The Use of a Digital Double for Effective Control of an Object with Many Destabilizing Nonlinear Feedbacks

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Abstract

Managing a nonlinear dynamic object is a rather complex and urgent task. This task is even more complicated if there are destabilizing feedbacks in it. If such a connection is global, i.e. connects the output of an object with its input, then if there is sufficiently accurate information about the mathematical model of such an object, such a connection is most simply compensated by an external additional contour, which coincides with the specified global connection according to the mathematical model, but has the opposite sign. If the stabilizing feedback is local, then its compensation becomes significantly more complicated. An additional difficulty is such nonlinear feedbacks, which are not always positive or negative, but their sign depends on other factors, for example, on the sign of the signal at the input of the object. An example of such feedbacks is a quadratic feedback, or, for example, a feedback formed by the product of an internal quantity by its derivative. Examples of such objects have been written in the literature, but reliable evidence of the solution of the problem has not been found in these sources: either the declared successful solution of such a problem is not confirmed by independent modeling, or a simplified model of the object is used for control, free from this ambiguity of negative connection. This article discusses a similar problem without this simplification. The only known sufficiently effective method is based on local negative connections, and if such are not possible, then an equivalent option is possible only with the use of a digital double of a controlled object, which gives the method of pseudolocal feedback. This article offers a solution to such a problem, known from the literature, using the proposed method.

Keywords

PID controller, nonlinear object, modeling, optimization, digital twin, VisSim

1. Introduction

The most effective method for controlling nonlinear dynamic objects is based on numerical optimization using specialized software. However, there are examples from the literature of objects for which such a method is ineffective, it does not provide the desired solution to the problem. Despite the fact that there are reports of an effective solution to this kind of problem, for example, [1], a detailed study using modeling shows that the reliability of such a message is not confirmed, as shown, for example, in [2] and [3]. In particular, in [3] a simplified problem is solved in which, instead of the product of the signal by its derivative, the product of the signal modulus by its derivative is used, and

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instead of the square of the signal, the product of the signal by its module is used. In this case, the negative relationship is always either positive or negative, but not necessarily variable, as in the original case, according to the problem statement in the article [1]. This article describes a method for solving this problem without using the specified simplification of the mathematical model of the object.

2. Problem statement

In the article [1] it is stated that the authors have designed a robust controller for an object described by the following differential equation:

$$\ddot{y} = a_1(t)y\dot{y} - a_2(t)y^2 + b(t)u + M(t). \tag{1}$$

Here y - is the output value of the control object, u - is the input value of the object, t is the time since the beginning of the transition process, M, a1 and a2 b(t) are the parameters of the object (1). These parameters, according to article [1], can vary in the range:

$$-2 \le a_1 \le 5;$$

$$0 \le a_2(t) \le 2;$$

$$4 \le b \le 6.$$

The claim that the authors [1] managed to design a robust controller for this object is refuted by modeling, as stated in [2], [3]. In [3], a simpler problem is solved, where by introducing an output signal module instead of this signal itself in one of the two multipliers, the feedback becomes unambiguously either positive or negative. The paper solves a simpler problem, where by introducing the output signal module instead of the actual signal in one of the two multipliers [3], the feedback becomes unambiguously either positive or negative. Thus, the object model takes the following form:

$$\ddot{y} = a_1(t) \cdot |y| \cdot \dot{y} - a_2(t) \cdot \mathrm{y} \cdot |y| + b(t)u + M(t). \tag{2}$$

We will limit the task of synthesizing the regulator due to an excessively broad formulation. Controlling even a stationary object with such nonlinear feedbacks is a rather complex problem, therefore we will set fixed parameter values, exactly those at which the exponential modeling was carried out in the publication [1], namely: a1 = -5, a2 = 2, b = 1. The numerical value of the last coefficient, if it is stationary, is not significant, because whatever it is, the problem can be reduced to a problem with a single value if a common coefficient equal to the inverse value is introduced into the sequential regulator, therefore the choice of b = 1 does not simplify or complicate the task in practice, but in modeling eliminates the extra block that provides this coefficient. If the system has a single negative feedback, it is not necessary to consider the impact of the disturbance on the system, since the response to the jump of the disturbance applied at the output of the object and the response to the jump of the task at the input of the system are uniquely interrelated by a simple ratio, therefore, without loss of complexity of the task [2], we can put M = 0. Thus, the equation of the object takes the following form:

$$\ddot{y} = -2y\dot{y} - 5y^2 + u. \tag{3}$$

The transfer function of the regulator should have the following form in the area of the Laplace transform:

$$W(s) = k_{\mathrm{P}} + \frac{1}{s}k_{\mathrm{I}} + sk_D. \tag{4}$$

Here k_{P}, k_{I}, k_D – coefficients of proportional, integrating and differentiating paths, s – Laplace transform operator.

In terms of Laplace transformations, an object cannot be represented as a transfer function, since this mathematical apparatus is applied only to linear objects.

For optimization, a target (cost) function based on the integral of the product of the control error modulus by a linearly increasing signal is used:

$$F_C(T) = \int_0^T \{t|e(t)| + f[e(t)]\} dt. \quad (5)$$

Here

$$f[e(t)] = 1000 \cdot \max\{0;\ e(t)\frac{d}{dt}e(t)\}. \quad (6)$$

The additional term $f[e(t)]$is introduced to further ensure stability. This is the positive part of the product of the error by its derivative. We will not justify this cost function, because it has been too often justified and explained in our earlier publications [4–9].

The simplest control system should contain a serial PID controller [2]. The diagram for modeling and optimizing such a system in the *VisSim* program is shown in the Figure. 1. Unfortunately, this method, which is very effective for controlling nonlinear objects, in this case does not allow to obtain a successful result. Optimization is interrupted due to the fact that the output signals in this system reach unacceptably large values. In addition, even if the optimization procedure led to a successful solution of this problem, it would be successful only for the special case of using such an input signal that is used in this procedure. In the case of optimizing regulators for a linear system, any value of the amplitude of the test effect can be taken with equal success, since the system is linear, an increase in input signals causes only the same increase in all other signals in the system. In nonlinear systems, the situation is different: a system that is stable with one set of signals will not necessarily be stable with other signals.

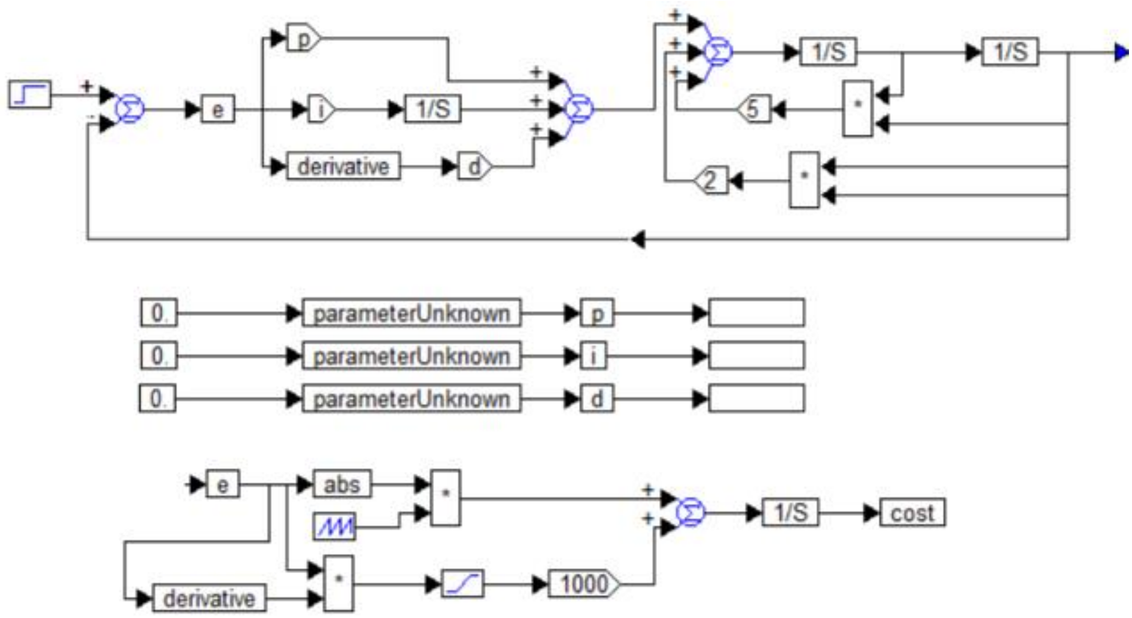

Figure 1: Scheme for modeling and optimization of a system with an object (3)

Indeed, if the test signal were taken very small, then both feedback signals would be even smaller. For example, if the input signal is reduced by introducing a small coefficient μ, then both feedback signals would decrease by a factor of $\mu 2$. This follows from the fact that the internal feedback is equal to the product of the signal y by its derivative, and the feedback signal of the external circuit is equal to the square of the value *y*. Thus, it is always possible to choose such a small value that the feedback will make a negligible contribution to the operation of the system.

Speaking about the problems of solving this problem, we should mention the biggest problem that arises if the internal feedback is negative with a sufficiently large coefficient, and the input signal also has a sufficiently large amplitude.

An additional and very significant problem is the presence of uncertainty in the feedback sign: if the feedback in any of the circuits, for example, is positive, then when the sign of the input signal changes, it turns into negative and vice versa. This follows from the rectification effect when taking the square of the signal. In the case of the product of a signal by its derivative, this product is positive if the system performs unstable movements moving away from the equilibrium state. If the system approaches an equilibrium state, then this product is negative.

Thus, the task is to create a system of effective management of the object (3), while there are no restrictions on the complexity of the structure, since with the modern development of electronic technology this is not a problem.

3. The principle of using local feedback

The block diagram of the object is shown in *Fig. 2*, where some important points are additionally indicated.

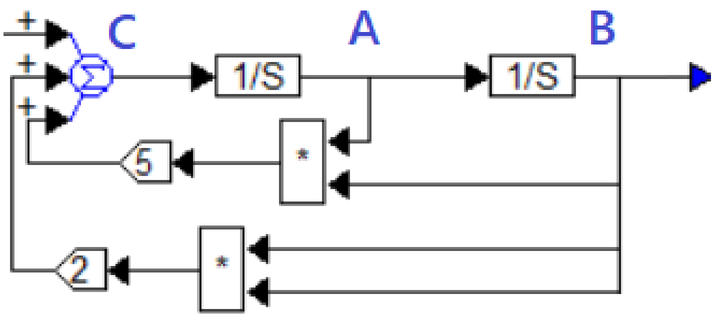

Figure 2: Object model with control points

Note that the signal at the output of the object at point *B* is available for measurement and use. An additional summing amplifier can be installed at the input of the summing element, so it can be argued that additional summing inputs are available for use at point *C*. For this reason, the effect of a global feedback that feeds the square of the output signal of an object to its input with a coefficient $a2 = 2$ can be effectively suppressed by adding an additional contour, the structure of which is identical, and the coefficient is opposite in value, but equal in magnitude, also $d2 = -2$. In this way, you can either completely compensate, that is, neutralize the effect of global feedback, or, if the specified coefficient is not known precisely enough, or it changes within small limits, then you can at least reduce the effect of this feedback to a value corresponding to the error of its estimation. For example, if the coefficient is known with an error of 1%, then the effect of such a connection can be suppressed 100 times.

If the signal from point *A* was available for measurement, it could be suppressed in exactly the same way, so the object as a whole could be made linear due to two compensating feedbacks, after which it would remain to design a regulator to control a linear object consisting of two sequentially connected integrators. Management of such an object can be carried out quite successfully. If such control would not work with a sufficiently successful result, then it would be additionally possible to introduce another local feedback so that the integrator could be covered, for example, by a proportional link with a negative coefficient. This would transform the integrator into an aperiodic link, after which it would be possible to use the method described above for controlling such an object, which is not significantly complicated.

The problem of implementing this method is the impossibility of receiving a signal from point *A*.

4. The use of a digital double and the pseudo-local feedback method based on it

If you create a digital or analog double of an object that will work in parallel and simultaneously with the object, then in this double you can access all the internal signals in it. In this copy of the object there are also points *A*, *B*, *C*. Further, compensating feedbacks can be carried out from these points, equal in magnitude, but opposite in sign. As a result, we get a composite object, the final transfer function of which is equivalent to the transfer function of the object without taking into account nonlinear feedbacks. Thus, we actually get a linear object. The control of a linear object can then be easily carried out by using a traditional PID controller, and the coefficients of this controller can be calculated by numerical optimization using the cost function (5), (6). Figure 3 shows a structure with a digital double and compensating feedbacks. In this figure, the object model is collapsed into a *Compound* block for a simpler view of the structure (the *VisSim* program allows you to apply such a graphical simplification). Figure 4 shows the complete model for optimization, where the optimization block is also shown in accordance with (5) and (6), and the digital twin of the object is also shown as a composite *Digital Twin block.*

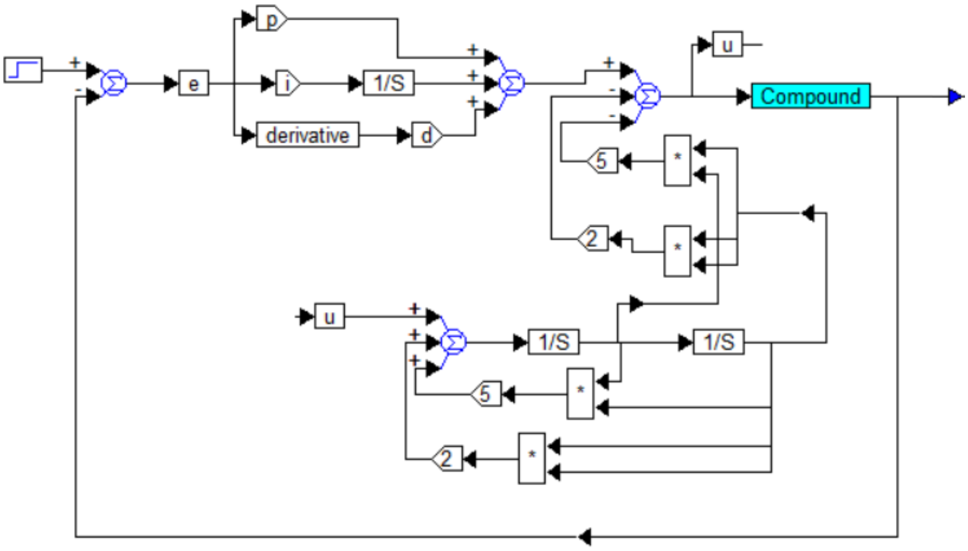

Figure 3: A model of a system with a digital double of the object and compensating feedbacks

The obtained coefficients of the regulator for this case (see Fig. 4) have the following values:

$$k_P = 6.61807 \approx 6.618,\ k_I = 4.66217 \cdot 10^{-4} \approx 0,\ k_D = 4.16013 \approx 4.16.$$

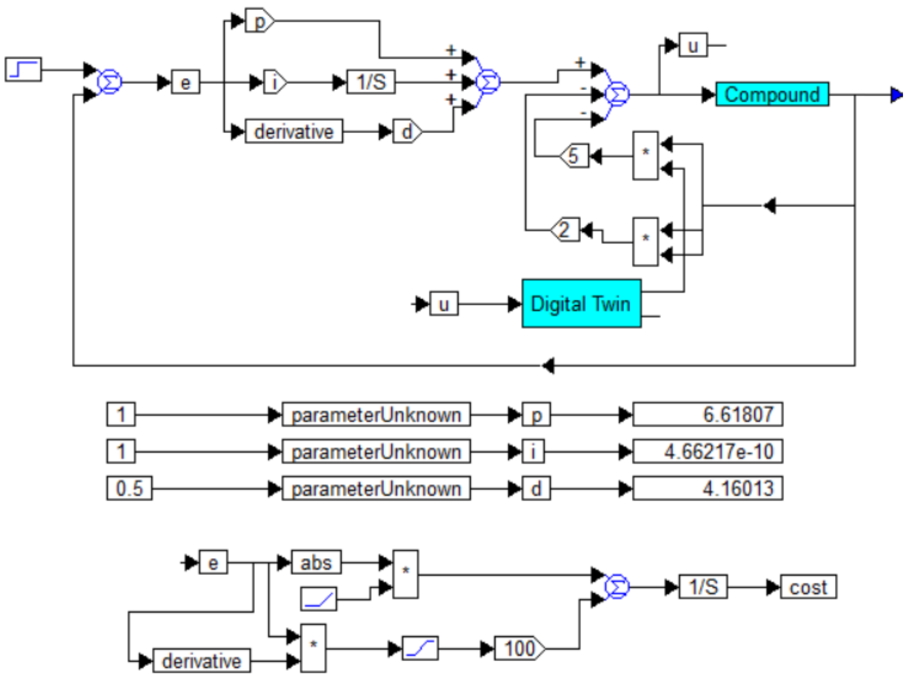

Figure 4: A model of a system with a digital double of the object and compensating feedbacks and with a block for optimizing the regulator in accordance with the cost function (5), (6)

The resulting transition process is shown in *Fig. 5*, it is close to ideal. Indeed, the duration of the process is about 3 seconds, the overshoot does not exceed 2%, the static error is zero. However, the transient process in response to a single signal variant, that is, in this case, to a single step jump, is an insufficiently complete characteristic of a nonlinear system, if the system were linear, this characteristic would be sufficient. In this case, it is necessary to build a family of transients corresponding to different amplitudes of the input effect. In addition, since the mathematical model of the object does not have symmetry with respect to the sign of the input signal, it is necessary to build not only responses to positive jumps, but also responses to negative jumps of the task. Such a family of transients is shown in *Fig. 6*. When the amplitude changes from a negative value of -1.2 to a positive value of 1.2, the transients are completely identical, they differ only in scale, which is determined by the amplitude of the input signal. The simulation showed that this property is global, also a further increase or decrease in the amplitude of the input signal does not change this pattern, the system behaves like a linear automatic control system with high control quality.

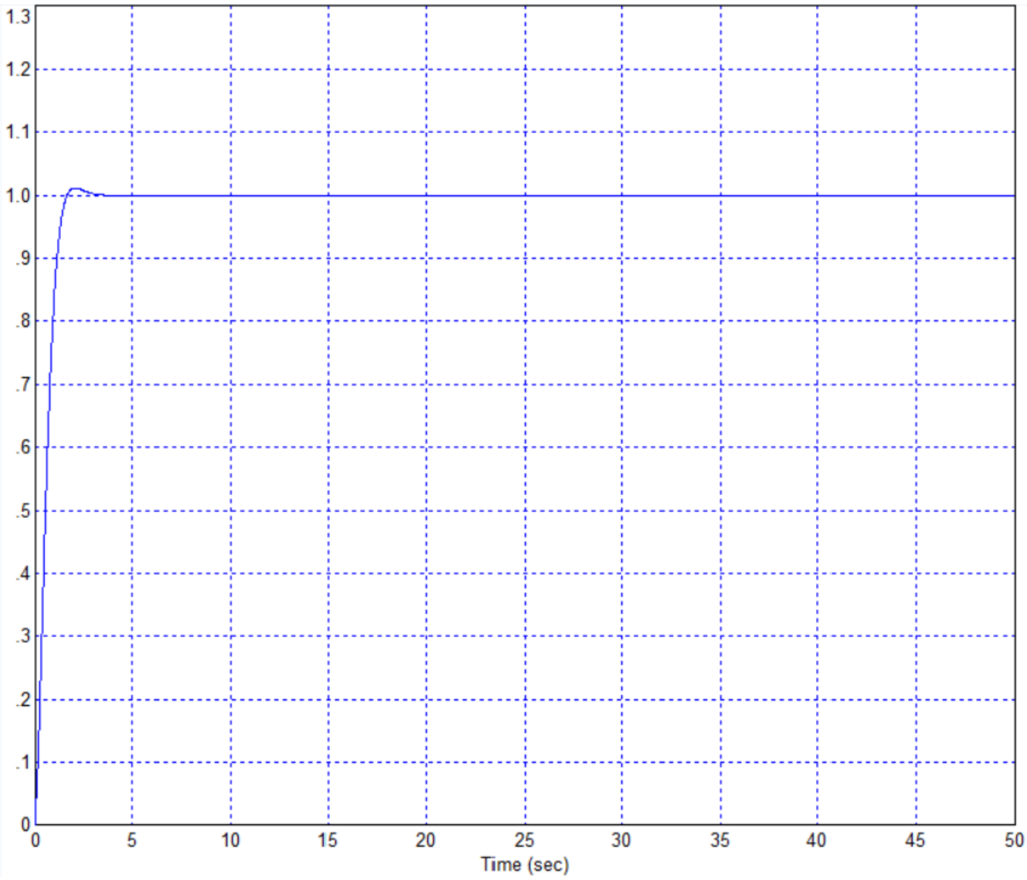

Figure 5: The resulting transition process in the system according to Fig. 4

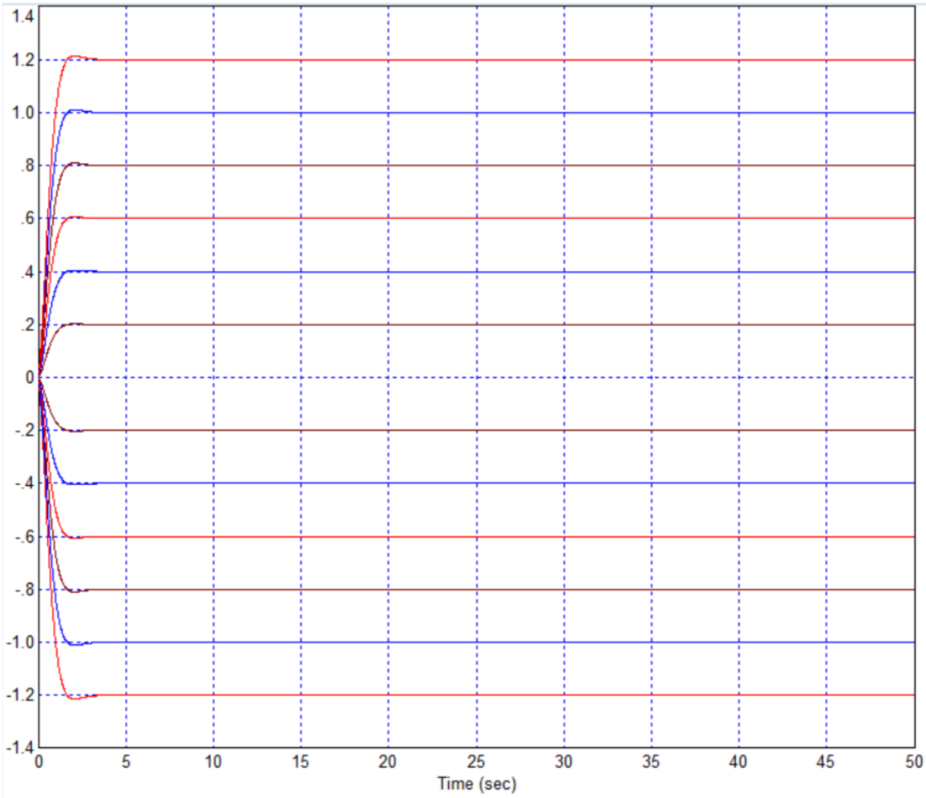

Figure 6: The resulting family of transients in the system according to Fig. 4 with an input signal amplitude from -1.2 to 1.2; the type of processes corresponds to a linear automatic control system

The article [10] shows that in systems with a nonlinear object of this kind, the following effect can occur: even with a satisfactory form of transients in the form of a response to step effects starting from zero, the system may have unsatisfactory responses in processes that are formed as a response to a step effect that ends with a zero value. This is the so-called instability of the system in the small, that is, at small values of the final steady-state value.

In the article [10], for this reason, it is recommended to use a stepwise effect as a test signal, which first increases abruptly from zero to some non-zero value, and then, when the process calms down, such an effect should return back to the zero steady value. Such test effects of different amplitudes were applied when modeling the resulting system, the resulting graphs are shown in *Fig. 7*. Transients correspond to processes in a linear system.

A check on the rudeness of the system was also carried out. For this purpose, each of the coefficients of the digital double changed by 1% in both directions, and they also changed simultaneously by 1% in different directions, both in the direction of increase and decrease. The feedback coefficients also changed from the digital double by 1%. The greatest influence was exerted by changes in the coefficient of the differentiating channel: a change in this coefficient by 1% led to a change in the transition process by 2.5%, as shown in *Fig. 8*, but at the same time the static accuracy, of course, did not change, i.e. changes were made only at the end of the dynamic component of the transition process. Changes in the remaining coefficients affect less than half or even less. Thus, the resulting system is quite crude, which is required in order for it to be implemented in practice.

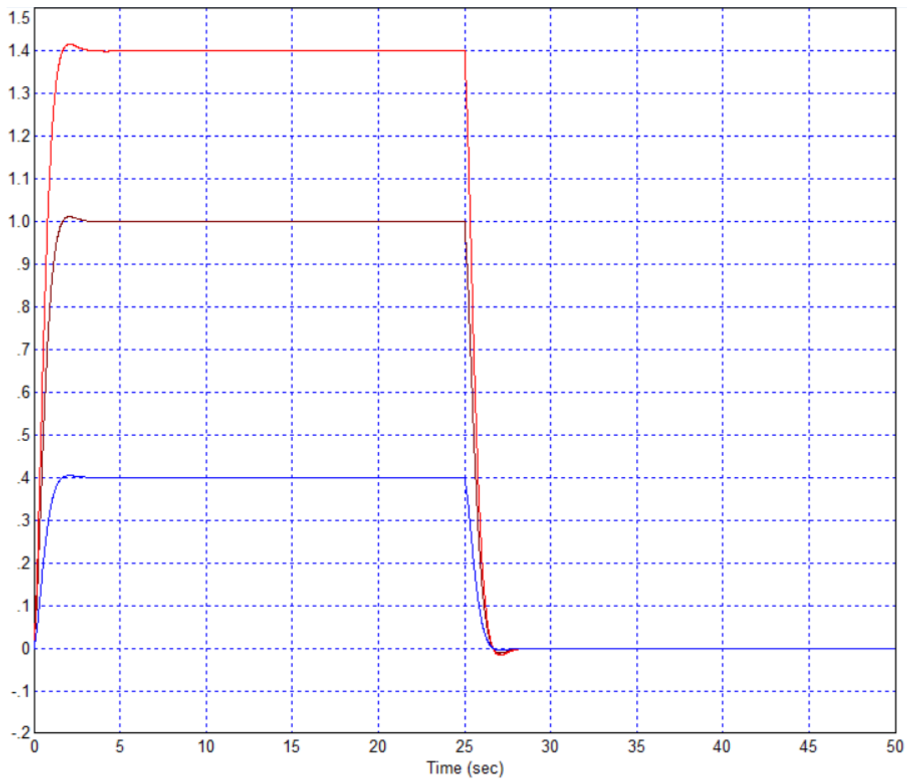

Figure 7: The resulting family of transients in the system according to Fig. 4, when given in the form of a pulse returning to the zero state, the amplitudes of the input signal varied from 0.4 to 1.4; the type of processes corresponds to a linear automatic control system

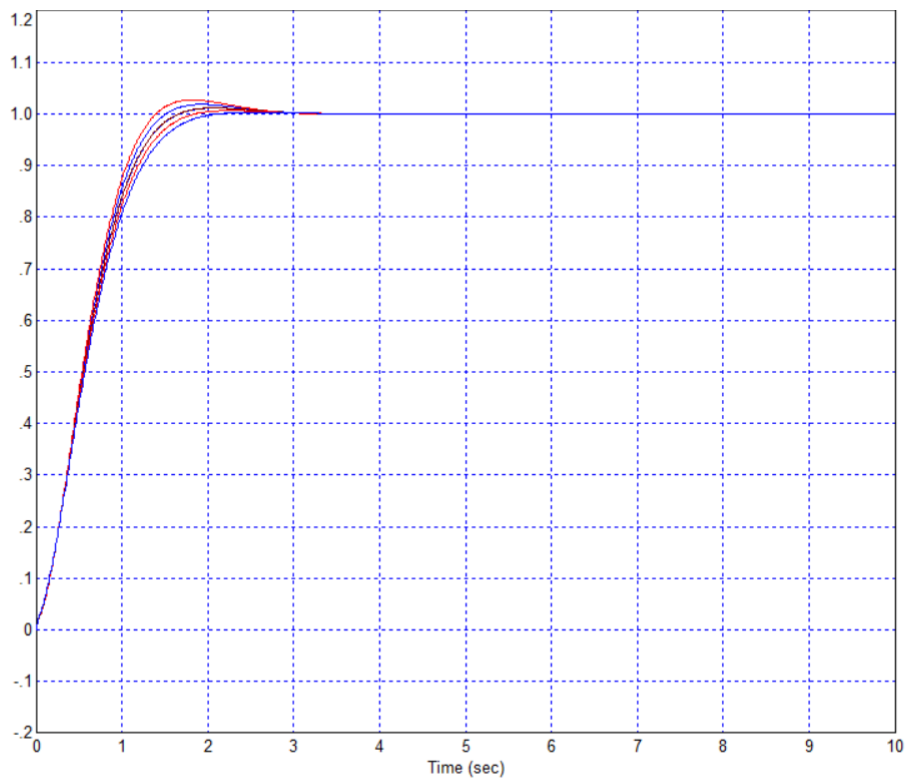

Figure 8: Changes in transients in the system according to Fig. 4 when changing the coefficient of the differentiating path of the regulator in both directions by up to 1%

5. Discussion and conclusions

In this article, the problem of controlling a substantially nonlinear object with two destabilizing feedbacks is solved. The task is complicated by the fact that these connections are not only nonlinear, but also change the sign of their effect depending on the sign of the input signal or interference, since they have the effect of rectification of this signal. Thus, if any of these connections, for example, is negative with a positive signal, then it will be positive with a negative signal, and vice versa. As you know, the concept of "negative feedback" or "positive feedback" does not indicate the sign of the signal coming through this connection, but the sign of the contribution of this signal to the interference that has arisen. Negative feedback, as a rule, for a linear system introduces a stabilizing effect, contributes to the fact that the object maintains its equilibrium state, and positive feedback, as a rule, introduces a destabilizing effect. However, in the case of nonlinear feedbacks, everything is far from so simple. In the works [2], [3], [10] it is demonstrated how difficult the task of controlling such an object is even if the feedback sign does not change, which can be done by introducing a rectifying amplifier into one of the feedback paths. This modification used in the works [2], [3], [10], it simplifies the task at least twice, because if such a modified system is stable, for example, with positive signals, it is automatically stable with negative signals, due to the symmetry of the mathematical dependence relative to the sign of the input signal. In these works there was not even an attempt to solve the problem without this simplification.

This article offers a solution to this problem without this simplification by forming pseudo-local feedbacks through the use of a digital double of the object model. As a result, the system is identical in its properties to a linear automatic control system due to the fact that pseudo-local connections make it possible to compensate for nonlinear effects. If the model of an object in a digital double differs by no more than 1% from the true model of the object, this method works quite effectively. Studies with a larger error value were not included in the task of this article. To further improve the system, pseudo-local coupling can be used not only to compensate for non-linearity, but also to cover the first part of the object model with proportional negative feedback, which further stabilizes the object. Such feedback will have the same effect as second-order differentiation.

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