Solving the Task of Topological Formation Intelligent Mobile «S-bots» for One «Swarm-bot» System

Gennady Krivoulya¹, Nikolay Koshevoy², Volodymyr Tokariev¹, Iryna Ilina¹, and David Dubinsky¹

¹Kharkiv National University of Radio Electronics, 14 Nauky Ave., Kharkiv, 61166, Ukraine
²National Aerospace University «Kharkov Aviation Institute», Kharkiv, 61070, Ukraine

Abstract
When controlling the movement of intelligent mobile «s-bots» that are part of one «Swarm-bot» system, unforeseen situations may arise in a physically disorganized environment. In such a case, it is necessary that the algorithm built into each «s-bot» be launched, which enables all intelligent mobile «s-bots» to rebuild their parameters and function stably in this physically disorganized environment. When such situations arise, prompt solutions are required. Solving such situations can include the task of overcoming any obstacle in a physically disorganized environment. Then, the built-in algorithm should include mechanisms for forming various topologies of intelligent mobile «s-bots». The authors of this work propose to use a mathematical apparatus, Poya’s enumeration, to solve such a problem. When solving the task of overcoming any obstacle in a physically unorganized environment, it is possible to program the «Swarm-bot» system in such a way that each variant of the formed topology for intelligent mobile «s-bots» corresponds to the shape of the obstacle.

Keywords 1
«Swarm-bot» systems, intelligent mobile «s-bot», physical unorganized environment, embedded system, embedded algorithm

1. Introduction

In the present day, intelligent mobile «s-bots» that are part of «Swarm-bot» systems have found wide usage. They play a high relevance in areas that are related to reducing the risk to human life. The advantage of using «Swarm-bot» systems is due to the fact that such systems have the properties of tunable structures and programmable logic, reconfiguration, and therefore resistance to failures. The use of «Swarm-bot» systems makes it possible to increase the radius of action of such systems, due to the increase in the number of intelligent mobile «s-bots» included in their composition, and the expanded set of tasks that can be performed significantly increases the probability of achieving the set goal. Today, «Swarm-bot» systems, which include intelligent mobile «s-bots», are capable of performing the following tasks:
- to protect the external and internal territory of the specified objects;
- for power structures in a physically disorganized environment;
- search and rescue;
- on providing assistance in the agricultural sector, and others.

Intelligent mobile «s-bots», having the properties of tunable structures and programmable logic, can more accurately determine the location of the desired target points, but today they have a drawback, which manifests itself in the low speed of detecting these ground target points. When managing...
intelligent mobile «s-bots» that are part of «Swarm-bot» systems, it is necessary to have highly specialized algorithms that are necessary for:
- motion control by intelligent mobile «s-bots», which does not take into account dynamic change in the physical unorganized environment;
- management of the formation of the topology of intelligent mobile «s-bots» while moving, provided that these intelligent mobile «s-bots» that are part of one «Swarm-bot» system have a certain status.

In practice, intelligent mobile «s-bots» move without a priori knowledge of the external physical unorganized environment and perform topology correction of the «Swarm-bot» system when solving a task or subtasks in order to achieve the set goal. The authors of this work solve the scientific problem of developing a highly specialized built-in algorithm for controlling intelligent mobile «s-bots» that are part of one «Swarm-bot» system. When solving the task, the built-in highly specialized algorithm controls the formation of the topology of intelligent mobile «s-bots» regardless of the stage of the life cycle of the entire «Swarm-bot» system. To achieve the set scientific task, it is necessary that intelligent mobile «s-bots» be able, using built-in algorithms:
- reconfigure their structures;
- rebuild your settings;
- to function in various conditions created by the physical unorganized environment [1,2].

2. Related Works

In the scientific paper «Mathematical Model for Finding Probability of Detecting Victims of Man-Made Disasters Using Distributed Computer System with Reconfigurable Structure and Programmable Logic», the authors consider issues related to the development of methods for planning the rescue stages of victims of man-made disasters, using computer distributed systems with programmable logic and reconfigurable structure. The paper examines the issues of forming a rescue «s-bot» team based on them, with the creation of control modules that perform search, task allocation and planning trajectories moving in an unorganized physical environment with obstacles. The authors solve the problem of exploring each cell in the time given for exploring the entire workspace, maximizing the probability of target detection and probabilistic characteristics of various search strategies. The authors conclude, that the solution of these tasks will allow the effective performance of rescue operations [3]. The authors of the scientific work «Algorithm of Iterations of Distribution of Subtasks Between «S-Bot» in One «Swarm-Bot» System» consider issues related to the study of the possibility of using a centralized particle swarm algorithm for the distribution of subtasks between «s-bot» and one «Swarm-bot» systems to solve the main task. On the basis of the centralized particle swarm algorithm, the authors developed a sequence of iterations, which showed that it is an effective algorithm, as it allows to find the best solution to the task much faster. It was established that the developed algorithm of iterations based on the centralized particle swarm algorithm is characterized by simplicity of operation, a small set of input parameters that need to be set at the first iteration, sufficiently acceptable accuracy, and what especially encouraged the authors is that the developed algorithm has a fast convergence to the optimal solution [4].

In the scientific work «Implementation of combined method in constructing a trajectory for structure reconfiguration of a computer system with reconstructible structure and programmable logic», the authors established that in order to neutralize threats and minimize losses caused by unusual, emergency, extraordinary and catastrophic situations, leading to the avalanche-like increase in degradation processes and the destruction of computer systems with a reconfigurable structure and programmable logic requires the development of new principles, approaches, methods and methods of operational monitoring, analysis and forecasting of situations, the development of options for management solutions, procedures for their selection and implementation within the framework of the theory of structural dynamics management. To solve the optimization task of creating scenarios for the structural reconfiguration of computer systems, the authors of the article proposed a method and an algorithm implementing it, the novelty of which consists in the combined use of the random directed search method and the method of cutting off unpromising variants of the structural reconfiguration of computer systems of the «branches and boundaries» type. The authors believe that the proposed
approach allows solving the optimization tasks of building scenarios for structural reconfiguration of computer systems [5]. The scientific publication "Coordinated Route Planning of Multiple Fuel-constrained Unmanned Aerial Systems with Recharging on an Unmanned Ground Vehicle for Mission Coverage" solves the difficult task of recharging unmanned aerial vehicles - UAVs. The scientific contribution of the authors of the article is to develop a heuristic for choosing the UAV route, without a significant increase in computation time [6].

The scientific publication "Vector Field based Control of Quadrotor UAVs for Wildfire Boundary Monitoring" solves the problem of monitoring the dynamic boundaries of forest fires using UAVs equipped with onboard cameras. The forest fire boundary is described using the zero level set function, and its change is modeled using the Hamilton-Jacobi equation [7].

In the scientific papers "A Novel Method for Distinguishing Indoor Dynamic and Static Semantic Objects Based on Deep Learning and Space Constraints in Visual-inertial SLAM" and "Bayesian Optimization-based Three-dimensional, Time-varying Environment Monitoring using an UAV" the authors consider questions interactions of the "Swarm-bot" system with a physical unorganized environment, using on-board cameras [8,9].

In the scientific work "Distributed Fault Estimation and Fixed-Time Fault-Tolerant Formation Control for Multi-UAVs subject to Sensor Faults", the authors consider the problems of reconfiguration and fault tolerance that may arise during the operation of unmanned aerial vehicles - UAVs [10]. In the scientific papers "Deep Learning for Safe Autonomous Driving: Current Challenges and Future Directions" and "T-GCN: A Temporal Graph Convolutional Network for Traffic Prediction", the authors use deep learning methods and graph theory to solve the “Swarm-bot” traffic optimization problem in system in a physical unorganized environment [11,12].

In the scientific work “Multi-fidelity black-box optimization for time-optimal quadrotor maneuvers”, the authors explore the problem of multi-point optimization as a black box for solving the problem of achieving time-optimal maneuvers of unmanned aerial vehicles - UAVs [13]. In the scientific papers "Kimera: From SLAM to spatial perception with 3D dynamic scene graphs" and "Lane Detection Method with Impulse Radio Ultra-Wideband Radar and Metal Lane Reflectors", the authors explore the problem of the interaction of the "Swarm-bot" system with a physical unorganized environment. [14,15]. In the scientific work "Assistive Robotic Technologies for Next-Generation Smart Wheelchairs", the authors explore the development trends of next-generation robotic technologies [16]. The scientific work "Resilient Trajectory Propagation in Multirobot Networks" explores the interaction between "s-bots" of one "Swarm-bot" system [17]. In the scientific work "Multi-UAV planning for cooperative wildfire coverage and tracking with quality-of-service guarantees", the authors investigate the problem of achieving the target functionality when using several unmanned aerial vehicles - UAVs [18]. In the scientific work "Modified Gray Codes for the Value (Time) Optimization of a Multifactor Experiment Plans", the authors explore the possibility of using the modified Gray code to optimize the cost (time) of plans when conducting multifactorial experiments [19]. In the scientific papers "Artificial intelligence in the Internet of things" and "Modelling and verification of reconfigurable multi-agent systems", the authors study models for reconfiguring multi-agent systems using artificial intelligence [20,21]. In the scientific work "Automatic calibration of dynamic and heterogeneous parameters in agent-based models", the authors study heterogeneous parameters in agent-based models during the interaction of the "Swarm-bot" system with a physical unorganized environment [22]. In the scientific papers "Guest Editorial: Special issue on recent developments in advanced mechatronics systems" and “Passive robust control for uncertain Hamiltonian systems by using operator theory”, the authors present material on the latest developments in the field of advanced mechatronics systems and explore models passive robust control for indefinite Hamiltonian systems using operator theory [23,24]. In scientific papers "Design, Testing, and Evolution of Mars Rover Testbeds: European Space Agency Planetary Exploration" and "UV-C Mobile Robots with Optimized Path Planning: Algorithm Design and On-Field Measurements to Improve Surface Disinfection Against SARS-CoV-2 » the authors investigated optimization models with path planning for mobile robots and described the evolution of the rover test benches [25,26]. In the materials of the conferences "IEEE Proceedings International Symposium on Multi-Robot and Multi-Agent Systems", a team of authors presents research on the interaction between intelligent mobile "s-bots" of one "Swarm-bot" system. Algorithms for joint multi-agent pathfinding and collision avoidance are described [27-29].
3. Methods

When controlling the movement of intelligent mobile “s-bots” that are part of one «Swarm-bot» system, unforeseen situations may arise in a physically disorganized environment. In such a case, it is necessary that an algorithm built into each «s-bot» be launched, which enables all intelligent mobile «s-bots» to rebuild their parameters and function stably in this physically disorganized environment. When such situations arise, prompt solutions are required. Solving such situations can include the task of overcoming any obstacle in a physically disorganized environment. Then, the built-in algorithm should include mechanisms for forming various topologies of intelligent mobile «s-bots». The authors of this paper propose to use the mathematical apparatus, the theory of Poya's enumeration, to solve such a problem. When solving the task of overcoming any obstacle in a physically unorganized environment, it is possible to program the «Swarm-bot» system in such a way that each variant of the formed topology for intelligent mobile «s-bots» corresponds to the shape of the obstacle. Let's consider the mathematical apparatus of Poya's enumeration theory. A group is a non-empty set G together with a binary operation (*), which combines any two elements a and b and forms a second element, which can be denoted as (a * b) or simply ab. To be identified as a group (G, *), four requirements, known as group axioms, must be met:

- the first requirement is closure:
  \[ \forall a, \forall b \in G, a * b \in G, \] (1)

- the second requirement is associativity:
  \[ \forall a, \forall b, \forall c \in G, (a * b) * c = a * (b * c), \] (2)

- the third requirement is the presence of a neutral element:
  \[ \forall a \in G, \exists e \in G, e * a = a * e = a, \] (3)

- the fourth requirement is the presence of an inverse element:
  \[ \forall a^{-1} \in G, a * a^{-1} = a^{-1} * a = e, \] (4)

Then suppose that a finite set T consists of n elements. The set of all one-to-one mappings of a set T onto itself is called the symmetric group Tₙ of degree n. Therefore, each such mapping is called a permutation, and the order of the symmetric group is equal to the number of permutations of elements, that is, |Tₙ| = n!

Let A be a permutation group acting on a set T. Those permutations in A leave the given element t in T fixed and form a subgroup of the group A. Then we obtain the number of orbits D determined by the permutation group A [30]:

\[ D = \frac{1}{|A|} \sum_{a \in A} J_1(a) \] (5)

where J₁(a) - is the number of elements fixed by substitution a. Finally, the cyclic index K(A) of the permutation group A is a polynomial in the variables t₁, t₂, t₃,...,tₙ, as defined by:

\[ K(A) = \frac{1}{|A|} \sum_{a \in A} t_1^{J_1(a)} t_2^{J_2(a)} t_3^{J_3(a)} ... t_n^{J_n(a)} = \frac{1}{|A|} \sum_{a \in A} \prod_{k=1}^{n} t_k^{J_k(a)}, \] (6)

Substituting the function c(x,y) into K(A) replaces each tᵢ with c(xᵢ, yᵢ), then for configurations the enumeration series is obtained by substituting the enumeration series for figures into the cycle index of the group of configurations:

\[ C(x, y) = K(A, c(x, y)) = \frac{1}{|A|} \sum_{a \in A} \prod_{k=1}^{n} (c(x^k, y^k))^{J_k(a)}, \] (7)

By virtue of expressions (6) and (7), the enumerating series of configurations is obtained from the cyclic index of some permutation group. Let us now turn to the enumeration series for the figures. Undoubtedly, the more common enumeration series for figures is:

\[ c(x) = 1 + x, \] (8)

Our main task is to determine the value of the function K(A, 1+x) for an arbitrary permutation group A and apply this function to individual groups. Next, we will perform the formation of a set of formation structures. Let us introduce the following notation. Consider intelligent mobile «s-bots» - in the amount of D objects that are part of one «Swarm-bot» system, designated by the vertex M = {1, 2,...,D}. Then, using graph theory, we get:

\[ G = (M, B), \] (9)
where M is the top of the graph G;
B is an edge of G.
Then edge B of graph G:
\[(i, j) \in B(i, j \in M \text{ and } i \neq j),\]
describes the relationship between intelligent mobile «s-bots» that are part of one «Swarm-bot» system.
The permutation group at the vertices of a graph G of order n is precisely the symmetric group \(T_n\). This group induces a permutation group acting on edges in a natural way, which the authors propose to denote as \(T_n^{(2)}\). Thus, different graphs are represented by different equivalence classes under the action of \(T_n\). Therefore, from (7) we obtain an enumerating polynomial for graphs with n vertices:
\[g_n(x) = K(T_n^{(2)}, 1 + x).\]

4. Experiment

For example, the «Swarm-bot» system includes three intelligent mobile «s-bots», which have some kind of status in relation to each other. According to (9):
\[G = (M, B),\]
we get:
\[M = \{1, 2, 3\},\]
\[B = \{(1, 2), (1, 3), (2, 3)\}.

Let the permutation group \(T_3\) for vertices be \(M\) and the permutation group \(T_3^{(2)}\) be induced by the permutations, then, for each:
\[\alpha \in T_3,\]
exists:
\[\alpha' \in T_3^{(2)},\]
such that:
\[\alpha'\{i, j\} = \{\alpha_i, \alpha_j\},\]
then we find the structure of the cycle \(S_3:\)
- firstly:
\[\alpha_0 = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix} = (1)(2)(3).\]
\[\alpha_1 = \begin{pmatrix} 2 & 3 & 1 \\ 1 & 2 & 3 \end{pmatrix} = (123).\]
\[\alpha_2 = \begin{pmatrix} 3 & 1 & 2 \\ 3 & 1 & 2 \end{pmatrix} = (132).\]
\[\alpha_3 = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix} = (1)(23).\]
\[\alpha_4 = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix} = (13)(2).\]
\[\alpha_5 = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix} = (12)(3).\]
Next, we find the structure of the \(T_3^{(2)}\) cycle:
- secondly:
\[\alpha_0'\{1, 2\} = \{1, 2\}.\]
\[\alpha_0'\{1, 3\} = \{1, 3\}.\]
\[\alpha_0'\{2, 3\} = \{2, 3\}.\]
\[\alpha_0'\{1, 3\} = \{1, 3\}.\]
then:
\[\alpha'_0 = \begin{pmatrix} 12 & 13 & 23 \\ 12 & 13 & 23 \end{pmatrix} = (12)(13)(23).\]
\[\alpha'_1 = \begin{pmatrix} 23 & 12 & 13 \\ 12 & 13 & 23 \end{pmatrix} = (12 23 13).\]
\[\alpha'_2 = \begin{pmatrix} 13 & 23 & 12 \\ 12 & 13 & 23 \end{pmatrix} = (12 13 23).\]
\[\alpha'_3 = \begin{pmatrix} 12 & 13 & 23 \\ 13 & 12 & 23 \end{pmatrix} = (12 13)(23).\]
\[\alpha'_4 = \begin{pmatrix} 12 & 13 & 23 \\ 23 & 13 & 12 \end{pmatrix} = (1223)(13).\]

\[\alpha'_5 = \begin{pmatrix} 12 & 13 & 23 \\ 12 & 23 & 13 \end{pmatrix} = (12)(1323).\]

The cycle index \(K(T_3)\) of the permutation group \(T\) is the average value:

\[\prod_{k=1}^{n} \alpha_k^{J_k(T)},\] throughout the permutation in the group. Therefore, the cycle indices of the symmetric group \(T_3\) and the pair group \(T_3^{(2)}\) are obtained from Table 1:

\[K(T_3) = K\left(T_3^{(2)}\right) = \frac{1}{6} (\alpha_1^3 + 2\alpha_3 + 3\alpha_1\alpha_2),\] applying formula (7) we get the graphs:

\[g_n(x) = \frac{1}{6} \left[(1 + x)^3 + 2(1 + x)^3 + 3(1 + x)(1 + x^2)\right] = 1 + x + x^2 + x^3,\]

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5. Results

As it was said above, if the «Swarm-bot» system includes three intelligent mobile «s-bots», then four topologies of intelligent mobile «s-bots» are possible. Each intelligent mobile «s-bot» in the «Swarm-bot» system has a certain status in relation to the other «s-bot». Then we can confirm Figure 1 in the graph \(G = (A, B)\) there is no parameter \((B)\), since every intelligent mobile «s-bot» in the «Swarm-bot» system has an equal status.

![Figure 1](image)

**Figure 1**: The topology of intelligent mobile "s-bot" a), b) and c) is presented as part of one "Swarm-bot" system. With such a topology, each "s-bot" has an equal status and is the leader, so the parameter \((B)\) from the formula for describing the graph \(G = (A, B)\) is absent.
Figure 2: The "Swarm-bot" system includes two intelligent mobile "s-bot" a) and b) whose status is higher (they are leading) in relation to the third "s-bot" c), but equal in relation to each other, so the parameter (B) from the graph description formula \( G = (A, B) \) is present as a single edge of the graph.

Figure 3: The "Swarm-bot" system has one intelligent mobile "s-bot" c), which has a higher status (it is the leader) in relation to the other two "s-bots" a) and b), but the statuses "s-bots" a) and b) are equal to each other, therefore the parameter (B) from the graph description formula \( G = (A, B) \) is present as two graph edges.

Figure 4: The "Swarm-bot" system has one intelligent mobile "s-bot" c), which has a higher status (it is the leader) in relation to the other two "s-bots" a) and b), but the statuses "s-bots" a) and b) are not equal to each other, therefore the parameter (B) from the graph description formula \( G = (A, B) \) is present in the form of three graph edges.

6. Discussions

After analyzing the data in Table 1, we can say that four topologies of intelligent mobile «s-bots» are possible. Each intelligent mobile «s-bot» in the «Swarm-bot» system has some status in relation to another «s-bot». Then we can assert:
- first, the presented topology in Figure 1, there are no edges in the graph, since each intelligent mobile «s-bot» in the «Swarm-bot» system has an equal status;
- second, the topology shown in Figure 2, the Swarm-bot system has two intelligent mobile «s-bots», which have a higher status than the third «s-bot», i.e. there is one graph edge;
- third, the topology shown in Figure 3, the «Swarm-bot» system contains one intelligent mobile «s-bot», which has a higher status in relation to the other two «s-bots», i.e. there are two edges of the graph;
- fourth, the topology shown in Figure 4 in the «Swarm-bot» system has one intelligent mobile «s-bot», which has a higher status in relation to the other two «s-bots», i.e. there are three edges of the graph.

The authors of this work, when solving the problem of overcoming some kind of obstacle in a physical unorganized environment, used a mathematical apparatus, the Poya enumeration theorium. The solution of such a problem showed that when overcoming some kind of obstacle in a physical unorganized environment, it is possible to program the «Swarm-bot» system in such a way that each variant of the formed topology for intelligent mobile «s-bots» correlates with the shape of the obstacle. The results obtained convinced the authors that the application of the Poya enumeration theory was fully justified. It is supposed to continue research in this direction and conduct a comparative analysis of algorithms to eliminate the disadvantage associated with the low speed of detection and location of ground target points. Based on the results of the analysis, develop recommendations on the appropriateness of using one or another algorithm in Swarm-bot systems.

7. Conclusions

In this work, a scientific problem was posed and successfully solved, which may arise when controlling movement in a physical unorganized environment by intelligent mobile «s-bots» that are part of one «Swarm-bot» system. In this case, it is necessary that the algorithm built into each «s-bot» be launched, which enables all intelligent mobile «s-bots» to rebuild their parameters and function stably in this physical unorganized environment. When such situations arise, they require a prompt solution. The solution of such situations can be attributed to the problem of overcoming any obstacles in a physical unorganized environment. Then, the built-in algorithm should include mechanisms for generating various options for the topology of intelligent mobile «s-bots».

The authors of this paper used a mathematical apparatus, the Poia enumeration, to solve this problem. The solution of such a task showed that when overcoming any obstacle in a physically unorganized environment, it is possible to program the «Swarm-bot» system in such a way that each variant of the formed topology for intelligent mobile «s-bots» corresponds to the shape of the obstacle. The obtained results convinced the authors that the application of Poya's enumeration theory was fully justified.

It is proposed to continue research in this direction and conduct a comparative analysis of algorithms to eliminate the shortcoming associated with the low speed of detecting and determining the location of ground target points. Based on the results of the analysis, develop recommendations on the expediency of using one or another algorithm in «Swarm-bot» systems.

8. References


