On Properties and Complexity of Incomplete Argumentation Framework

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Abstract

Dung's Argumentation Framework (AF) has been extended in several directions, including the possibility of representing unquantified uncertainty about the existence of arguments and attacks. The framework resulting from such an extension is called *incomplete Argumentation Framework* (iAF). In this paper, we discuss three satisfaction problems recently proposed for iAF, namely *totality*, *determinism* and *functionality* [1], and analyze their complexity under several semantics. We then show that any iAF can be rewritten into an equivalent one where either only (unattacked) arguments or only attacks are uncertain. Finally, we relate iAF to probabilistic argumentation framework, where uncertainty is quantified.

Keywords

Formal Argumentation Theory, Uncertainty, Computational Complexity.

1. Introduction

Formal argumentation has emerged as one of the important fields in Artificial Intelligence [2, 3, 4]. In particular, Dung's abstract Argumentation Framework (AF) is a simple yet powerful formalism for modeling disputes between two or more agents [5]. An AF consists of a set of *arguments* and a binary *attack* relation over the set of arguments that specifies the interactions between arguments: intuitively, if argument a attacks argument b, then b is acceptable only if a is not. Hence, arguments are abstract entities whose status is entirely determined by the attack relation. An AF can be seen as a directed graph, whose nodes represent arguments and edges represent attacks. Several argumentation semantics—e.g. *grounded* (gr), *complete* (co), *preferred* (pr), *stable* (st) [5], and *semi-stable* (sst) [6]—have been defined for AF, leading to the characterization of σ-*extensions*, that intuitively correspond to the sets of arguments that can be collectively accepted under semantics $\sigma \in \{\text{gr}, \text{co}, \text{st}, \text{pr}, \text{sst}\}$.

Various proposals have been made to extend the Dung framework with the aim of better modeling the knowledge to be represented [7, 8, 9, 10, 11, 12, 13, 14]. For the representation of uncertain information, two main extensions have been proposed: *incomplete AF* (iAF), where arguments and attacks may be uncertain [15, 16], and *probabilistic AF* (PrAF), where arguments and attacks are associated with a probability [17, 18, 19]. In this paper we focus on iAF, but we
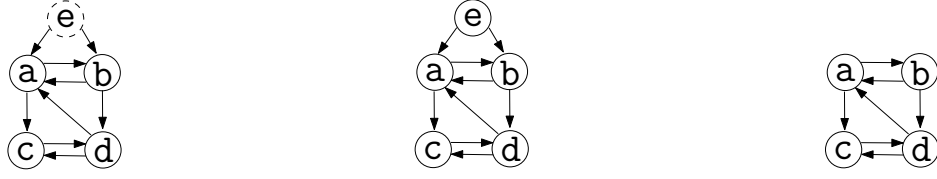
Figure 1: iAF Δ of Example 1(left) and its completions Λ_1(center) and Λ_2(right).

will also study its connection with PrAF.

Example 1. Consider an iAF $\Delta = \langle \{a, b, c, d\}, \{e\}, \{(a, b), (b, a), (a, c), (b, d), (c, d), (d, c),$ $(d, a), (e, a), (e, b)\}, \emptyset \rangle$ whose corresponding graph is shown in Figure 1(left), where dashed nodes represent uncertain arguments (see Definition 1 for the formal definition). Δ describes the following scenario. A party planner invites Alice, Bob, Carl, David, and Erik to join a party. Due to their old rivalry (*i*) Alice replies that she will join the party if Bob, David and Erik do not; (*ii*) Bob replies that he will join the party if Alice and Erik do not; (*iii*) Carl replies that he will join the party if Alice and David do not; (*iv*) David replies that he will join the party if Bob and Carl do not; and (*v*) Erik replies that he is not sure whether he can join the party. This situation can be modeled by iAF Δ, where an argument x states that "*(the person whose names' initial is)* x *joins the party*", and argument e =*"Erik joins the party"* is uncertain. $\qquad\square$

The semantics of an iAF is given by considering all *completions*, i.e. AFs obtained by removing consistent subsets of the uncertain elements, and for each completion its σ-extensions under a given semantics σ. The completions of iAF Δ of Example 1 are the AF Λ_1 and Λ_2 shown in Figure 1, center and right-hand side respectively, where Λ_1 is the AF obtained from Δ by keeping all the arguments and attacks, while completion Λ_2 is obtained from Δ by removing the uncertain argument e and, consistently with this, attacks (e, a) and (e, b) starting from e.

Since most of the argumentation semantics defined for AF are *non-deterministic* (i.e. *multiple-status*, as they prescribe more than one extension), the notions of credulous and skeptical acceptance have been defined. Thus, given a semantics σ, an argument is *credulously accepted* if it occurs in at least one σ-extension, whereas it is *skeptically accepted* if all σ-extensions contain it. Acceptance problems have been recently extended to iAF: an argument is *possibly credulously* (resp. *skeptically*) accepted if there exists a completion where it is credulously (resp. skeptically) accepted; an argument is *necessarily credulously* (resp. *skeptically*) accepted if for all completions it is credulously (resp. skeptically) accepted.

Contributions. Our main contributions are as follows.

- We present three satisfaction problems for iAF called *determinism* (DS), *totality* (TS), and *functionality* (FS) (Section 3). Intuitively, totality tells us if an argument can be decided whatever the considered scenario (i.e. completion/world) is, while determinism tells us if we can safely make a decision in every scenario. Functionality combines totality and determinism and thus tells us if an argument can be firmly decided whatever the considered scenario is, that is if the information at hand is sufficient to make a decision in every possible scenario.

- We investigate the complexity of the problems of checking determinism, totality, and functionality for general iAF and odd-cycle free iAF (a summary of the results is reported in Table 1).

- We show that an iAF Δ can be rewritten into an "equivalent" one Δ', such that for every σ-extension E of Δ, there is a σ-extension E' of Δ' coinciding with E, modulo some meta-arguments added in the rewriting (Section 4). Particularly, Δ' can be of one of the following forms: i) only arguments or only attacks are uncertain (Δ' is said to be an *arg-iAF* or *att-iAF*, respectively); or ii) uncertainty is only on arguments that are not attacked by any other argument (Δ' is said to be an *farg-iAF*). We show that an *arg-iAF* can be translated into an *farg-iAF*. It can be shown that the results in Table 1 also hold for these restricted forms of iAFs.

- Finally, we study the relationships between iAF and PrAF and relate the acceptance problems in iAF to probabilistic acceptance in PrAF (Section 5). We derive a PrAF corresponding a given iAF and show that computing the probability of acceptance of an argument is $FP^{\#P}$-hard even restricting to acyclic iAFs.

We assume the reader is familiar with the complexity classes used in the paper [20].

2. Preliminaries

In this section we recall the (abstract) Argumentation Framework (AF) and the incomplete AF.

2.1. Argumentation Framework

An abstract *Argumentation Framework* (AF) [5] is a pair $\langle A, R \rangle$, where A is a set of *arguments* and $R \subseteq A \times A$ is a set of *attacks*.

Given an AF $\Lambda = \langle A, R \rangle$ and a set $S \subseteq A$ of arguments, an argument $a \in A$ is said to be *i) defeated* w.r.t. S iff there exists $b \in S$ such that $(b, a) \in R$, and *ii) acceptable* w.r.t. S iff for every argument $b \in A$ with $(b, a) \in R$, there is $c \in S$ such that $(c, b) \in R$. The sets of arguments defeated and acceptable w.r.t. S are as follows (where Λ is understood):

- $Def(S) = \{a \in A \mid \exists b \in S \,.\, (b, a) \in R\}$;
- $Acc(S) = \{a \in A \mid \forall b \in A \,.\, (b, a) \in R \Rightarrow b \in Def(S)\}$.

Given an AF $\langle A, R \rangle$, a set $S \subseteq A$ of arguments is said to be *conflict-free* iff $S \cap Def(S) = \emptyset$. Moreover, a set $S \subseteq A$ is said to be a *complete* (co) extension iff it is conflict-free and $S = Acc(S)$. A complete extension $S \subseteq A$ is said to be:

- *preferred* (pr) iff it is $\subseteq$-maximal;
- *stable* (st) iff it is a total preferred extension, i.e. a preferred extension such that $S \cup Def(S) = A$;
- *semi-stable* (sst) iff it is a preferred extension with a maximal set of decided elements;
- *grounded* (gr) iff it is $\subseteq$-minimal.

In the following, if not specified otherwise, σ denotes any semantics in $\{\texttt{gr}, \texttt{co}, \texttt{st}, \texttt{pr}, \texttt{sst}\}$. For any AF Λ and semantics σ, $\sigma(\Lambda)$ denotes the set of σ-extensions of Λ. All the above-mentioned semantics except the stable admit at least one extension (i.e. $\sigma(\Lambda) \neq \emptyset$), and the grounded admits exactly one extension (i.e. $|\texttt{gr}(\Lambda)| = 1$) [5, 6]. The grounded semantics is called *deterministic* (or *unique status*), whereas the other semantics are called *non-deterministic* (or *multiple status*). The stable semantics is said to be *total* as each argument belongs to either E or $Def(E)$ (i.e. it is either *true* or *false*) for every extension E. For any AF Λ, it holds that $\texttt{st}(\Lambda) \subseteq \texttt{sst}(\Lambda) \subseteq \texttt{pr}(\Lambda) \subseteq \texttt{co}(\Lambda)$, and $\texttt{gr}(\Lambda) \subseteq \texttt{co}(\Lambda)$. An AF is *acyclic* (resp. *odd-cycle free*) if the associated graph is acyclic (resp. odd-cycle free). For acyclic AFs all the considered semantics coincide.

In the following we consider the acceptability of *argument literals* (or simply *literals*). A literal is either an argument a or its negation $\neg a$. A set of literals S is said to be *consistent* if it does not contain two literals a and $\neg a$. We use $\neg S$ to denote the set $\{\neg a \mid a \in S\}$, and S^* to denote $S \cup \neg S$. To deal with literals, we define the notion of *fulfillment* of an extension. Given an AF $\Lambda = \langle A, R \rangle$ and an extension E for it, the *fulfillment* of E is $E \cup \neg Def(E)$. Clearly, the fulfillment of an extension is a consistent set of literals. Herein, arguments in E are represented as positive literals (i.e. interpreted as *true*), while arguments defeated by E are represented as negative literals (i.e. interpreted as *false*). *In the rest of the paper, with a little abuse of notation, whenever we refer to an extension we mean its fulfillment.*

For any AF $\Lambda = \langle A, R \rangle$, semantics σ, and literal $a \in A^*$, we say that a is *credulously* (resp. *skeptically*) accepted (under semantics σ), denoted as $CA_\sigma(\Lambda, a)$ (resp. $SA_\sigma(\Lambda, a)$) if a belongs to at least a (resp. every) σ-extension of Λ. We use CA_σ (resp. SA_σ), or simply CA (resp. SA) whenever σ is understood, to denote the credulous (resp. skeptical) acceptance problem, that is, the problem of deciding whether a literal is credulously (resp. skeptically) accepted. Clearly, for the grounded semantics, which has exactly one extension, these problems are identical (i.e. $\mathsf{CA}_{\texttt{gr}} \equiv \mathsf{SA}_{\texttt{gr}}$).

Example 2. Consider the AF $\Lambda = \langle \{\texttt{a}, \texttt{b}, \texttt{c}, \texttt{d}\}, \{(\texttt{a}, \texttt{b}), (\texttt{b}, \texttt{a}), (\texttt{a}, \texttt{c}), (\texttt{b}, \texttt{c}), (\texttt{c}, \texttt{d}), (\texttt{d}, \texttt{c})\} \rangle$. Λ has 4 complete extensions: $E_0 = \emptyset$, $E_1 = \{\texttt{a}, \neg \texttt{b}, \neg \texttt{c}, \texttt{d}\}$, $E_2 = \{\neg \texttt{a}, \texttt{b}, \neg \texttt{c}, \texttt{d}\}$ and $E_3 = \{\neg \texttt{c}, \texttt{d}\}$. That is, $\texttt{co}(\Lambda) = \{E_0, E_1, E_2, E_3\}$. E_0 is the grounded extension, E_1 and E_2 are stable (preferred and semi-stable) extensions. Thus, $\texttt{a}, \texttt{b}, \texttt{d}$, as well as $\neg \texttt{a}, \neg \texttt{b}, \neg \texttt{c}$, are credulously accepted for all semantics except the grounded; only $\texttt{d}$ and $\neg \texttt{c}$ are skeptically accepted for stable, preferred and semi-stable semantics. $\qquad \square$

2.2. Incomplete Argumentation Framework

We now recall the incomplete Argumentation Framework (iAF) [15].

Definition 1 (Incomplete AF). An *incomplete Argumentation Framework* (iAF) is a tuple $\Delta = \langle A, B, R, T \rangle$, where A and B are disjoint sets of arguments, and R and T are disjoint sets of attacks between arguments in $A \cup B$. Arguments in A and attacks in R are said to be *certain*, while arguments in B and attacks in T are said to be *uncertain*.

Certain arguments in A are definitely known to exist, while uncertain arguments in B are not known for sure: they may occur or may not. Analogously, certain attacks in R are definitely

known to exist if both the incident arguments exist, while for uncertain attacks in T it is not known for sure if they hold, even if both the incident arguments exist.

An iAF compactly represents alternative AF scenarios, or possible worlds, called *completions*.

Definition 2 (Completion). A *completion* for an iAF $\Delta = \langle A, B, R, T \rangle$ is an AF $\Lambda = \langle A', R' \rangle$ where $A \subseteq A' \subseteq A \cup B$ and $R \cap (A' \times A') \subseteq R' \subseteq (R \cup T) \cap (A' \times A')$.

The set of completions of Δ is denoted by $comp(\Delta)$.

An iAF $\langle A, B, R, T \rangle$ is *acyclic* (resp. *odd-cycle free*) iff the AF $\langle A \cup B, R \cup T \rangle$ is acyclic (resp. odd-cycle free).

To define acceptability and properties satisfaction of literals for iAFs, we refine the definition of fulfillment of extensions. Given an iAF $\Delta = \langle A, B, R, T \rangle$ and an extension E for a completion $\Lambda = \langle A', R' \rangle$ of Δ, the *fulfillment* of E is $E \cup \neg Def(E) \cup \neg(B \setminus A')$. Herein, arguments in E are represented as positive literals (i.e. interpreted as *true*), while arguments attacked by E as well as uncertain arguments not occurring in Λ are represented as negative literals (i.e. interpreted as *false*).

Example 3. The only preferred extension of the completion Λ_2 for the iAF Δ of Example 1 is $\{b, c\}$, whose fulfillment is $\{\neg a, b, c, \neg d, \neg e\}$. □

Acceptance problems. Credulous and skeptical acceptance for iAF have been recently proposed in [21], where the goal, i.e. the element for which acceptance is checked, is an argument. As we focus on fulfillment of extensions, our goal is a literal.

Definition 3 (Possible/Necessary Credulous/Skeptical Acceptance). Let $\Delta = \langle A, B, R, T \rangle$ be an iAF and $\sigma \in \{\text{gr}, \text{co}, \text{st}, \text{pr}, \text{sst}\}$. Then, a literal $g \in A^* \cup B^*$ is said to be:

1. *possibly credulously accepted* under σ, denoted as $PCA_\sigma(\Delta, g)$, iff there exists a completion Λ of Δ and a σ-extension E of Λ such that $g \in E$;
2. *possibly skeptically accepted* under σ, denoted as $PSA_\sigma(\Delta, g)$, iff there exists a completion Λ of Δ such that g occurs in every σ-extension of Λ;
3. *necessarily credulously accepted* under σ, denoted as $NCA_\sigma(\Delta, g)$, iff for every completion Λ of Δ, there exists a σ-extension E of Λ such that $g \in E$;
4. *necessarily skeptically accepted* under σ, denoted as $NSA_\sigma(\Delta, g)$, iff for every completion Λ of Δ, g occurs in every σ-extension of Λ.

We use PCA_σ (resp. PSA_σ, NCA_σ, NSA_σ), or simply PCA (resp. PSA, NCA, NSA) whenever σ is understood, to denote the problem of deciding acceptance according to Item 1 (resp. 2, 3, and 4) of Definition 3. For the grounded semantics we have $PCA_{\text{gr}} \equiv PSA_{\text{gr}}$ and $NCA_{\text{gr}} \equiv NSA_{\text{gr}}$.

Example 4. Consider the iAF $\Delta = \langle \{a, b, d\}, \{c\}, \{(a, b), (b, a)\}, \emptyset \rangle$ shown in Figure 2 (left), where bidirectional arrows represent mutual attacks. Δ has 2 completions: $\Lambda_1 = \langle \{a, b, d\}, \{(a, b), (b, a)\} \rangle$ and $\Lambda_2 = \langle \{a, b, c, d\}, \{(a, b), (b, a)\} \rangle$ (see Figure 2). Under semantics $\sigma \in \{\text{st}, \text{pr}, \text{sst}\}$, Λ_1 has the two extensions $E_1' = \{a, \neg b, \neg c, d\}$ and $E_1'' = \{\neg a, b, \neg c, d\}$; moreover, Λ_2 has the two extensions $E_2' = \{a, \neg b, c, d\}$ and $E_2'' = \{\neg a, b, c, d\}$. Thus, $a, b, c, d, \neg a, \neg b, \neg c$ satisfy PCA; $c, d, \neg c$ satisfy PSA; $a, b, d, \neg a, \neg b$ satisfy NCA and only d satisfies NSA. □
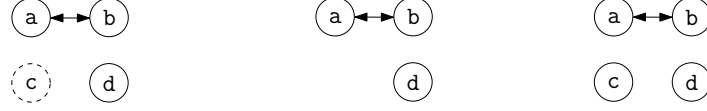
Figure 2: iAF Δ of Example 4 (left) and its completions Λ_1 (center) and Λ_2 (right).

3. Totality, Determinism and Functionality

In this section we present three satisfaction problems for iAF called *determinism* (DS), *totality* (TS), and *functionality* (FS) that have been recently proposed in [1]. Informally, an argument is said to be *deterministic* if in every completion all extensions assign the same status (either accepted, rejected, or undecided) to it. Moreover, an argument is said to be *total*, if for all extensions it is either accepted or rejected (i.e. it is never undecided). Totality is inspired by the criterion leading to stable semantics [5]. In fact, it requires the same property as the stable semantics but for a given goal argument (instead of all arguments). When both totality and determinism hold for a given argument, we say that it is *functional*. Thus, for iAF, totality tells us if an argument can be decided whatever the considered scenario (i.e. completion/world) is, while determinism tells us if we can safely make a decision in every scenario (completion). Functionality combines totality and determinism and thus tells us if an argument can be firmly decided whatever the considered scenario is, as shown in the following example.

Example 5. Continuing with Example 1, suppose the party planner is interested to know if, whatever Erik decides to do, *he has sufficient information to decide* whether or not Alice (resp. Bob) joins the party. The preferred (stable/semi-stable) extensions of Λ_1 and Λ_2 are $\{\{c, e\}, \{d, e\}\}$ and $\{\{b, c\}\}$, respectively. As a and b are functional, the party planner concludes that a decision on the fact that Alice (resp. Bob) joins or not the party can be taken in both the scenarios that Erik joins or not the party.

Note that such questions cannot be answered by possible/necessary credulous/skeptical acceptance. In fact, a is functional but not possible credulously accepted, and b is functional but not necessary skeptically accepted. $\square$

Definition 4 (Total, deterministic, functional literals). Let $\Delta = \langle A, B, R, T \rangle$ be an iAF, $\sigma \in \{\mathtt{gr}, \mathtt{co}, \mathtt{st}, \mathtt{pr}, \mathtt{sst}\}$, and $g \in A^* \cup B^*$ a goal literal. We say that g is:

- *total* under σ, denoted as $TS_\sigma(\Delta, g)$, if for each $\Lambda \in comp(\Delta)$, *i)* $\sigma(\Lambda) \neq \emptyset$ and *ii)* for each $E \in \sigma(\Lambda)$, either $g \in E$ or $\neg g \in E$;
- *deterministic* under σ, denoted as $DS_\sigma(\Delta, g)$, if for each $\Lambda \in comp(\Delta)$, *i)* $\sigma(\Lambda) \neq \emptyset$, and *ii)* either $SA_\sigma(\Lambda, g)$ or $SA_\sigma(\Lambda, \neg g)$ or $(\neg CA_\sigma(\Lambda, g) \wedge \neg CA_\sigma(\Lambda, \neg g))$;
- *functional* under σ, denoted as $FS_\sigma(\Delta, g)$, if g is total and deterministic under σ, that is, for each $\Lambda \in comp(\Delta)$, *i)* $\sigma(\Lambda) \neq \emptyset$ and *ii)* either $SA_\sigma(\Lambda, g)$ or $SA_\sigma(\Lambda, \neg g)$.

The next result immediately derives from definitions.

Fact 1. *Let Δ be an iAF, $\sigma \in \{\mathtt{gr}, \mathtt{co}, \mathtt{st}, \mathtt{pr}, \mathtt{sst}\}$ and g a literal. We have that i) $TS_\sigma(\Delta, g) \equiv TS_\sigma(\Delta, \neg g)$, ii) $DS_\sigma(\Delta, g) \equiv DS_\sigma(\Delta, \neg g)$, and iii) $FS_\sigma(\Delta, g) \equiv FS_\sigma(\Delta, \neg g)$ hold.*

Figure 3: iAF Δ of Example 6 (left), its completions Λ_1 and Λ_2 (center), and its derived PrAF Δ^p (right).

Example 6. Consider the iAF $\Delta = \langle A = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$, $B = \{\mathtt{d}\}$, $R = \{(\mathtt{a}, \mathtt{b}), (\mathtt{b}, \mathtt{a}), (\mathtt{b}, \mathtt{c})$, $(\mathtt{c}, \mathtt{c}), (\mathtt{d}, \mathtt{a}), (\mathtt{d}, \mathtt{b})\}$, $T = \emptyset \rangle$ reported in Figure 3, left side. Δ has two completions $\Lambda_1 = \langle A$, $R \setminus \{(\mathtt{d}, \mathtt{a}), (\mathtt{d}, \mathtt{b})\} \rangle$ and $\Lambda_2 = \langle A \cup B, R \rangle$, also shown in Figure 3 (center). For Λ_1 there are three complete extensions (recall that we refer to the fulfillments): $E_0 = \{\neg \mathtt{d}\}$, $E_1 = \{\mathtt{a}, \neg \mathtt{b}, \neg \mathtt{d}\}$ and $E_2 = \{\neg \mathtt{a}, \mathtt{b}, \neg \mathtt{c}, \neg \mathtt{d}\}$. The grounded extension is E_0, while E_1 is preferred, and E_2 is stable, preferred, and semi-stable. For Λ_2 there is only one complete extension $E_3 = \{\neg \mathtt{a}, \neg \mathtt{b}, \mathtt{d}\}$, which is grounded, preferred, and semi-stable. The goal $\mathtt{b}$ is i) total only for semantics $\mathtt{pr}$ and $\mathtt{sst}$ (it is not total for $\mathtt{st}$ as Λ_2 does not have stable extensions) and ii) deterministic for semantics $\mathtt{gr}, \mathtt{st}$ and $\mathtt{sst}$ (it is not deterministic for $\mathtt{pr}$ as the two preferred extensions, E_1 and E_2, of Λ_1 are such that E_1 contains $\neg \mathtt{b}$ while E_2 contains $\mathtt{b}$). Thus, $\mathtt{b}$ is functional for $\mathtt{sst}$ only. $\square$

We use TS_σ (resp. DS_σ, FS_σ), or simply FS (resp. TS, DS) whenever σ is understood, to denote the problem of deciding whether a literal is total (resp. deterministic, functional).

The next proposition provides conditions under which unattacked arguments are functional.

Proposition 1. *For any iAF $\Delta = \langle A, B, R, T \rangle$ and unattacked argument $g \in A \cup B$, it holds that g is functional i) under semantics $\sigma \in \{\mathtt{gr}, \mathtt{co}, \mathtt{pr}, \mathtt{sst}\}$, and ii) under semantics $\sigma = \mathtt{st}$ if Δ is odd-cycle free.*

Thus, under semantics $\sigma \in \{\mathtt{co}, \mathtt{gr}, \mathtt{pr}, \mathtt{sst}\}$, every unattacked argument is total and deterministic. However, this does not hold for general iAFs under stable semantics, as there could be completions prescribing no extensions.

We conclude this section by providing the complexity of the problems of deciding whether a literal is total, deterministic, and functional for the case of general and odd-cycle free iAFs.

Theorem 1. *For general/odd-cycle free iAFs, the complexity results of Table 1 hold.*

The odd-cycle free property guarantees that, except for the grounded and complete semantics, the complexity of checking determinism and functionality decreases by one level of the polynomial hierarchy whereas checking totality becomes trivial.

4. Equivalent Forms of iAFs

In this section we show that iAFs can be rewritten into equivalent iAFs where uncertainty is restricted to either attacks or (unattacked) arguments.

Definition 5. An iAF $\Delta = \langle A, B, R, T \rangle$ is said to be *argument-incomplete* (arg-iAF for short) if $T = \emptyset$, whereas it is said to be *attack-incomplete* (att-iAF for short) if $B = \emptyset$.

	(General) iAF			odd-cycle free iAF		
σ	TS	DS	FS	TS	DS	FS
gr	coNP-c	trivial	coNP-c	coNP-c	trivial	coNP-c
co	coNP-c	coNP-c	coNP-c	coNP-c	coNP-c	coNP-c
st	Π_2^p-c	Π_2^p-c	Π_2^p-c	trivial	coNP-c	coNP-c
pr	Π_2^p-c	Π_2^p-c	Π_2^p-c	trivial	coNP-c	coNP-c
sst	Π_2^p-c	Π_2^p-c	Π_2^p-c	trivial	coNP-c	coNP-c

Table 1
Complexity of totality (TS), determinism (DS), and functionality (FS) problems for iAFs. For any complexity class $\mathcal{C}$, $\mathcal{C}$-c means C-complete.

Given an iAF Δ, we denote by $arg(\Delta)$ the arg-iAF derived from Δ by replacing every uncertain attack (a, b) with the certain attacks $(a, \alpha_{ab}), (\alpha_{ab}, \beta_{ab}), (\beta_{ab}, b)$, where α_{ab} (resp. β_{ab}) is a fresh certain (resp. uncertain) argument.

Analogously, we denote by $att(\Delta)$ the att-iAF derived from Δ as follows: for each uncertain argument b, make b certain and add an uncertain attack (α, b), where α is a fresh certain argument—it is sufficient to add only one fresh argument α.

Example 7. Consider the iAF $\Delta = \langle\{\mathsf{b}, \mathsf{c}\}, \{\mathsf{a}\}, \{(\mathsf{a}, \mathsf{b}), (\mathsf{b}, \mathsf{a})\}, \{(\mathsf{b}, \mathsf{c})\}\rangle$ shown in Figure 4 (left). The arg-iAF derived from Δ is $\Delta' = \langle\{\mathsf{b}, \mathsf{c}, \alpha_{\mathsf{bc}}\}, \{\mathsf{a}, \beta_{\mathsf{bc}}\}, \{(\mathsf{a}, \mathsf{b}), (\mathsf{b}, \mathsf{a}), (\mathsf{b}, \alpha_{\mathsf{bc}}), (\alpha_{\mathsf{bc}}, \beta_{\mathsf{bc}}), (\beta_{\mathsf{bc}}, \mathsf{c})\}, \emptyset\rangle$ shown in Figure 4 (center), whereas the att-iAF derived from Δ is $\Delta'' = \langle\{\mathsf{b}, \mathsf{c}, \mathsf{a}, \alpha\}, \emptyset, \{(\mathsf{a}, \mathsf{b}), (\mathsf{b}, \mathsf{a})\}, \{(\mathsf{b}, \mathsf{c}), (\alpha, \mathsf{a})\}\rangle$ shown in Figure 4 (right). □

The transformations described above to eliminate uncertain attacks/arguments are inspired by those to eliminate attacks/arguments with probability less than 1 in probabilistic AF [22].

We now introduce a special class of arg-iAFs.

Definition 6. An arg-iAF $\Delta = \langle A, B, R, \emptyset\rangle$ is said to be *fact-uncertain* (farg-iAF) iff $\forall(a, b) \in R$, $b \notin B$.

Observe that the iAF of Example 1 is an farg-iAF.

Given an arg-iAF Δ, $farg(\Delta)$ denotes the farg-iAF derived from Δ as follows: for each uncertain argument b which is attacked in Δ, make b certain and add the attacks $(b^u, b^c), (b^c, b)$, where b^c (resp. b^u) is a fresh certain (resp. uncertain) argument. With a little abuse of notation, for any iAF Δ we use $farg(\Delta)$ to denote $farg(arg(\Delta))$.

Example 8. Considering the iAF Δ of Example 7, the derived farg-iAF $farg(\Delta)$ is shown in Figure 5. The completions of $farg(\Delta)$ are obtained by selecting all subsets of the set $\{\mathsf{a}^u, \beta_{\mathsf{bc}}^u\}$ of uncertain arguments. □

Given an iAF $\Delta = \langle A, B, R, T\rangle$, let $\varphi \in \{arg, att, farg\}$, for any $\Lambda' = \langle A', R'\rangle \in comp(\varphi(\Delta))$, $af_\Delta(\Lambda')$ denotes the AF $\Lambda'' = \langle A'', R''\rangle \in comp(\Delta)$ with:

- $A'' = A \cup ((B \cap A') \setminus \{a \mid (\alpha, a) \in R' \vee a^u \notin A'\})$, and
- $R'' = (R \cap (A'' \times A'')) \cup ((T \cap (A'' \times A'')) \setminus \{(a, b) \mid (\beta_{ab} \notin A') \vee (\beta_{ab}^c \in A' \wedge \beta_{ab}^u \notin A')\})$.
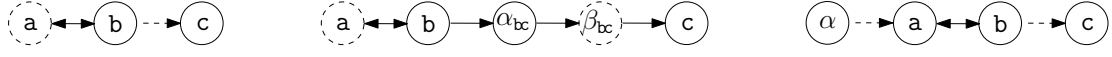
Figure 4: iAF Δ of Example 7 (left), arg-iAF Δ' derived from Δ (center), and att-iAF Δ'' derived from Δ (right).
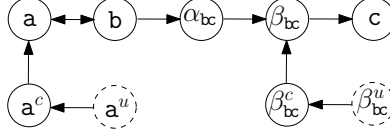

Figure 5: Farg-iAF of Example 8.

Herein, the set $\{a \mid (\alpha, a) \in R' \vee a^u \notin A'\}$ is used to avoid considering arguments either (i) attacked by α in $comp(att(\Delta))$ or (ii) always false in $comp(farg(\Delta))$ as $a^u \notin A'$. Analogously, $\{(a, b) \mid (\beta_{ab} \notin A') \vee (\beta_{ab}^c \in A' \wedge \beta_{ab}^u \notin A')\}$ is used to avoid considering uncertain attacks that are chosen to not occur in either (i) $comp(arg(\Delta))$, as $\beta_{ab} \notin A'$, or (ii) $comp(farg(\Delta))$ as $(\beta_{ab}^c \in A' \wedge \beta_{ab}^u \notin A')$.

Example 9. Consider the iAF Δ of Example 7 and its farg-iAF $farg(\Delta)$ of Example 8. For $\Lambda = \langle A \cup \{a^u\}, R \setminus \{(\beta_{bc}^u, \beta_{bc}^c)\}\rangle \in comp(farg(\Delta))$, containing the uncertain argument a^u but not β_{bc}^u, $af_\Delta(\Lambda) = \langle\{a, b, c\}, \{(a, b), (b, a)\}\rangle \in comp(\Delta)$ that contains the uncertain argument a and does not contain the uncertain attack (b, c). $\qquad\square$

Lemma 1. *For any iAF Δ and $\varphi \in \{arg, att, farg\}$, $af_\Delta : comp(\varphi(\Delta)) \to comp(\Delta)$ is a surjective function.*

The next theorem states the 'equivalence' between iAFs and the iAFs derived by applying the previous mappings.

Theorem 2. *Let $\Delta = \langle A, B, R, T\rangle$ be an iAF, $\sigma \in \{\mathtt{gr}, \mathtt{co}, \mathtt{st}, \mathtt{pr}, \mathtt{sst}\}$, and $\varphi \in \{arg, att, farg\}$. Then, $comp(\Delta) = \{af_\Delta(\Lambda) \mid \Lambda \in comp(\varphi(\Delta))\}$, and*
$\sigma(\Lambda') = \{E \cap (A \cup B) \mid \Lambda \in comp(\varphi(\Delta)) \wedge \Lambda' = af_\Delta(\Lambda) \wedge E \in \sigma(\Lambda)\} \ \forall \Lambda' \in comp(\Delta)$.

Thus, any iAF Δ is equivalent to an arg-iAF (resp. farg-iAF, att-iAF) Δ' in the sense that there is mapping between the completions of $\varphi(\Delta)$ and the completions of Δ, and for any pair of AFs for which the mapping holds, the two AFs have the same (modulo arguments added in the rewriting) set of σ-extensions, for $\sigma \in \{\mathtt{gr}, \mathtt{co}, \mathtt{st}, \mathtt{pr}, \mathtt{sst}\}$. This result entails that arg-iAFs (resp. farg-iAF, att-iAF) have the same expressivity of general iAFs, though arg-iAFs (resp. farg-iAF, att-iAF) have a simpler structure. The results of Theorem 2 with those of Theorem 1 entail that the complexity results for the problems of checking totality, determinism, and functionality for iAFs in Table 1 also hold for att-iAF, arg-iAF and f-arg-iAF.

Finally, as shown next, the restricted forms of iAF allow to establish a tight connection between iAF and Probabilistic AF, and simplify the presentation as it suffices to focus on arg-iAF and an analogous form of restricted Probabilistic AF where only arguments are uncertain.

5. Relationship with Probabilistic Acceptance

Probabilistic Argumentation Framework (PrAF) has been investigated in the recent years [18, 23, 24, 25, 26, 27, 28]. Incomplete AF is tightly connected to PrAF, as every completion of an iAF corresponds to a so-called *possible world* in PrAF. Here we highlight this relationship and relate acceptance problems in iAF, to probabilistic acceptance in PrAF.

W.l.o.g. we focus on PrAF where only arguments are uncertain (and attacks are certain, i.e. their probability is 1), since as shown in [22] a PrAF with probabilities on arguments and attacks can be transformed into an equivalent PrAF where only arguments may have probability lower than 1 (that is, the probability of arguments is greater than 0 and lower than or equal to 1, while that of attacks is 1). Here, an argument $a \in A$ is viewed as a probabilistic event that is independent from events associated with other arguments $b \in A$ (with $b \neq a$).

Analogously to what is said above, since any iAF is equivalent to an arg-iAF, w.l.o.g. in the following we consider arg-iAFs and denote them by triples $\langle A, B, R \rangle$ (i.e. omitting the empty set of uncertain attacks).

Definition 7 (PrAF). A Probabilistic Argumentation Framework (PrAF) is a triple $\langle A, R, P \rangle$ where $\langle A, R \rangle$ is an *AF*, and P is a function assigning a non-zero probability value to every argument in A, that is, $P : A \to (0, 1]$.

Observe that assigning probability equal to 0 to arguments is useless. Basically, the value assigned by P to any argument a represents the probability that a actually occurs. Moreover, every attack (a, b) occurs with conditional probability 1, that is, a attacks b whenever both a and b occur.

The meaning of a PrAF is given in terms of *possible worlds*. Given a PrAF $\nabla = \langle A, R, P \rangle$, a possible world of ∇ is an AF $\Lambda = \langle A', R' \rangle$ such that $A' \subseteq A$ and $R' = R \cap (A' \times A')$. We use $pw(\nabla)$ to denote the set of all possible worlds of ∇. An *interpretation* for a PrAF $\nabla = \langle A, R, P \rangle$ is a probability distribution function (PDF) I over the set $pw(\nabla)$ of the possible worlds. Each $\Lambda = \langle A', R' \rangle \in pw(\nabla)$ is assigned by I probability $I(\Lambda)$, where:

$$I(\Lambda) = \prod_{a \in A} P(a) \cdot \prod_{a \in A \setminus A'} (1 - P(a)).$$

We now define a PrAF Δ^p encoding an iAF Δ.

Definition 8 (Derived PrAF). Given an arg-iAF $\Delta = \langle A, B, R \rangle$, the PrAF derived from Δ is $\Delta^p = \langle A \cup B, R, P \rangle$, where $P : A \cup B \to \{1/2, 1\}$ with $P(a) = 1$ for $a \in A$ and $P(b) = 1/2$ for $b \in B$.

It is easy to check that, given an arg-iAF $\Delta = \langle A, B, R \rangle$, for every $\Lambda \in pw(\Delta^p)$, $I(\Lambda)$ is equal to either 0 or $\frac{1}{2^{|B|}}$. As stated next, non-zero probability possible worlds of derived PrAF Δ^p one-to-one correspond to completions of Δ.

Proposition 2. *For any arg-iAF Δ, $comp(\Delta) = \{\Lambda \mid \Lambda \in pw(\Delta^p) \wedge I(\Lambda) > 0\}$.*

Example 10. Consider the arg-iAF Δ of Example 6 (see Figure 3). The derived PrAF $\Delta^p = \langle A \cup B, R, P \rangle$, with $P(x) = 1$ for $x \in \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$ and $P(\mathtt{d}) = 1/2$, is shown in Figure 3 (right). There are only two possible worlds with probability greater than 0: $\Lambda_1 = \langle A, R \setminus \{(\mathtt{d}, \mathtt{a}), (\mathtt{d}, \mathtt{b})\} \rangle$ and $\Lambda_2 = \langle A \cup B, R \rangle$ with $I(\Lambda_1) = I(\Lambda_2) = 1/2$. $\square$

Given a PrAF, the *probabilistic acceptance* provides the probability that a given goal is accepted [28]. Specifically, given a PrAF ∇, a semantics σ, and a goal literal g, the probability that g is accepted can be computed by associating to every extension $E \in \sigma(\Lambda)$, with $\Lambda \in pw(\nabla)$, a probability $Pr(E, \Lambda, \sigma)$ so that $\sum_{E \in \sigma(\Lambda)} Pr(E, \Lambda, \sigma) = 1$ (the sum of the probabilities of the σ-extensions of Λ is equal to 1). More in detail, a PrAF Δ^p derived from a given iAF Δ is considered, and a PDF over the set $\sigma(\Lambda)$ of the extensions of each possible world $\Lambda \in pw(\Delta^p)$ is required. This means that the condition $\sigma(\Lambda) \neq \emptyset$ must hold. To ensure this, *in the rest of this section, if not specified otherwise, whenever we write semantics σ and (arg-)iAF Δ we mean that either i) $\sigma \in \{\text{gr}, \text{co}, \text{pr}, \text{sst}\}$ and Δ is an (arg-)iAF without restrictions or ii) $\sigma = \text{st}$ and Δ is odd-cycle free*; in the latter case, the existence of an extension is guaranteed since $\text{st}(\Lambda) = \text{sst}(\Lambda) = \text{pr}(\Lambda) \neq \emptyset$ for all $\Lambda \in pw(\Delta^p)$.

Definition 9 (Probabilistic Acceptance). Given an arg-iAF $\Delta = \langle A, B, R \rangle$ and a literal $g \in A^* \cup B^*$, the probability $PrA_\Delta^\sigma(g)$ that g is acceptable w.r.t. semantics σ is:

$$PrA_\Delta^\sigma(g) = \sum_{\substack{\Lambda \in pw(\Delta^p) \wedge \\ E \in \sigma(\Lambda) \wedge g \in E}} I(\Lambda) \cdot Pr(E, \Lambda, \sigma)$$

where $Pr(\cdot, \Lambda, \sigma)$ is a PDF over the set $\sigma(\Lambda)$.

Hereafter, we consider the uniform PDF assigning to every σ-extension of Λ the same probability ($1/n$ where n is the number of σ-extensions). Our results still hold for other PDFs over $\sigma(\Lambda)$, such as that used in [28].

Given an arg-iAF Δ and a literal g, the problem of computing the value $PrA_\Delta^\sigma(g)$ (under a given semantics σ) is denoted by PrA_σ, or simply PrA whenever σ is understood. We recall that PrA_σ is defined after choosing an arbitrary but fixed PDF over the set of extensions. As stated next, PrA_σ is FP$^{\#\text{P}}$-hard, regardless of the chosen PDF and semantics σ.

Theorem 3. PrA_σ *is FP$^{\#\text{P}}$-hard, even for acyclic arg-iAF and for any chosen PDF.*

We observe that, consistently with our intuition, the complexity of the probabilistic counterpart of the verification problem, that is computing the probability that a set of arguments is a σ-extension for a PrAF [24], is not harder than that of PrA_σ which focuses on acceptance.

The following propositions highlight the relations between iAFs and PrAFs (with uniform PDF over extensions).

Proposition 3. *For any goal g, we have that: $PCA_\sigma(\Delta, g)$ is false iff $PrA_{\Delta^p}^\sigma(g) = 0$; and $NSA_\sigma(\Delta, g)$ is true iff $PrA_{\Delta^p}^\sigma(g) = 1$.*

The connection between iAF and PrAF is also investigated in [16], where the PrAF associated to an iAF assigns a probability in $(0, 1)$ to each uncertain argument. Moreover, PCA, PSA, NCA, and NSA are related to the concept of *probabilistic credulous/skeptical acceptance* [29], which is different from that of probabilistic acceptance of Definition 9. Indeed, the probability that an argument g is credulously (resp. skeptically) accepted is the sum of the probabilities of the possible worlds where g is credulously (resp. skeptically) accepted, according to a given semantics σ. Hence, the probability of a world Λ is added to the summation iff g belongs to at

least one (resp. every) σ-extension of Λ. In contrast, with the aim of offering a more granular approach, Definition 9 uses the probabilities assigned to σ-extensions by the PDF $Pr(\cdot, \Lambda, \sigma)$. For instance, taking the PrAF Δ^p of Example 10 (see Figure 3), under the complete semantics the probabilistic skeptical (resp. credulous) acceptance of b is 0 (resp. $1/2$), while $PrA_{\Delta^p}^{co}(\text{b}) = 1/6$ as b belongs to one of three extensions of one of the two worlds (see Example 11 below). Although the conditions of Proposition 3 are similar to those identified in [16], they refer to different notions of probabilistic acceptance. In fact, while probabilistic skeptical and credulous acceptance define an interval, Definition 9 provides a precise value in that interval.

The next proposition considers acyclic arg-iAFs, a subclass of odd-cycle free iAFs.

Proposition 4. *Let* $\Delta = \langle A, B, R \rangle$ *be an acyclic arg-iAF and g a goal. It holds that:*
- $PSA_\sigma(\Delta, g) \equiv PCA_\sigma(\Delta, g)$ *and* $NSA_\sigma(\Delta, g) \equiv NCA_\sigma(\Delta, g)$;
- $PSA_\sigma(\Delta, g)$ *is true iff* $PrA_{\Delta^p}^\sigma(g) \geq \frac{1}{2^{|B|}}$;
- $NSA_\sigma(\Delta, g)$ *is false iff* $PrA_{\Delta^p}^\sigma(g) \leq 1 - \frac{1}{2^{|B|}}$.

Our last result relates the satisfaction of properties (e.g. totality) to probabilistic acceptance.

Theorem 4. *A goal g is total iff* $PrA_\Delta^\sigma(g) + PrA_\Delta^\sigma(\neg g) = 1$; *and deterministic iff* $\forall \Lambda \in comp(\Delta), \sum_\alpha Pr(E, \Lambda, \sigma) = 1$, *where either:*
$(i)\ \alpha = E \in \sigma(\Lambda) \wedge g \in E$ *or*
$(ii)\ \alpha = E \in \sigma(\Lambda) \wedge \neg g \in E$ *or*
$(iii)\ \alpha = E \in \sigma(\Lambda) \wedge (g \notin E \wedge \neg g \notin E)$.

Example 11. Consider again the iAF of Example 6. Assuming a uniform distribution for the probabilities of extensions, the probabilistic acceptance of literals is as reported in the following table, where a grey cell means that the literal of the cell's column is total, under the semantics of the cell's row.

	a	¬a	b	¬b	c	¬c	d	¬d
gr	0	1/2	0	1/2	0	0	1/2	1/2
co	1/6	2/3	1/6	2/3	0	1/6	1/2	1/2
st	0	1/2	1/2	0	0	1/2	0	1/2
pr	1/4	3/4	1/4	3/4	0	1/4	1/2	1/2
sst	0	1	1/2	1/2	0	1/2	1/2	1/2

$\square$

6. Conclusions and Future Work

We have presented the totality, determinism and functionality properties for iAFs, and provided complexity bounds for the problems of checking whether these properties hold under five well-known argumentation semantics [1]. Analogously to several other computational approaches in formal argumentation, these problems generally suffers from high computational complexity [30, 31, 32, 33, 28, 34, 35, 36, 37]. We have also identified equivalent forms of iAFs (in terms of extensions), investigated possible and necessary variants of the credulous and skeptical acceptance

problems for iAFs. We have also explored the relationship between acceptance problems in iAFs and probabilistic acceptance in PrAFs.

As future work we plan to consider other notions of acceptance that, as totality, determinism, and functionality, require the existence of an extension (e.g. skeptical acceptance under stable semantics requiring non-emptyness [38]). We envisage that SAT-based approaches for computing credulous and skeptical acceptance [39] could be used to define algorithms for checking functionality, totality and determinism since these properties can be defined in terms of credulous and skeptical acceptance for AFs. More elaborated solutions need to be investigated for iAF. Considering the connections between iAFs and Control AFs [40, 41, 42, 43], we plan to investigate totality, determinism, and functionality also in that context. Finally, it would be interesting to consider the concepts of totality, determinism, and functionality in the presence of some kinds of preferences, such as those in [10, 11, 14, 44, 45].

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