Basic Theoretical Provisions of Entropy Approach for Intelligent Air Transportation Management

Andriy V. Goncharenko

National Aviation University, 1, Liubomyra Huzara Avenue, Kyiv, 03058, Ukraine

Abstract
The paper is dedicated to the elaboration of the principal theoretical provisions for the optimal governing of the intelligent air transportation management system. Subjective entropy maximum principle developed for active systems control is an initial postulate. The optimization theory of the subjective individuals’ preferences is the working hypothesis. Based upon the Jaynes’ principle of the entropy maximum the preferences functions are being found in the explicit view. Conditional optimization of the subjective preferences functions entropy, as the uncertainty measure modeling the available operational multi-alternativeness, allows organizing some rational air transportation system’s managerial efforts. Illustrative example simulation is performed. Necessary diagrams are plotted.

Keywords
Entropy, preferences, uncertainty, air transport, optimization, variation, management, concept, objective functional.

1. Introduction

Air transportation management operates in conditions with a rather uncertainty degree. On one hand there must be all necessary aircraft maintenance and repair procedures performed [1] including the most complex airplane structural parts such as the aircraft powerplants [2]. The due course airplane maintenance carried out in the due scope and due time scheduled ensures the aeronautical engineering operational reliability and decreases the risks of the probable aviation incidents and accidents happening [3, 4].

It is undoubtedly that the air transportation management chasing the goals of their business profitable running considers expected utilities of such kind of activity [5]. The complexity of this operational situation makes an impact upon the individual choice behavior [6].

The famous Jaynes’ principle of the entropy maximum [7 – 9] allows taking into account the uncertainty degree in not only purely physical systems, and the entropy based research penetrated all spheres of science [10], but the entropy paradigm is also applicable to economic activity studies [11] in the context of the subjective analysis of active systems [12].

The scientific gap of the art-on-the-day investigations contains the lack of the uncertainty degree conditional optimization problem settings that relate with the statistical estimations in the aviation radio equipment reliability parameters monitoring [13], novelty smart materials creation [14], neural network modelling targeted at the transportation challenging tasks performance prediction [15]. The entropy ideas could be applicable to the neuro-fuzzy network synthesis [16] optimization as well.

Thus, the ideology of an entropy based theoretical research stream [7 – 10, 12, 17 – 20] should be prolonged further with the help of the previously elaborated provisions of [12], which has already been successfully implemented in [12, 17 – 20].
2. Theoretical contemplations

In order to simulate functioning of some air transportation management intelligent system it is proposed to use the entropy approach [7 – 9] developed for the active systems modeling [12].

2.1. Active systems concept

Considering an intelligent air transportation management system as an active one it is proposed to compile the objective functional in the view of [7 – 9, 12, 17 – 20]:

\[ \Phi_\pi = \alpha H_\pi + \beta \varepsilon + \gamma N, \]

(1)

where \( \alpha, \beta, \) and \( \gamma \) are corresponding structure parameters that can be considered at different problems settings as the uncertain Lagrange multipliers, some coefficients or weight coefficients. Here they are interpreted as the internal intelligent air transportation management system’s parameters dealing with the intelligent artificial subject’s “estimation” of the effectiveness of the alternatives available at the operational situation. \( H_\pi \) is entropy of the alternatives preferences \( \pi; \varepsilon \) is a function of the effectiveness that together with the alternatives preferences entropy \( H_\pi \) determines conditions of the attainable alternative preferences \( \pi \) distribution optimality; \( N \) is normalizing condition.

It is important to realize that the regularities of the artificial subject’s intelligence existence and functioning have to resemble the natural intellectual properties of the individual responsible for making the managerial decisions in the considered sphere of the activity.

Therefore, the structure parameters of \( \alpha, \beta, \) and \( \gamma \) introduced in the objective intelligent air transportation management system functional (1), in some respect, could be defined as endogenous parameters reflecting certain features and properties of psych [12].

One of the active system’s peculiarities is the system’s controllability (being managed conditions) through the active element and by the active element of that active system. In particular, in the aviation and air transportation field of industry, it is the uncertainty of the operational situations that can lead to some dangerous states and conditions. That is why the active element (an individual or a subject making the responsible controlling or managerial decisions) is forced to take actions in the conditions of some multi-alternativeness and conflicts. It happens because of the necessity to act at the presence of the lack of the resources (such as, for example, the deficiency of time, limitations of the altitudes, distances etc.).

There is no doubt that safety in aviation is the top priority. Thus, it is obvious that there is great urgency, importance, and actuality in studies touching the directions related with the influence of the notorious human factor upon safety. Such problematic is important also because the stated problems solutions require expressing individuals’ subjective preferences functions with the help of some functions taken in the explicit view.

The systematic fundamental research of the multi-alternativeness in the framework of the approach connected with the concept of the problem-resource situation development has been apparently initiated in the monograph [12]. There, it has been considered mainly two adjacent problems of the general formulation; for the presented theoretical study those problems are coupled with the objective intelligent air transportation management system functional (1), [12]:

2.1.1. Conditional extremum of entropy

This formulation following [12] deals with the necessity to obtain the optimal distribution of the subjective preferences functions of the active element of the active system (individual’s subjective preferences functions) \( \pi(\sigma_i) \); at the set of the achievable for the individual’s goals alternatives of \( \sigma_i, \)

\( i = 1, \ldots, N, \)

where \( N \) is the number of the attainable for the individual’s goals alternatives; that are delivering a conditional extremum value to the entropy \( H_\pi \) of the subjective preferences functions of \( \pi(\sigma_i) \), [12]:
\( \varepsilon(\pi, U, \ldots) = \varepsilon_0; \quad P_\pi(\pi) = A_0; \quad \pi_{\text{extr}}(\cdot) = \arg \max_{\pi \in \Pi} \Phi_\pi, \) \hfill (2)

where \( \varepsilon \) is a corresponding function of the subjective effectiveness; \( U \) is a corresponding function of the subjective utility; \( \varepsilon_0 \) is “isoperimetric constraints” for the function of the subjective effectiveness of \( \varepsilon \); \( P_\pi(\pi) \) is a certain subjective “perimetric function” for the functions of the subjective preferences of \( \pi(\sigma_i) \); \( A_0 \) is “isoperimetric constraints” for the subjective “perimetric function” of \( P_\pi(\pi) \); \( \Pi \) is the class of the subjective preferences functions of \( \pi(\sigma_i) \), at which the extremal distribution of the \( \pi_{\text{extr}}(\cdot) \) functions is chosen.

### 2.1.2. Conditional extremum of effectiveness

The second formulation is also after [12]; and it is about the counter problem setting: to find the distribution of the subjective preferences functions of \( \pi(\cdot) \), delivering the conditional extremal value to the function of the subjective effectiveness of \( \varepsilon(\pi, U, \ldots) \), subject to the “isoperimetric” constraints, [12]:

\[ H_0 = H_0; \quad P_\pi(\pi) = A_0; \quad \pi_{\text{extr}}(\cdot) = \arg \max_{\pi \in \Pi} \Phi_\pi, \] \hfill (3)

where \( H_0 \) is a corresponding “isoperimetric constraints” for the subjective entropy of \( H_\pi \).

Thus, in the view of the above formulations of the expressions of (2) and (3), it is postulated in the framework of the subjective analysis [12] (the theory of subjective preferences, or the entropy paradigm of the subjective individuals’ preferences functions with respect to the available alternatives effectiveness functions, or the theory of active systems) that, for the active (in the presented case intelligent) control or management of the intelligent air transportation management system, the optimized objective functional could be constructed in the rather general form of (1), [12].

### 2.1.3. Generalization

As the kind of the variational problem generalization in regards to the objective intelligent air transportation management system functional (1), [12], the attempt has been made in the view of the integral form of the objective functional. The objective functional of the type of (1) in the sense of the problem setting of (2) could be represented for some research cases in the view of

\[ \Phi_\pi = \int_{0}^{\infty} \left( -\sum_{i=1}^{N} \pi_i(t) \ln \pi_i(t) + \beta \sum_{i=1}^{N} \pi_i(t) F_i + \gamma \left[ \sum_{i=1}^{N} \pi_i(t) - 1 \right] \right) dt, \] \hfill (4)

where \( t \) is time;

\[ -\sum_{i=1}^{N} \pi_i(t) \ln \pi_i(t), \] \hfill (5)

is entropy \( H_\pi \) of the subjective preferences functions of \( \pi_i(t) \); where \( \beta, \gamma \) are the structural parameters, analogous to those ones introduced above in the objective intelligent air transportation management system functional of (1); but here, they are already reduced by \( \alpha \); and the previously introduced designations for such parameters therefore are just deliberately kept the same for the conceptual perceptions easiness; \( F_i \) is the function of the subjective effectiveness of the \( i \)-th available alternative taken for the set of the alternatives consideration on the purpose of the subjective preferences functions entropy conditional optimization; here, these are the elements which correspond with the utility functions, but with a specific meaning related to the particular case problem setting; the final under-integral member linked with:

\[ \sum_{i=1}^{N} \pi_i(t) = 1, \] \hfill (6)

of the identity to zero value of the normalizing condition, is one more of the conventional constraints described above.
Now, the objective intelligent air transportation management system functional constructed in the integral form view of (4), comprising such under-integral members as those of the expressions represented in (5) and (6), encompasses the properties of both subjective analysis postulated form (1) and dynamical optimization characteristics through the time developing processes as it is stated with the form described by (4).

Also, some of the particular cases, for the problem settings in the dynamical framework of the variational problem, could be considering the intensive and extensive parameters optimizations; which is a constantly urgent problematic for the rational intelligent air transportation management system functioning too.

For a simplest variational problem setting, it could be taken into account such intensive and extensive parameters as, for instance, $x(t)$ and $\dot{x}(t)$.

Where

\[
\dot{x}(t) = \frac{dx}{dt},
\]  

is an intelligent air transportation management system effectiveness controlling function (the first derivative with respect to time) of the controlled parameter of $x(t)$.

In case of (4) – (7), the functions of $x(t)$ and $\dot{x}(t)$, as the functions of the subjective effectiveness of the two attainable alternatives, should have the corresponding subjective preferences functions of the active elements (subjects or individuals responsible for making the intelligent air transportation management system governing decisions): $\pi_1(t)$, $\pi_2(t)$.

As to the modifications of the objective intelligent air transportation management system functional with the elements of both intensive and extensive parameters, it could be proposed a model based upon, let us say, a combinations of the functions of $x(t)$ and $\dot{x}(t)$, for example, one of such construction might be as follows:

\[
\Phi_z = \int_{t_0}^{t_f} \left[ -\sum_{i=1}^{N} \pi_i(t) \ln \pi_i(t) + \beta [\pi_1(t)x(t) + \alpha_2 \pi_2(t)\dot{x}(t) + \alpha_3 \pi_3(t)x(t)\dot{x}(t) + \\
+ \alpha_4 \pi_4(t)\frac{\dot{x}(t)}{x(t)}] + \gamma \left[ \sum_{i=1}^{N} \pi_i(t) - 1 \right] \right] dt,
\]  

where $\alpha_2$, $\alpha_3$, and $\alpha_4$ are the coefficients that take into account the differences in the measurement units and dimensions for the corresponding elementary functions, pertaining to the achievable alternatives effectiveness functions, described with the expressions of the introduced for the consideration combinations:

\[
x(t), \quad \dot{x}(t), \quad x(t)\dot{x}(t) \quad \text{and} \quad \frac{\dot{x}(t)}{x(t)}.
\]

For the objective intelligent air transportation management system functional (8) with the elements of (9), there might be 11 particular cases of the (9) expressions combinations assessing some preferable properties.

**2.1.4. Subjective optimality postulate**

Formulation of the subjective optimality postulate is based upon the vision of the active systems governing and management [12]. In the considered problematic interpretation this is the intelligent air transportation system reasonable management.

If the controlled information income flow is processed effectively and adequately enough, then the participants of the intelligent air transportation system managerial process (the air transportation system operation, economical activity processes etc.) have a possibility, taking into their own consideration the resources (for example, technical, financial and so on) required for the given process, their own business running, to choose one or another attainable alternative at the available problem-resource situation that has been formed.
Along with this, following the theoretical contemplations and the theories of the described above ideas, traced with the expressions of (1) – (9), the active element of either artificial or natural intelligent governing system (the subject, individual, person responsible for the decision making, key-element of the active system) forms her/his own alternative preferences functions distributions for the achievable for her/his own purposes alternatives with the application of the objective functional taken in the fairly general view (1), [12].

Remarkable here is that constructing the objective intelligent air transportation management system functional in the view of, [12]:

$$
\Phi^- = -\sum_{i=1}^{N} \pi^- (\sigma_i) \ln \pi^- (\sigma_i) - \beta \sum_{i=1}^{N} \pi^- (\sigma_i) L(\sigma_i) + \gamma \sum_{i=1}^{N} \pi^- (\sigma_i) - 1,
$$

where $$\pi^- (\sigma_i)$$ is a corresponding negative subjective individual’s preference function of the responsible decision making person; $$L(\sigma_i)$$ is the corresponding function of the personally estimated losses related with the available alternatives; the individual distinguishes a certain one-sided attitude to the managerial process.

On the contrary, formulating the objective functional as, [12]:

$$
\Phi^+ = -\sum_{i=1}^{N} \pi^+ (\sigma_i) \ln \pi^+ (\sigma_i) + \beta \sum_{i=1}^{N} \pi^+ (\sigma_i) U(\sigma_i) + \gamma \sum_{i=1}^{N} \pi^+ (\sigma_i) - 1,
$$

where $$\pi^+ (\sigma_i)$$ is a corresponding positive subjective individual’s preference function of the responsible decision making person; $$U(\sigma_i)$$ is the corresponding function of the personally estimated utility related with the available alternatives; the individual distinguishes a certain second-sided attitude to the managerial process.

Both formulation of (10) and (11) that is both the negative (losses, harmfulness) and positive (utility, usefulness) estimations are possible.

And the both-sided estimated managerial process of (10) and (11) is extremized with the use of the necessary conditions for the subjective preferences functions entropy conditional optimization in the view of

$$
\frac{\partial \Phi^-}{\partial \pi^- (\sigma_i)} = 0, \quad \frac{\partial \Phi^+}{\partial \pi^+ (\sigma_i)} = 0, \quad \forall i \in \{1, N\}.
$$

For both senses of (10) and (11) the conditions of (12) yield the so-called canonical distributions of the subjective preferences functions [12].

In cases of the object (item, thing) subjective preferences these canonical distributions of the preferences functions are conventionally called: the subjective preferences functions distributions of the first kind [12].

The optimal distributions are as follows, [12]:

$$
\pi^- (\sigma_i) = \frac{e^{-\beta_L L(\sigma_i)}}{\sum_{j=1}^{N} e^{-\beta_L L(\sigma_j)}}, \quad \pi^+ (\sigma_i) = \frac{e^{\beta_U U(\sigma_i)}}{\sum_{j=1}^{N} e^{\beta_U U(\sigma_j)}},
$$

where $$\beta_L$$ and $$\gamma_L$$ are corresponding coefficients with the subscript for the objective intelligent air transportation management system functional of (10) having related with the negative sense alternatives $$\sigma_i$$ effectiveness estimations; and $$\beta_U$$ and $$\gamma_U$$ are corresponding coefficients if the subjective attention is drawn to the positive features, in generally speaking terms to the same set of the alternatives.

2.2. Simulation

An example of the calculations results with the data of

$$
\sigma = 0…100, \quad \beta_L = 0.009, \quad L_L(\sigma) = 10\sigma, \quad L_U(\sigma) = 15\sigma, \quad L_U(\sigma) = 20\sigma,
$$

is illustrated in the Figures 1 – 4.
The designations in the Figure 1 are obvious. The diagrams of the losses functions of the Figure 1 are plotted for the calculations with the first equation of (13) and by the last three equations of the data set supposed with (14).

\[ \sigma L_1(\sigma) \]
\[ \sigma L_2(\sigma) \]
\[ \sigma L_3(\sigma) \]

**Figure 1:** Example figure for losses functions

The subjective preferences functions diagrams (see the Figure 2) are plotted by the corresponding losses calculations with the first equation of (13); the designations in the Figure 2 are also easily identified and readable. The subjective preferences functions normalizing condition expressed with the equation (6) is visible in the Figure 2 too.

\[ \sigma L_1(\sigma) \]
\[ \sigma L_2(\sigma) \]
\[ \sigma L_3(\sigma) \]
\[ \sigma L_1(\sigma) + \sigma L_2(\sigma) + \sigma L_3(\sigma) \]

**Figure 2:** Example figure for preferences functions

Similarly to the diagrams shown in the Figure 2, one can notice (see the Figure 3) the equivalent character of the phase portraits (phase diagrams) of the subjective individuals’ preferences functions with respect to the other phase coordinate of the corresponding losses (harmfulness) functions. The designations used in the Figure 3 are the same as those ones of the Figures 1 and 2. Hence, there is no necessity to dwell to much on the phase curves designations; as well as the normalizing condition (6) is illustrated in the Figure 3.

At last in the Figure 4, it is represented the subjective entropy computed by formula (5). The maximal value of the subjective individuals’ preferences uncertainty degree (subjective entropy), which equals

\[ H_s = \ln(3), \quad (15) \]

for the modeled three-alternative case, is shown in the Figure 4 as well.
The simplicity of the example modeled with the formula (5), first equation of (13), and the data of (14) demonstrates the standard character of the dependencies in the abstract three-alternative case when the maximal preferences are allotted to the lowest losses.

In theoretical aspects the computational results shown in the Figures 1 – 4 are emphasizing the differences to the theoretical contemplations discussed in [12]; therefore, it might be considered a theoretical development in the understanding of the concept.

A bit different example of the calculations results with the differing data of
\[
L_1(\sigma) = 10\sigma + 600, \quad L_2(\sigma) = 15\sigma + 200, \tag{16}
\]
is illustrated in the Figures 5 – 8.

The designations in the Figures 5 – 8 are kept the same as those ones implemented for the diagrams shown in the Figures 1 – 4.
3. Discussion

The models based upon the second equation of (13) should provide greater subjective individuals’ preferences functions values to the greater values of the utility (usefulness) functions. This is logically explainable.

As a whole, the entropy approach modeled with the linear combination for the objective intelligent air transportation management system functional (1) might include a much greater number of constraints than it has been described herewith.
Moreover, the normalization conditions for “one”, introduced with relation of (6), are not so obligatory. Sometimes these conditions could be considered as the subsidiary constraints supplementing the specified problem setting for the perceptual easiness.

Endogenous parameters implemented in the presented study entropy paradigm models are presupposed to be predetermined as some external factors impact. Their influence is simplified in principle. Although, such parameters might drastically change the optimal distributions provided the parameters are different and interrelating.

The isoperimetric functions described with the expressions of (2) and (3) might ensure some extremal value (either maximum or minimum) to the objective functional or not because the solutions in the view of the canonical distributions of the subjective individuals’ preferences functions (13) are being obtain just based upon the first order (necessary) extremum existence conditions (12). In order to clarify that issue (the deliverance of either maximum or minimum to the objective functional value by the found distributions of the subjective individuals’ preferences functions) some additional research should be conducted.

The entropy member, described with the expression (5), of the objective functionals may have the same optimal “point” with the objective functionals themselves and may be not. That depends upon the specific cases modeled in different problem formulations.

As for the integral form of the objective functionals themselves, there are endless numbers of the problems stated in the framework of the calculus of variations that could be solved with the help of the Euler-Lagrange theory. However, the noted with the intensive versus extensive parameters models of equations indicated above as (7) and (8), as well as with the expressions of (9), allow conducting dynamical modeling.

The mean value of the uncertainty for the period of consideration is proposed to be estimated in the ideology of the equation of (8) mathematical statement.

The losses versus utilities dilemmas expressed with (10) and (11), as well as the simplest illustrated simulations results represented in the Figures 1 – 8, show the prospects of the entropy paradigm approach.

Nonetheless, the simplified problem setting discussed herewith is brightly highlighting the most important features: theoretical considerations for the optimization of the intelligent air transportation management system in conditions of the operational multi-alternativeness; and the caused because of the operational multi-alternativeness the operational situation of uncertainty measured with the entropy of the subjective preferences.

4. Conclusion

Computer modelling with the application of the subjective entropy paradigm implemented to an air transportation management intelligent system makes it possible to conduct a reasonably optimal governing activity based upon the theoretically substantiated provisions and results.

Aviation transport management rational organization should take into account the uncertainty of the decision making responsible individuals’ preferences related with the alternatives of the air transportation means operation.

Specifically the theoretical developments covered herein are proposed to be used in the field of transport, management, and logistics.

Further research endeavours are envisaged to be taken in the studies dealing with the necessity of the systems intellectual potential optimal growth findings.

5. References