$

Then

 $w = \sum_{i=1}^{n} (\alpha_i - \alpha_i^*) x_i, \quad b = y_j - \sum_{i=1}^{n} (\alpha_i - \alpha_i^*) (x_i \cdot x_j) + \varepsilon, \quad i = 1, 2, \dots, j = 1, 2, \dots n$ The objective regression function is obtained as follows:

$$f(x) = \sum_{i=1}^{n} (\alpha_i - \alpha_i^*) K(x, x_i) + b$$
(17)

3.2. Particle swarm optimization

The PSO algorithm first initializes a group of particles in the feasible solution space, each of which represents a potential optimal solution to extremal optimization. The particle characteristics are indicated by three indexes: position, velocity, and fitness value. The fitness value can be calculated by the fitness function, which reflects the advantages and disadvantages of particles. The moving particle in the solution space updates its individual position by tracking the individual extremum Pbest and the group extremum Gbest; while the individual extremum Pbest refers to the optimal position of fitness value calculated from the position experienced by the individual, and the group extreme Gbest means

the optimal position of fitness searched by all particles in the population. The fitness value will be calculated upon each update of the particle position, and the positions of individual extremum Pbest and group extremum Gbest are updated by comparing the fitness value of the new particle with those of individual extreme and group extremums.

Assume that a D-dimensional space for searching has a population $X = (X_1, X_2, ..., X_n)$ which includes n number of particles, among which the i-th particle is represented as a D-dimensional vector $X_i = [x_{i1}, x_{i2}, \dots, x_{iD}]^T$, representing the position of the i-th particle in the D-dimensional space as well as a candidate solution. The fitness value corresponding to the position X_i of each particle can be calculated by the objective function. The velocity of the i-th particle is $V_i = [V_{i1}, V_{i2}, \dots V_{iD}]^T$, its individual extremum is $P_i = [P_{i1}, P_{i2}, \dots P_{iD}]^T$, and the group extremum of the population is $P_g =$

 $\left[P_{g1}, P_{g2}, \dots P_{gD}\right]^{T}$

In each iteration, the particle updates its velocity and position through the individual extremum and group extremum under the following formula:

$$V_{id}^{k+1} = \omega V_{id}^{\bar{k}} + c_1 r_1 (P_{id}^k - X_{id}^k) + c_2 r_2 (P_{gd}^k - X_{id}^k)$$
(18)
$$X_{id}^{k+1} - X_{id}^{k} + V_{id}^{k+1}$$
(19)

$$X_{id}^{\kappa+1} = X_{id}^{\kappa} + V_{id}^{\kappa+1}$$
(19)

Wherein ω refers to the inertia weight with d=1, 2, ..., D; i = 1,2,..., n; k means the current number of iterations; V_{id} indicates the velocity of particles; c_1 and c_2 are non-negative constants called acceleration factors; and r_1 and r_2 are random numbers distributed between [0,1].

The particle swarm optimization (PSO) algorithm is shown in Figure 1.



Figure 1: Flowchart for PSO algorithm

Popular parameter optimization algorithms include genetic algorithms and swarm intelligence optimization algorithms. To verify that the PSO-SVR model is more efficient than the GA-SVR model, this paper compares their training performance and prediction accuracy (see Table 1), and figures out that the calculation efficiency of PSO is about 7 times higher than the GA-SVR model with the same accuracy.

Model	GA-SVR		PS	S0-SVR
RMS	Train	Predict	Train	Predict
North	0.191	0.386	0.137	0.343
East	0.250	0.444	0.346	0.415
Height	0.004	0.026	0.035	0.027
Mean	0.148	0.285	0.172	0.262
Time(s)	s) 368			57

 Table 1

 Comparison of root mean square error and calculation duration between GA-SVR and PSO-SVR models

4. PSO-SVR aided robust EKF algorithm

The process of the PSO-SVR aided robust EKF algorithm is shown in Figure 2. On the condition of no abnormality in GNSS data, we can enter the PSO-SVR training mode. The input of training samples is the position, speed, and attitude calculated by INS mechanization equations, as well as the acceleration and angular rate data output by INS. The output of training samples is the position increment of the navigation solution. We use the input and output of training samples to train the PSO-SVR model. When the abnormality occurs in GNSS, the training will stop and PSO-SVR prediction mode starts, in which the input is still the position, velocity, and attitude output by INS mechanization equations, as well as the acceleration and angular rate output by INS. Next, the input data will be put into the well-trained PSO-SVR model to predict the increment of navigation solution; finally, the error correction will be carried out in the position, velocity, and attitude calculated by INS to obtain the estimated values.



Figure 2: PSO-SVR-aided robust EKF algorithm **Note:** The blue line represents the training part and the green line suggests the prediction part

5. Experiment

5.1. Data acquisition

To verify how the PSO-SVR-aided robust EKF model can resist outliers when the abnormality occurs in GNSS, we collected the data using the vehicle-running test method on November 21, 2021 in an open factory area located at the South Fifth Ring Road of Beijing (see Figure 3).



Figure 3: Track of vehicle

We adopted high-precision real-time positioning and navigation modules independently developed by the group (see Figure 3), which integrates the Sensor's high-precision inertial measurement unit STIM300 and NovAtel's OEM719 GNSS. Their performance parameters are shown in the following Table 2. The modules are installed in the trunk of the vehicle, the GNSS antenna is mounted on the top of the vehicle to attach with the odometer through cables, enabling the experimental data to be transmitted to the laptop. RTK is supported by the domestic CORS, and the real-time positioning and navigation module, testing vehicles, and equipment installation is suggested in Figure 4.

Table 2

Performance parameters of GNSS and MEMS IMU

Equipment	Performance				
GNSS	Signal Tracking	BDS: B1/B2 GPS: L1/L2 GLONASS: L1/L2 GALILEO:E1/E5b			
	Single point positioning (RMS)	Horizontal: $\pm 1.5m$ Vertical: $\pm 3.0m$			
	DGPS(RMS)	Horizontal: $\pm 0.4m$ Vertical: $\pm 0.8m$			
	RTK(RMS)	Horizontal: ±8cm Vertical: ±15cm			
	Updating Frequency(Hz)	5			
MEMS IMU	Bias Stability of accelerometer(mg)	2			
	Velocity random walk (ug/sqrt (Hz))	120			
	Bias Stability of gyroscope(deg/h)	5			
	Angle random walk(deg/sqrt(h))	0.4			







Figure 4: Diagrams for real-time positioning and navigation module, testing vehicles, and equipment installation

5.2. Experimental analysis

To verify the effectiveness of PSO-SVR aided robust EKF model, we design two groups of experiments below: (1) single point abnormality of GNSS: add outliers of 25 meters, 20 meters, 15 meters, and 10 meters in the East at the time of 192s to compare the robustness of REKF and PSO-SVR-REKF algorithms in dealing with outliers of different sizes; (2) continuous abnormality of GNSS: add the outlier of 25 meters in the East during 192s to 202s (lasting for 10s) to compare the robustness of REKF and PSO-SVR-REKF.

1. Single point abnormality of GNSS

Figure 5(a) shows the performance comparison of REKF and EKF after adding the 20-meter outlier to GNSS. It's obvious that the 20-meter outlier is a significant outlier and the robust EKF performs 5(a), the robust EKF will carry out suspicious section to weaken the effect of outliers on results (see Figure 5(b)). If the outlier is within the normal range, EKF will perform standard section to deliver the same results as obtained by standard EKF (see Figure 5(c)). Hence, robust EKF will improve the accuracy most significantly by eliminating the abnormal data without the assistance of PSO - SVR. However, when the outliers gradually reduce, the diagnosis ability of robust EKF will decrease accordingly, which largely affects the accuracy of results in the end. As can be referred to in Table 3, the robust EKF cannot resist the effect of outliers when the observation outlier equals 10 meters.



Figure 5: Comparison of REKF and EKF (with different outliers)

To verify the effectiveness of the PSO-SVR-aided robust EKF algorithm, we adopt the data with 20meter, 15-meter, and 10-meter outliers to train and predict PSO-SVR. The input-out data of the first 150s are taken as PSO-SVR training samples to train the model and predict the position increment after the 150s as indicated in Figure 6. As can be seen, the abnormality occurs in the position increment of REKF at the time of 192s, and the prediction increment of PSO-SVR can effectively resist the abnormality. We replace the original REKF solution increment with prediction increment and restore it to the position data, the trajectory of which is suggested in Figure 7. Obviously, the PSO-SVR-aided robust EKF model can further resist the influence of outliers based on the robust solution results. Referring to Table 3, PSO-SVR-REKF has the highest accuracy, followed by REKF, and the standard EKF. Compared with EKF and REKF, the PSO-SVR-REKF model significantly improves the accuracy.



Figure 6: Prediction increment and deviation



Table 3

Comparison of largest outliers in EKF, REKF, and PSO-SVR-REKF

Methods Outlier(m)	EKF	REKF	PSO-SVR- REKF
25m	7.210	0.002	0.0018
20m	5.620	1.550	0.0017
15m	4.440	2.850	0.0019
10m	2.980	2.980	0.0018

2. Continuous abnormality of GNSS

To investigate the effect of robust EKF when the abnormality continues in GNSS, we add the 25meter outlier during 192-201s (lasting for 10s). As a result, the robust EKF performs elimination section to resist outliers to the greatest extent (see Figure 8). However, when the abnormality continues longer, affected by the observation covariance matrix of the Kalman filter, the abnormality diagnosis ability of REKF declines with time, the robustness decreases gradually, and even the results have a divergent trend (see Figure 9).



Figure 8: The abnormality lasts for 10s in GNSS



Figure 9: The abnormality lasts for 11s in GNSS

To determine the robustness of PSO-SVR-REKF in the continuous abnormality, according to the PSO-SVR model training and prediction scheme in accordance with Point (1), the PSO-SVR-REKF solution results are obtained and then compared with REKF. As pointed out in Figure 11, the maximum deviation of the REKF solution is 17.35 meters, and the maximum deviation of the PSO-SVR-EKF solution is 6.18 meters. When the abnormality of GNSS lasts more than 10s, the accuracy of PSO-SVR-REKF will be improved by 64% compared with REKF.



Figure 10: Prediction increment and deviation when the abnormality lasts for 11s



Figure 11: Comparison between PSO-SVR-REKF and REKF (when the abnormality lasts for 11s)

6. Conclusion

After analyzing the performance of robust EKF in dealing with small and continuous outliers, this paper builds up a PSO-SVR-aided robust EKF algorithm along with its process. The main conclusions are as follows:

1) When obvious abnormalities occur in GNSS, REKF can suppress the outliner to the largest extent. However, the anomaly discrimination ability of robust EKF weakens with the reduction of outliers. When the outliner of GNSS is equal to or above 10m, REKF will lose its robustness; when the abnormality lasts over 10s in GNSS (the outliner is 20m), the robustness of REKF will gradually decrease. In other words, the less significant the outliner becomes, the longer it will last, and the weaker the robustness of REKF will be.

2) Whether the abnormality is significant or not, the PSO-SVR-REKF model can resist the outliner to the largest extent. When the abnormality of GNSS lasts more than 10s, it can also resist the influence of outliers, which means, the PSO-SVR-REKF algorithm can be less affected by the significance and duration of abnormal data than REKF, and can resist outliers below 10 meters. When the abnormality of GNSS continues for 10s, the accuracy will be improved by 64%.

7. References

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