## Research on Auxiliary Train Location Method Based on Trajectory Constraint

Yushuai Ning, Cuiran Li<sup>\*</sup>, Jianli Xie

Lanzhou Jiaotong University, Lanzhou, China

#### Abstract

Aiming at the problem that satellite signals are easily blocked by surrounding buildings, mountains and tunnels, which makes global navigation satellite system (GNSS) unable to work normally. In order to obtain the real-time position information of high-speed trains in the sections where the satellite navigation system fails, this paper proposes a method based on the combination of trajectory constraints and wireless sensor network (WSN) to assist train positioning. First, the initial position information of the train is obtained through the WSN location algorithm time difference of arrival (TDOA), then the initial position is modified using the trajectory constraint aided location model. Finally, Kalman Filter (KF) is used for information fusion to achieve accurate train positioning. The simulation results show that the auxiliary positioning method with motion track constraint can improve the positioning accuracy of the train very well, and the average positioning accuracy can be improved by 2~3metres.

#### Keyword

auxiliary positioning; trajectory constraint; time difference of arrival; Kalman filter

### 1. Introduction

As the main artery of national economy, railway plays an important role in the process of national economic development. With the continuous improvement of transportation requirements, train positioning technology has gradually become a research hotspot <sup>[1]</sup>. The current train positioning technology mostly relies on GNSS for positioning, and realizes accurate positioning by combining with other sensors. However, this method has some limitations. In an open environment, it can achieve accurate train positioning

This method will fail in the environment where satellite signals are blocked for a long time. Such as mountain canyons, tunnels, etc. This puts forward higher requirements for train positioning technology.

In view of the situation that the satellite signal is blocked for a long time and the train positioning information is incomplete, scholars at home and abroad have carried out relevant research. Reference [2] proposed a GNSS/INS integrated navigation scheme based on extended KF assisted by Long Short-Term Memory (LSTM). Using LSTM to learn the mathematical relationship between the error of integrated navigation system and the INS solution result. Under the GNSS failure environment, the error state of integrated navigation is predicted and corrected to achieve accurate train positioning. This method can improve the positioning accuracy of trains, but the algorithm complexity is high, which is not conducive to real-time positioning. Reference [3] proposed an integrated navigation method of polarized light/SINS/Beidou satellite navigation system (BDS)/geomagnetism, using federated Kalman filter for multi-sensor data fusion. This method combines a variety of information to achieve highprecision positioning of trains through information complementation. However, as the amount of information increases, there will be contradictions between the information, which will affect the positioning effect. Reference [4] proposed a GNSS/INS integrated navigation model based on Ultra Wide Band (UWB) technology. This algorithm can improve the positioning accuracy, but due to the limited UWB communication distance, it is only applicable to a small range of application scenarios. It is not applicable to train positioning.

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EMAIL: \* licr@mail.lzjtu.cn (Cuiran Li)

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In order to solve the above problems, this paper proposes a research idea of using WSN to assist train positioning. WSN has the advantages of flexible networking, strong adaptability, low cost and high reliability <sup>[5]</sup>. WSN can be deployed in the restricted area of GNSS to provide continuous accurate positioning for trains.

# System model WSN deployment and positioning principle

In order to obtain accurate train position information, the location method based on distance measurement is generally adopted. Among the common methods, the received signal strength (RSSI) is vulnerable to the environment. Angle of arrival (AOA) requires additional hardware support, which increases the overall cost. The time of arrival (TOA) requires the anchor point to synchronize with the target node clock, which requires high requirements. TDOA does not require additional hardware support or clock synchronization. It can be used as the positioning method of trains in WSN.

WSN is a distributed sensor network, as shown in Figure 1. It is composed of sensors distributed around railway lines and receivers on trains.

In a two-dimensional plane, the sensor that has known its own position information is called an anchor point, and the coordinates of the anchor point i are  $(x_i, y_i)$ , i = 1, 2, ..., M. The unknown node to be located is called the target node (i.e. train), and its coordinate is (x, y).



Figure 1 WSN Distribution

Set the anchor *i* receiving the observation signal of the target node as  $u_i(t)$ , and record its transmission delay as  $t_i$ , and take anchor 1 as the reference node. The difference between the transmission delay of the target node to other anchor points and the transmission delay to the reference node can be recorded as  $t_{i1}$ .

$$t_{i1} = t_i - t_1, \quad i = 2, 3, ..., M$$
 (1)

Since the wireless signal travels at the speed of light, the distance difference between other anchors and the target node is  $d_{i1}$ .

$$d_{i1} = d_i - d_1 = c \times t_{i1}, \quad i = 2, 3, ..., M$$
 (2)

Where *c* is the speed of light, and  $d_i$  is the distance from the target node to the anchor point. Equation (3) is obtained according to distance difference and anchor position information.

$$x + y_{i1}y + d_{i1}d = \frac{1}{2}(K_i - K_1 - d_{i1}^2), \quad i = 2, 3, ..., M$$
 (3)

Among them  $x_{i1} = xi - x_1$ ,  $y_{i1} = y_i - y_1$ ,  $K_i = x_i^2 + y_i^2$ .

Therefore, equation (3) becomes a pseudo linear system of equations about x, y. when M = 3, the equations have unique solutions. when  $M \ge 4$ , the equation was overdetermined<sup>[6]</sup>. The estimated position  $\hat{p}_0$  can be obtained from the equations.

$$\hat{\boldsymbol{p}}_0 = \begin{bmatrix} \hat{x}_0 \\ \hat{y}_0 \end{bmatrix}$$
(4)

### 2.2. Error model of TDOA

The communication between the train and the anchor point is not all LOS propagation, and there is always a slight delay. However, due to the existence of high mobility, even a small delay will cause deviation to the estimated position, Even a small delay will cause deviation to the estimated position, so the extra delay caused by NLOS propagation is recorded as  $\tau_e$ , the additional delay follows an exponential distribution<sup>[7]</sup>, and its probability density function is

$$P(\tau_e) = \begin{cases} 1/\tau_{rmsi} \exp(-\tau_e/\tau_{rmsi}) & \tau \ge 0\\ 0 & \tau < 0 \end{cases}$$
(5)

Where  $\tau_{\rm rmsi}$  is root mean square delay extension, which can be expressed as

$$\tau_{rmsi} = T_1 d_i^{\varepsilon} \xi \tag{6}$$

In the above formula,  $T_1$  is the  $\tau_{rmsi}$  middle value of d = 1000 m time,  $d_i$  is the distance between the target node and the anchor point  $i, \varepsilon$  is the exponential component with a value between 0.5 and 1, and the standard deviation  $\sigma_{\xi}$  is the lognormal distribution of random variables between 4 and 6 dB.

Then the time when the detection signal of the target node reaches the anchor point is

$$t_i = t_i^0 + \tau_e \tag{7}$$

Where  $t_i^0$  is the time of signal LOS propagation. Then the time difference between signal arrival at anchor point *i* and *j* signal arrival can be expressed as

$$\Delta t_{ii} = (t_i^0 - t_i^0) + (\tau_{ie} - \tau_{ie}), \quad i, j = 1, 2, 3, \dots n \quad (8)$$

As the extra delay is related to the environment and has a large randomness, so the extra delay is related to the environment and has a large randomness, the quasi normal distribution can be used to fit the extra delay to a certain extent <sup>[7]</sup>, then the signal arrival time difference between the two anchor points is

$$\Delta t_{ii} = \Delta t_{ii}^0 + \mu \tag{9}$$

In the above formula,  $\Delta t_{ij}^0$  is the time difference between signals arriving at two anchor points. The error part  $\mu$  obeys normal distribution  $\mu \sim N(0, \sigma^2)$ .

# Auxiliary train location method based on trajectory constraint Track constraint

The movement mode of the train is special, which is different from the general vehicle movement. The train always runs on the track. Therefore, track information can be used to constrain WSN location information to achieve multi information fusion and improve location accuracy.

First, set the positioning result of the train measured within a period of time to  $(x_t, y_t)$ , t = 1, 2, 3, ..., n. Then fit the motion track in this period of time, and set the motion track as

$$\hat{y} = ax + b \tag{10}$$

Equation (11) represents the sum of squared errors of the system, which is a binary function about a and b.

$$J(a,b) = \sum_{t=1}^{n} [y_t - (a+bx_t)]^2$$
(11)

According to the method of finding extreme value of binary function, the following formula can be obtained

$$\begin{cases} nb + a\sum_{t=1}^{n} x_{t} = \sum_{t=1}^{n} y_{t} \\ b\sum_{t=1}^{n} x_{t} + a\sum_{t=1}^{n} x_{t}^{2} = \sum_{t=1}^{n} x_{t} y_{t} \end{cases}$$
(12)

The value of parameter a, b can be solved by equation (13), that is the train's motion track in this period of time.

Assume that the train speed along the navigation coordinate system is and respectively  $v_x$ ,  $v_y$ , and the observation time interval is  $\Delta t \cdot x_0$ ,  $y_0$  is the train position coordinate obtained from the previous positioning. Then the pseudo observations  $p_{\text{limit}}$  can be obtained by using the constraint information of the motion trajectory.

$$\begin{bmatrix} \tilde{x} \\ \tilde{y} \end{bmatrix} = \begin{bmatrix} x_0 + v_x \Delta t \\ y_0 + v_x \Delta t \end{bmatrix}$$
(13)

### 3.2. Trajectory constraint aided positioning model

Next, we will revise the TDOA estimation results according to  $p_{\text{limit}}$ . As shown in Figure 4, it is the auxiliary positioning model of motion trajectory constraint. Its principle is to use the difference between the pseudo observation position information determined by the trajectory constraint and the position information obtained by TDOA as the observation, input it into the KF to obtain the optimal estimation of the positioning error, and then feed it back to the positioning result of TDOA for correction, so as to improve the positioning accuracy of TDOA.



Figure 2 Trajectory constraint aided positioning model

Suppose that the train positioning result obtained from TDOA is  $P_{\text{TDOA}} = [P_{\text{TDOA},x} \quad P_{\text{TDOA},y}]^{\text{T}}$ , and the pseudo

observation value obtained from the motion track constraint information is  $\boldsymbol{P}_{\text{limit}} = [P_{\text{limit},x} \quad P_{\text{limit},y}]^{\text{T}}$ . The difference  $\Delta \boldsymbol{P} = [\Delta x \quad \Delta y]^{\text{T}}$  between the two estimation methods is taken as the state vector of KF, the optimal estimation of position error is obtained through filtering, and then the estimated position of TDOA is corrected, finally the estimated position  $\boldsymbol{P}_{\text{L-TDOA}}$  of the motion trajectory constrained auxiliary positioning model is obtained.

First, the state vector of the system can be expressed as

$$\boldsymbol{X}_{\text{L-TDOA}k} = \begin{bmatrix} \delta \boldsymbol{P}_{x} & \delta \boldsymbol{P}_{y} & \delta \boldsymbol{v}_{x} & \delta \boldsymbol{v}_{y} \end{bmatrix}^{T}$$
(14)

Where  $\delta P_x$ ,  $\delta P_y$  represents the position error,  $\delta v_x$ ,  $\delta v_y$  represents the speed difference of the train, and the state equation can be expressed as

$$\boldsymbol{X}_{\text{L-TDOA}k} = \boldsymbol{F} \cdot \boldsymbol{X}_{\text{L-TDOA}k-1} + \boldsymbol{\omega}_{k-1}$$
(15)

 $X_{L-TDOAk}$  is the state vector at time k,  $X_{L-TDOAk-1}$  is the state vector at time k-1, F is the state transition matrix,  $\boldsymbol{\omega}_{k-1}$  is the white noise component, and obeys the Gaussian distribution.

The state transition matrix  $\boldsymbol{F}$  is

$$\boldsymbol{F} = \begin{bmatrix} 1 & 0 & \Delta t & 0 \\ 0 & 1 & 0 & \Delta t \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$
(16)

The observation equation is

$$\boldsymbol{Z}_{\text{L-TDOA}k} = \boldsymbol{H} \cdot \boldsymbol{X}_{\text{L-TDOA}k} + \boldsymbol{\nu}_k \tag{17}$$

Where  $Z_{L-TDOAk}$  is the observation vector at time k, H is the measurement matrix, and  $v_k$  is the observation noise

The measurement matrix  $\boldsymbol{H}$  is

$$\boldsymbol{H} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$
(18)

Next, carry out the Kalman filtering process.

### 4. Simulation experiment

In order to verify the TDOA assisted train positioning effect of motion track constraint, this section uses Matlab2022a software to conduct simulation experiments. The experiment compares the localization performance of L-TDOA localization, TDOA localization and motion track constraint. The experimental simulation assumes that the train passes through a 10 km satellite signal shielded area, the train is in high-speed operation, and the train moves in a straight line with variable speed between 240 km/h and 280 km/h, only considering the position change of the two-dimensional plane. The wireless sensor anchor points are evenly deployed on both sides of the railway. The distance between the anchor points on one side is 200meters, and the distance between the anchor points and the rail is 15metres.

In the experiment, only the acceleration and deceleration

Order Number	Motion State	Duration/s
1	Uniform velocity	30
2	accelerate	10
3	Uniform velocity	20
4	Decelerate	20
5	Uniform velocity	20
6	accelerate	10
7	Uniform velocity	30

Table 1	Parameter Setting
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motion state of the train is considered, but the steering motion state of the train is not considered. Table 1 shows the parameter settings of train motion status. In addition, white noise is added to the TDOA positioning process of the train, and its variance is 1. In addition, error iss introduced into the measurement of the time difference of signal arrival, and the error is $1 \times 10^{-8}$ . Generally, root mean square error

(RMSE) and cumulative distribution function are used to evaluate the accuracy of positioning performance.



Figure 3 RMSE of TDOA



Figure 4 RMSE of trajectory-limit





The above figure shows the RMSE obtained through 100 simulation experiments. Figure 3 shows the RMSE of TDOA, which is within 8 meters. Figure 4 shows the RMSE of track limit positioning, and the error value is within 7m. Figure 5 shows the RMSE of L-TDOA, which is within 5 meters. Through comparison, it can be seen that the positioning result of L-TDOA is superior to the other two positioning methods. With the auxiliary positioning function of motion track constraint, its positioning accuracy is significantly improved by 3 meters.



Figure 6 Cumulative distribution function

Figure 6 shows the cumulative distribution function of TDOA, track constraint and L-TDOA positioning error. The cumulative distribution function can more intuitively reflect the statistical distribution of positioning error. It can be seen from Figure 6 that the positioning effect of L-TDOA is more accurate. The positioning error of L-TDOA is basically within 5 meters, while the positioning error of TDOA is within 8 meters. The experimental results show that the positioning accuracy of TDOA for trains can be improved by using motion track constraint assisted positioning, and the average positioning accuracy can be improved by 2~3m.

### 5. Conclusion

In order to obtain the real-time position information of high-speed trains in the sections where the satellite navigation system fails, this paper proposes an auxiliary train positioning method based on trajectory constraints. First, the initial positioning information of the train is obtained through WSN, and then the initial position is modified by using the auxiliary positioning model of motion track constraint, Finally, KF is used for information fusion to achieve accurate train positioning. The simulation results show that the auxiliary positioning method with motion track constraint can improve the positioning accuracy of TDOA for trains. Through comparison, it is found that L-TDOA can improve the positioning accuracy of 2-3 meters on average.

Next, we will further study the combination of the trajectory constraint positioning model and other on-board sensors, such as SINS, to further improve the positioning accuracy of the train.

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### 7. Literature

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