# Improving Accuracy by Ensuring Invariance of Two-Dimensional Binary Images in Intelligent Systems 

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#### Abstract

Technical vision systems in intelligent mobile robots directly increase productivity and visibly reduce costs related to the quality error factor by providing flexible-automated control over the quality of the manufactured product. However, when the system recognizes the images of the object, difficulties arise due to the linear movement of the image (rotation of the image around the center of gravity and displacement in the coordinate plane). Such linear displacements cause methodological errors in the estimation of proximity measures between reference and recognizable objects. Since such destabilizing factors reduce the reliability of image recognition, it is important to overcome the issue of invariance to linear displacements of images. In the proposed algorithm, the contour points of the two-dimensional binary images of objects at the output of the vision system are displayed as coordinates on the Cartesian coordinate plane of the screen. The values of the indicated coordinates are not invariant to the linear displacement and rotation of the image. Therefore, in order to correctly recognize such images, it is necessary to ensure that the image points are invariant with displacement and rotation. In order for the image to be invariant to orthogonal displacement, the coordinate system must be moved to the center of gravity of the defined image. Then, the rotation angle of the reference object relative to the initial position is determined by the moments of inertia relative to the coordinate axes of the image. After the rotation angle of the image is estimated, the coordinates of the contour points are found by rotating the reference image in the computer memory by this angle. Then the coordinates of the contour points of the current image are compared with the coordinates of the contour points of the rotated reference image. This comparison provides accurate information on whether the current image is the same or different from the reference image. Thus, the proposed algorithm allows invariant recognition of twodimensional binary images. The higher the level of invariance, the lower the average risk ratio. The proposed algorithm was simulated on a computer and positive results were obtained.


## Keywords

Pattern recognition, invariance, linear displacement, image rotation, vision system, average risk

## 1. Introduction

Depending on the actual problems in technical vision systems of digital industry and intelligent robots, there are problems of perception and recognition of object regularities, as well as technological processes for processing and preparation of relevant data. The effectiveness and efficiency of the systems directly depends on the reliability of the accurate performance of the measurement process, the compatibility of the parameters of the recognition and reference images, and the adoption of the necessary decision with the obtained results related to pattern recognition. Such processes occur due to

[^0]the presence of destabilizing factors, because due to the influence of these factors, errors occur in the adjustment of measurement and imaging parameters [1,2]. It is precisely such errors that make it difficult for intelligent robots to make the right decisions. The decision-making of robot complexes has an influence of the degree of proximity between the recognized and reference images included in the database in image recognition. In modern industry, technical vision systems have become an alternative to the human factor in performing visual or manual quality control operations of products. Therefore, robotics companies are committed to increasing productivity and reducing costs associated with quality errors that can occur during human supervision. Basically, when recognizing images of objects in recognition systems, certain difficulties arise due to their linear displacement (rotation of the center of gravity of the image or change of its position in the coordinate plane). Thus, important problems arise when the manufactured product rotates on the conveyor line or changes its direction after production. Obviously, determining the spatial orientation of objects in an industrial setting is both a complex and economically expensive task. These factors cause changes in the number of products, absolute values of coordinate parameters, random measurement error in the values of object parameters. These and other unstable factors reduce the accuracy of object recognition, so the recognition system should be invariant to changes in the object's position [3,4].

Various methods and tools have been proposed in various literatures to ensure the invariance of the object's rotation around the center of gravity and the change of the object's position. However, these existing methods cannot provide the most accurate invariance in image recognition. In these works, in order to ensure the invariance of the linear displacement in the object images, the main attention is paid to the static moments, which are considered their main feature. The analysis of these methods showed that in this case the symbols are very complex and therefore the reliability is low. Therefore, research aimed at finding the best methods and tools to achieve image recognition variability in technical vision systems remains relevant [5,6,7].

## 2. Problem statement

During the recognition of images of objects in robotic complexes, certain difficulties arise due to the rotation, displacement and scaling of the image around the center of gravity. Such problems lead to the loss of information about the number of objects, their location and the absolute value of the proximity measure between objects, and random errors in the calculation of values. Since such destabilizing factors reduce the reliability of image recognition, it is important to overcome the issue of invariance to linear displacements of images $[8,9]$.

Various methodologies and tools have been proposed to ensure the invariance of the rotation of object images around the center of gravity and large-scale changes in the image. However, these methods and techniques cannot provide the most accurate invariance in object recognition. Therefore, research aimed at finding the best methodologies and tools to achieve invariance in image recognition remains relevant $[10,11]$.

## 3. Comparative analysis

In order to clarify the solution to the problem of image detection and measurement of its parameters, the methods of recognition in the applied system were analyzed. Unlike all the methods in the table, more attention was paid to its geometric features. There are several ways to achieve sensitivity in transformations in the field of recognition systems, in particular, two groups of approaches can be distinguished among the most commonly applied transformation transitions. The methods of the first group include spatially insensitive properties (for example, the method of moments, the Fourier method of images). On the other hand, those who adopt an alternative approach work with object models and try to combine the objects observed and used in training by choosing parameters [12,13,14].

The method based on the analysis of the amplitudes of the individual harmonics of the Fourier spectrum of images has a number of advantages, such as a small number of important features, an unambiguous relationship between the rotation of the image or the corresponding rotation, the scale of the spectrum. The harmonic shift of the spectrum can be used to measure the corresponding shift in the image [ $15,16,17$ ].

Mellin and Fourier-Mellin transforms also reduce the number of features like Fourier transforms, which simplifies the recognition scheme. The double-scale invariance of the latest method allows to stabilize the verticality of the statistical characteristics of the measurement images based on them, thereby increasing the accuracy of the measurement. These methods are performed only in coherent optical systems and require fairly sophisticated image analyzers to achieve some degree of invariance [18,19,20].

The secant method is used to recognize images that are large enough to be contoured using segments or straight sequences. For this method, it is not enough to draw the contours of the images, but also to divide the angular area of the image into segments that can contain several or more objects. The most important condition when using this method is the stability of the visible shape of the object [21,22,23].

Table 1.
Mutual analysis of the methods used

| The method of obtaining the main features of the image |  | Fourier transform (analysis of harmonic Fourier spectrum) | Transfo rmatio n of Mellina | FourierMellina transforma tion | Secant method | Geometric moments method | Method of geometric moments of spacefrequency spectra of the image | Optical correlation method |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Early signs |  | Harmonic amplitude | Harmo nic amplitu de | Harmonic amplitude | Distribution <br> of secant lengths and angles between them | Geometric moments of the image | Geometric moments of the individual harmonic Fourier transform of the image | The basic maximum of the correlation function and its position |
|  | Displace ment | + (+/-) | - | + (+/-) | + | + (+/-) | + (+/-) | + (+/-) |
|  | Rotation | - | - | - | +( length) <br> - (corners) | + (+/-) | + (+/-) | + (+/-) |
|  | To change the scale | - | + | + | + (corners) <br> - (length) | + (+/-) | + (+/-) | + (+/-) |

After comparing different algorithms and schemes for solving the problems of recognition and identification of known objects, it can be concluded that among the most promising schemes are the characteristics of the object determined (controlled) by synchronous detection of the center of the image and the geometric moments of its image. its Fourier transform is used, or one of the methods of determining the position of the main maximum of the correlation function of an object description, which correlates schemes using a priori synthesized discriminant functions. However, when there are sufficiently arbitrary and a priori unknown changes in geometric parameters (properties), for example, the scale and shape of its description, for example, the use of the recognized methods of consideration is not effective enough [24,25].

## 4. Problem solving

Space has a $\mathrm{X}_{0} \mathrm{y}_{0}$ coordinate system and a certain description. When the coordinate axes are rotated, the coordinates of the image also change. Therefore, the task is to determine the angle of rotation of the coordinate axes and, accordingly, at what angle the image is rotated. According to the task, the following sequence of steps was performed.

1. The initial coordinates of the image are set $\mathrm{x}_{0}, \mathrm{y}_{0}$
2. The new image coordinates are calculated by the following formulas when the axes are rotated

$$
\begin{align*}
& x 1=y \sin \alpha+x \cos \alpha  \tag{1}\\
& y 1=y \cos \alpha-x \sin \alpha \tag{2}
\end{align*}
$$

3. The initial axial and centrifugal moments of inertia of the figure relative to the axes are determined according to the following formulas

$$
\mathrm{OX}_{0} \text { və } \mathrm{OY}_{0}
$$

$$
\begin{gather*}
J_{x}=\sum_{i=1}^{n}\left(x_{o i}-x_{s r}\right)^{2}  \tag{3}\\
J_{y}=\sum_{i=1}^{n}\left(y_{o i}-y_{s r}\right)^{2}  \tag{4}\\
J_{x y}=\sum_{i=1}^{n}\left(x_{o i}-x_{s r}\right)\left(y_{o i}-x y_{s r}\right) \tag{5}
\end{gather*}
$$

Where

$$
\begin{align*}
& x_{s r}=\frac{1}{n} \sum_{i=1}^{n} x_{i}  \tag{6}\\
& y_{s r}=\frac{1}{n} \sum_{i=1}^{n} y_{i}
\end{align*}
$$

4. The axial and centrifugal moments of inertia of the figure relative to the rotating axes $\mathrm{OX}_{1}$ and $\mathrm{OY}_{1}$ were found

$$
\begin{gather*}
J_{x 1}=J_{x 0} \cos ^{2} \alpha+J_{y 0} \sin ^{2} \alpha-J_{x 1 y 1} \sin 2 \alpha \\
J_{y 1}=J_{y 0} \cos ^{2} \alpha+J_{x 0} \sin ^{2} \alpha+J_{x 1 y 1} \sin 2 \alpha  \tag{7}\\
J_{x 1 y 1}=\frac{J_{x 0}-J_{y 0}}{2} \cdot \sin 2 \alpha+J_{x 0 y 0} \cdot \cos 2 \alpha
\end{gather*}
$$

The aim of the study is to find the angle of rotation of the coordinate axes. In other words, it is necessary to find the angle by which the figure is rotated relative to the original $\mathrm{X}_{0} \mathrm{y}_{0}$ coordinate axes.

The Arctg function was used to solve the problem:
Solving equation (7), it is rewritten in the following form:

$$
\begin{equation*}
\frac{J_{x 0}-J_{y 0}}{2} \cdot \sin 2 \alpha+J_{x 0 y 0} \cdot \cos 2 \alpha=J_{x 1 y 1} \tag{8}
\end{equation*}
$$

The following substitution has been applied:

$$
a=\frac{J_{x 0}-J_{y 0}}{2} ; \quad b=J_{x 0 y 0} ; \quad c=J_{x 1 y 1} .
$$

Then equation (8) becomes the following expression:

$$
\begin{equation*}
a \cdot \sin 2 \alpha+b \cdot \cos 2 \alpha=c \tag{9}
\end{equation*}
$$

To solve this equation, the double angle formula for sine and cosine is applied through the tangens function.

$$
\sin 2 \alpha=\frac{2 \operatorname{tg} \alpha}{1+\operatorname{tg}^{2} \alpha}
$$

$$
\cos 2 \alpha=\frac{1-\operatorname{tg}^{2} \alpha}{1+\operatorname{tg}^{2} \alpha}
$$

This time

$$
\begin{gather*}
\frac{2 \cdot a \cdot \operatorname{tg} \alpha}{1+\operatorname{tg}^{2} \alpha}+\frac{1-\operatorname{tg}^{2} \alpha}{1+\operatorname{tg}^{2} \alpha}=c \\
2 \cdot a \cdot \operatorname{tg} \alpha+b-b \cdot \operatorname{tg}^{2} \alpha=c+c \cdot \operatorname{tg}^{2} \alpha \\
(c+b) \cdot \operatorname{tg}^{2} \alpha+(-2) \cdot a \cdot \operatorname{tg} \alpha+(c-b)=0 \tag{10}
\end{gather*}
$$

The following substitution has been made:

$$
\begin{equation*}
\mathrm{a}_{1}=\mathrm{c}+\mathrm{b}, \mathrm{~b}_{1}=-2 \mathrm{a} \quad, \quad \mathrm{c}_{1}=\mathrm{c}-\mathrm{b}, \quad \mathrm{x}=\operatorname{tg} \alpha \tag{11}
\end{equation*}
$$

Then equation (10) can be written in the following form:

$$
a_{1} x^{2}+b_{1} x+c=0
$$

As can be seen, the quadratic equation is obtained. The roots of a quadratic equation are found by the following formula:

$$
\begin{equation*}
x_{1,2}=\frac{-b_{1} \pm d_{1}}{2 a_{1}} \tag{12}
\end{equation*}
$$

Here

$$
\begin{equation*}
d_{1}=\sqrt{b_{1}^{2}-4 a_{1} c_{1}} \tag{13}
\end{equation*}
$$

Then, when formula (11) is used, formulas (12) and (13) will be written as follows:

$$
\begin{gather*}
d_{1}=2 \sqrt{a^{2}+b^{2}-c^{2}} \\
x_{1}=\frac{2 a+d_{1}}{2 \cdot(c+b)}  \tag{14}\\
x_{2}=\frac{2 a-d_{1}}{2 \cdot(c+b)}
\end{gather*}
$$

According to

$$
\begin{align*}
& \alpha_{1}=\operatorname{ArcTg} \frac{2 a+d_{1}}{2 \cdot(c+b)} \\
& \alpha_{2}=\operatorname{Arctg} \frac{2 a-d_{1}}{2 \cdot(c+b)} \tag{15}
\end{align*}
$$

Returning to the original equation and the beginning, without substitution, is finally obtained

$$
\begin{gather*}
\alpha_{1}=\operatorname{Arctg} \frac{\left(J_{x 0}-J_{y 0}\right)+\sqrt{\left(\frac{J_{x 0}-J_{y 0}}{2}\right)+J_{x 0 y 0}^{2}-J_{x 1 y 1}^{2}}}{2 \cdot\left(J_{x 1 y 1}+J_{x 0 y 0}\right)}  \tag{16}\\
\alpha_{2}=\operatorname{Arctg} \frac{\left(J_{x 0}-J_{y 0}\right)-\sqrt{\left(\frac{J_{x 0}-J_{y 0}}{2}\right)+J_{x 0 y 0}^{2}-J_{x 1 y 1}^{2}}}{2 \cdot\left(J_{x 1 y 1}+J_{x 0 y 0}\right)}
\end{gather*}
$$



Figure 1. The coordinates of the figure relative to the new axes inclined at an angle $\alpha$ to the original coordinate axes

Here, $\mu$ is the angle that takes into account in which quadrant the figure will be located in the Cartesian coordinate plane as a result of the rotation of the coordinate axes (table 1).

Table 2.
Table of determination of angle $\mu$

| quarter <br> number | Quarter value in <br> angles | The value of the <br> angle $\mu$, taking into <br> account a quarter |
| :---: | :---: | :---: |
| I | $0 \leq \alpha<90$ | 0 |
| II | $90 \leq \alpha<180$ | 90 |
| III | $180 \leq \alpha<270$ | 180 |
| IV | $270 \leq \alpha<360$ | 270 |

Here, as well as in the solution of option 1 (arctg), one should take into account in which quarter the figure is located as a result of the rotation.

Based on the received formulas (16), a program was written to confirm the correctness of the received formulas. For example, the rotation of the quadrilateral, whose coordinates are listed in the table below, was considered (Table 3)

Table 3.
Coordinates of the rotated quadrilateral in the Cartesian plane

| x | 20 | 36 | 51 | 51 | 51 | 36 | 20 | 20 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| y | 50 | 50 | 50 | 34 | 17 | 17 | 17 | 35 |

## 5. Computer simulation

In the proposed algorithm, the contour points of the two-dimensional binary images of objects at the output of the vision system are displayed as coordinates in the Cartesian coordinate plane of the display. The values of the indicated coordinates are not invariant to the linear displacement and rotation of the image. Therefore, in order to correctly recognize such images, it is necessary to ensure that the image points are invariant with displacement and rotation. In order for the image to be invariant to orthogonal displacement, the coordinate system must be moved to the center of gravity of the defined image. Then, the rotation angle of the reference object relative to the initial position is determined by the moments of inertia relative to the coordinate axes of the image. After the rotation angle of the image is estimated, the coordinates of the contour points are found by rotating the reference image in the computer memory by this angle. Then the coordinates of the contour points of the current image are compared with the coordinates of the contour points of the rotated reference image. This comparison provides accurate information on whether the current image is the same or different from the reference image. Thus, the proposed algorithm allows invariant recognition of two-dimensional binary images.

A block diagram of the algorithm for computer simulation is given in the figure. The main program consists of subroutine input, figure coordinates input after rotation, rotation angle calculation subroutine, and result print blocks. The process of entering the initial data received in the subprogram entry into the computer takes place. Cartesian coordinates of a plane figure are taken as initial data. Then we rotate the arbitrarily drawn plane figure at a certain angle. The new coordinates of the plane figure, which has changed its position, are entered into the computer. A subroutine is used to calculate the angle formed during rotation. Also, the subroutine used interacts directly with the coordinate input block after rotation.


Figure 2: Block diagram of the proposed Algorithm
Calculation of new coordinates during rotation is determined according to the following formulas:

$$
\begin{aligned}
& x 1=y \sin \alpha+x \cos \alpha \\
& y 1=y \cos \alpha-x \sin \alpha
\end{aligned}
$$

The implementation of the rotation angle calculation subroutine consists of the following formulas:

$$
\begin{gathered}
J x 1=J x 0 \cos 2 \alpha+J y 0 \sin 2 \alpha-J x 1 y 1 \sin 2 \alpha \\
J y 1=J y 0 \cos 2 \alpha+J x 0 \sin 2 \alpha+J x 1 y 1 \sin 2 \alpha \\
J_{x 1 y 1}=\frac{J_{x 0-} J_{y 0}}{2} \cdot \sin 2 \alpha+J_{x 0 y 0} \cdot \cos 2 \alpha
\end{gathered}
$$

$$
\begin{aligned}
& \alpha_{1}=\operatorname{Arctg} \frac{\left(J_{x 0}-J_{y 0}\right)+\sqrt{\left(\frac{J_{x 0}-J_{y 0}}{2}\right)+J_{x 0 y 0}^{2}-J_{x 1 y 1}^{2}}}{2 \cdot\left(J_{x 1 y 1}+J_{x 0 y 0}\right)} \\
& \alpha_{2}=\operatorname{Arctg} \frac{\left(J_{x 0-} J_{y 0}\right)-\sqrt{\left(\frac{J_{x 0}-J_{y 0}}{2}\right)+J_{x 0 y 0}^{2}-J_{x 1 y 1}^{2}}}{2 \cdot\left(J_{x 1 y 1}+J_{x 0 y 0}\right)}
\end{aligned}
$$

In the program, the coordinates of the figure when rotated according to the above were calculated. And when calculating according to the formulas (16), the following results were obtained. In the tables, the coordinates of the plane figure when rotated from 0 to 360 degrees are given.

## Table 4

The coordinates of the plane figure when it is rotated by the first and second quadrants in the Cartesian coordinate system

| 0 $\boldsymbol{x}$ | ${ }^{0}$ | 30 $x$ | 30 $y$ | $\begin{gathered} 60^{\circ} \\ x \end{gathered}$ | $\begin{gathered} 60^{\circ} \\ y \end{gathered}$ | 90 $\boldsymbol{x}$ 0 | 90 $y$ | $\begin{gathered} 120^{\circ} \\ x \end{gathered}$ | $\underset{v}{120^{\circ}}$ | $\begin{gathered} 150^{\circ} \\ x \end{gathered}$ | 150 |  | $\begin{gathered} 180^{\circ} \\ y \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 20 | 50 | 42 | 33 | 53 | 8 | 20 | 50 | 42 | 33 | 53 | 8 | 20 | 50 |
| 36 | 50 | 56 | 25 | 61 | -6 | 36 | 50 | 56 | 25 | 61 | -6 | 36 | 50 |
| 51 | 50 | 69 | 18 | 69 | -19 | 51 | 50 | 69 | 18 | 69 | -19 | 51 | 50 |
| 51 | 34 | 61 | 4 | 55 | -27 | 51 | 34 | 61 | 4 | 55 | -27 | 51 | 34 |
| 51 | 17 | 53 | -11 | 40 | -36 | 51 | 17 | 53 | -11 | 40 | -36 | 51 | 17 |
| 36 | 17 | 40 | -3 | 33 | -23 | 36 | 17 | 40 | -3 | 33 | -23 | 36 | 17 |
| 20 | 17 | 26 | 5 | 25 | -9 | 20 | 17 | 26 | 5 | 25 | -9 | 20 | 17 |
| 20 | 35 | 35 | 20 | 40 | 0 | 20 | 35 | 35 | 20 | 40 | 0 | 20 | 35 |

Table 5
The coordinates of the plane figure when it is rotated by the third and fourth quadrants in the Cartesian coordinate system

| $2 \mathbf{2 0}^{\circ}$ | $\mathbf{2 1 0} 0^{\circ}$ | $\mathbf{2 4 0} 0^{\circ}$ | $\mathbf{2 4 0}^{\circ}$ | $\mathbf{2 7 0}^{\circ}$ | $\mathbf{2 7 0}^{\circ} \mathbf{3 0 0}^{\circ}$ | $\mathbf{3 0 0}^{\circ}$ | $\mathbf{3 3 0}^{\circ}$ | $\mathbf{3 3 0}^{\circ}$ | $\mathbf{3 6 0}^{\circ}$ | $\mathbf{3 6 0}^{\circ}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\boldsymbol{x}$ | $\boldsymbol{y}$ | $\boldsymbol{x}$ | $\boldsymbol{y}$ | $\boldsymbol{x}$ | $\boldsymbol{y}$ | $\boldsymbol{x}$ | $\boldsymbol{y}$ | $\boldsymbol{x}$ | $\boldsymbol{y}$ | $\boldsymbol{x}$ | $\boldsymbol{y}$ |
| 42 | 33 | 53 | 8 | 20 | 50 | 42 | 33 | 53 | 8 | 20 | 50 |
| 56 | 25 | 61 | -6 | 36 | 50 | 56 | 25 | 61 | -6 | 36 | 50 |
| 69 | 18 | 69 | -19 | 51 | 50 | 69 | 18 | 69 | -19 | 51 | 50 |
| 61 | 4 | 55 | -27 | 51 | 34 | 61 | 4 | 55 | -27 | 51 | 34 |
| 53 | -11 | 40 | -36 | 51 | 17 | 53 | -11 | 40 | -36 | 51 | 17 |
| 40 | -3 | 33 | -23 | 36 | 17 | 40 | -3 | 33 | -23 | 36 | 17 |
| 26 | 5 | 25 | -9 | 20 | 17 | 26 | 5 | 25 | -9 | 20 | 17 |
| 35 | 20 | 40 | 0 | 20 | 35 | 35 | 20 | 40 | 0 | 20 | 35 |

As can be seen in Figure 3, as a result of the applied algorithm, the rotation angles received the same values. Therefore, the proposed algorithm performs the necessary operations by returning the angle $\alpha$ that has fallen to other quadrants to the first quadrant each time. Thanks to this, 2D binary images can be recognized as rotation invariant, regardless of the rotation angle.

| RESULT |  |  |
| :--- | :--- | :--- |
|  | ArcTan-alfa1 | ArcTan-alfa2 |
| 0 | 3 | 0 |
| 30 | 33 | 30 |
| 60 | 63 | 60 |
| 90 | 93 | 90 |
| 120 | 123 | 120 |
| 150 | 153 | 150 |
| 180 | 183 | 180 |
| 210 | 213 | 210 |
| 240 | 243 | 240 |
| 270 | 273 | 270 |
| 300 | 303 | 300 |
| 330 | 333 | 330 |
| 360 | 363 | 360 |

Figure 3. Determination of angles during the application of the proposed algorithm
The first column of the table specifies the rotation angles of the figure. Columns 2 and 3 show the rotation angles calculated by formula (16) based on the arctg function.

## 6. Conclusion

A comparative analysis of algorithms for invariant recognition of two-dimensional binary images showed that there is no method that can fully solve this problem. Instead, there are algorithms that can partially solve the problem within certain constraints. One of the reasons for not solving the problem is that the trigonometric functions of the rotation angle take different signs in different quarters during image rotation. Therefore, the obtained result is not adequate to each other in different quarters. Therefore, the proposed method completely overcomes this drawback. Theoretical and computer modeling results show that the proposed formula gives accurate results only in the first quarter. Since trigonometric functions have different signs in different quadrants, the same formula does not give correct results in other quadrants. Therefore, the proposed algorithm performs the necessary operations by returning the angle $\alpha$ that has fallen to the other quarters to the first quarter each time. Because of this, 2D binary images are known to be rotation invariant regardless of the rotation angle. Computer modeling proves that the proposed method is correct.

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