Method of Sensor Network Functioning under the Redistribution Condition of Requests between Nodes

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Abstract
The results of previous studies show that relaying a significant number of requests between sensor network nodes leads to a decrease in functional stability and an increase in the number of failures. In most cases, a sensor network is built with predefined and described functions. However, if it is necessary to scale a network segment, it is necessary to define conditions for the redistribution of requests between nodes to ensure security. As is known, the reconfiguration of an information transmission system between nodes and relaying of messages are based on the construction of optimal routes for the transmission of messages, as well as an introduction of efficiency criteria and minimizing loss of data arrays.

Keywords
Sensor network, functional stability, query, node, attacks, flooding, method, model, reordering, routing.

1. Introduction

When modeling sensor networks, it is necessary to solve tasks of evaluating the performance of communication nodes. The requirements of network protocols often determine a certain order of packet transmission, which is preserved when requests pass from node to node, from sensor to sensor [1].

A situation often arises when requests received by an intermediate node cannot be forwarded to subsequent nodes due to unprocessed previous packets. The creation of such situations can be qualified as DoS (DDoS) attacks or flooding threats [2-4].

2. Main Part

The paper considers an example of organizing redistribution of requests in a communication node with a total buffer memory capacity equal to r.

It is assumed that information transmission lines have different bandwidths. Selection of requests from the Buffer Memory (BM) for transmission is carried out in order of their arrival in the BM.

Disruption of the order of requests at the output of the sending node occurs due to different bandwidths of lines and random lengths of packets of requests [5]. The reordering delay is the length of time required to restore the order of further transmission, determined by sending node, at the receiving node.

A single streaming dual-channel mass service system with shared capacity storage is used to estimate packet ordering delay r.

The query flow is assumed to be Poisson with parameter λ, a duration of service requests on the node is equal to i and has an exponential distribution with parameter μᵢ, i = 1, 2.

Without limitation of commonality, it is expected that μ₁ > μ₂ and it is also expected that the first node is fast and the second one is slow. It is also assumed that requests received by the
system go to the fast node. Requests are selected from the queue in order of their arrival to the system, that is, by the order determined by the routing protocol of the sensor network. Requests may be lost when the drive is full.

Let $\tau_n$ be a moment of issuing a request with a number $n$ from the node.

$$\Delta_{\text{res},n} = \begin{cases} \tau_{n-1} - \tau_n, & \text{if } \tau_{n-1} \geq \tau_n; \\ 0, & \text{in the opposite case.} \end{cases}$$

Then a random variable $\Delta_{\text{res},n}$ specifies the request redistribution delay $n$, associated with waiting for a request to exit node $n - 1$.

Requests that require reordering are accumulated at the output of the node in the so-called reorder buffer (Fig. 1) after serving a request with a number less than the number of requests waiting in the buffer (if any), the latter is instantly emptied.

![Figure 1: An example of forming requests in the rearrangement buffer](image)

Scientists have already obtained a matrix-geometric solution for the stationary distribution of queues taking into account the effect of reordering, which allows the calculation of the average number of applications in the reordering buffer [6].

It is assumed that the intervals between requests and the duration of their service are independent and have a phase-type distribution. At the same time, the rearrangement buffer is not taken into account when describing the model. For this, it is necessary to obtain the Laplace-Stiltjes transformation.

$\Delta_{\text{res},n}$ is stationary node operation mode, when $n \to \infty$, and even an expression for initial times of the rearrangement.

At the same time, it is necessary to separately calculate the recurrence relations for the factorial moments of the number of requests in the rearrangement buffer, which do not require solving the original system of equilibrium equations [7].

This allows us to significantly reduce the consumption of machine time and memory of sensor nodes compared to the requirements. In addition, the obtained relations relate factorial moments of the number of unordered requests to initial delays of reordering.

### 3. System of Equilibrium Equations

We can say that a sensor network segment is in an orderly state: if a request is served on a fast node $i$, on slow—request $j$ and $i < j$ in the opposite case, that is, when $i > j$—the network segment is out of order.

A sensor network segment is considered to be in an ordered state if it has only one request served on a fast node, and in an unordered state if a request is served on a slow node [8].

The stochastic behavior of a network segment can be described by a homogeneous Markov process $X(t), t \geq 0$ over a multitude of states

$$X = \bigcup_{k=0}^{R} X_k,$$

where

$$X_0 = \{(0)\}, X_k = X_{k1} \cup X_{k2}, R \geq k \geq 1, R = r + 2,$$

$$X_{k1} = \{(k, i, l) | l \geq 0\}, i = 1, 2.$$

For some point in time $t: X(t) = (0)$, if the system is empty; $X(t) = (k, i, l)$, if there are in the system (in the drive and the nodes) $k$ requests and reordering in the buffer $l$ requests, at the same time, when $i = 1$, the system is disordered when $i = 2$—ordered.

In the assumption that if $0 < \lambda, \mu_1, \mu_2 < \infty$, final probabilities exist, are strictly positive, do not depend on the initial distribution, and coincide with stationary probabilities.

$$p_x = \lim_{t \to \infty} P\{X(t) = x\}, x \in X.$$

Stationary probabilities of macrostates $X_{kl}$ do not take into account the state of the buffer, and $X_k$, which also do not take into account the orderliness of the system and determine only the number of requests in it that can be marked $p_{kl}$ and $p_k$ accordingly.

Stationary probabilities $p_x, x \in X$, is the only solution of the system of equilibrium equations.

$$\lambda p_0 = \mu_1 p_{12} + \mu_2 p_{11},$$

(1)
\[(\lambda + \mu_{3-i})p_{1 i} = u(1 - 1)[u(i - 1)\lambda p_0 + \mu_1 p_{2,3-i}] + u(i)\mu_i p_{2i,1-i}, \quad i = 1,2,1 \geq 0,\]

\[(\lambda + \mu)p_{kdl} = u(1 - 1)\mu_1 p_{k+1,3-i} + u(i)\mu_1 p_{k+1,1-i} + \lambda p_{k-1,il}, \quad k = 2,r + 1, i = 1,2,1 \geq 0,\]

\[\mu p_{Ril} = \lambda p_{r+1,il}, i = 1,2,1 \geq 0.\]

With the condition of rationing
\[p_0 + p_1 + \ldots = 1.\]

If \(\mu = \mu_1 + \mu_2,
\[u(x) = \begin{cases} 1, x > 0; \\ 0, x \leq 0. \end{cases}\]

Stationary probabilities of macrostates. Summing up equations (2–4) for \(l = 0,1, \ldots,\) the result will be obtained:
\[(\lambda + \mu_{3-i})p_{1 i} = u(1 - 1)\lambda p_0 + \mu_1 p_{2,1}, \quad i = 1,2,\]

\[(\lambda u(R - k) + \mu)p_{k 1} = \lambda p_{k-1,i} + u(R - k)\mu_1 p_{k+1}, k = 2,R, i = 1,2.\]

The system of equations (6–7) and (1) is a
\[p_0 = \frac{\mu_1 \mu_2 (2\lambda + \mu)}{\lambda^2 (\lambda + \mu_2)} p_2, \quad p_{1 1} = \frac{\mu_1}{\lambda + \mu_2} p_2, \quad p_{1 2} = \frac{\mu_2 (\lambda + \mu)}{\lambda (\lambda + \mu_2)} p_2,\]

\[p_k = \rho^{k-2} \left[\frac{\mu_1 \mu_2 (2\lambda + \mu) + \lambda \mu (\lambda + \mu_2)}{\lambda^2 (\lambda + \mu_2)} + \frac{1 - \rho^{r+1}}{1 - \rho}\right], k = 2,R,\]

where \(\rho = \frac{\lambda}{\mu}.

At the distribution \(\{p_0, p_{1 1}, p_{1 2}, p_k, k = 2,R\}\) probability \(p_{k0}, k = 2,R,\) can be computed from the recurrence relations which follow from (7) trivially.

If you enter vectors \(p_k^T = (p_{k1}, p_{k2}), k = 1,R,\)

you can get explicit statements about them presented in the matrix-geometric form. Indeed, from (7) \(p_k = 2,R + 1,\) taking into account the obvious ratio \(\lambda p_k = \mu p_{k+1}\) will be received:
\[(\lambda + \mu)p_{k 1} - \mu p_k = \lambda p_{k-1,i}, k = 2,R + 1, i = 1,2.\]

Taking into account that \(p_k = p_{k-1}\) the system of equations (9) concerning the unknowns is written \(p_{k1}\) and \(p_{k2}\) in matrix form:

\[Bp_k = \lambda p_{k-1}, k = 2,R + 1,\]

where \(B = \left(\begin{array}{cc} \lambda + \mu - \rho\mu_1 & -\rho\mu_1 \\
-\rho\mu_2 & \lambda + \mu - \rho\mu_2 \end{array}\right).\]

Reversed to \(B\) matrix \(B^{-1}\) looks like
\[B^{-1} = \frac{1}{\mu(\lambda + \mu)} \left(\begin{array}{cc} \lambda + \mu - \rho\mu_1 & \rho\mu_1 \\
\rho\mu_2 & \lambda + \mu - \rho\mu_2 \end{array}\right).\]
\[ F_{kl}(z) = \sum_{|z| = 1}^{\infty} p_{kl} z^1, \quad k = \overline{1, R}, i = 1,2. \]

Usually, it is not difficult to obtain a system of equations for the generating function from (1–4) \( F_{kl}(z) \):

\[ (\lambda u(R - k) + \mu) F_{kl}(z) = u(R - k)\mu zF_{k+1,i}(z) + \lambda F_{k-1,i}(z), \quad k = 2, R, \quad i = 1,2. \]

\[ \nu_{kiv} = \frac{dF(u)}{dz} \bigg|_{z=1} = \sum_{i=0}^{z} (i)_u p_{kil}, \quad k = \overline{1, R}, i = 1,2, v \geq 0. \]

It should be emphasized that \( \nu_{kiv} = p_{ki}, k = \overline{1, R}, i = 1,2, \) and the values \( \nu_v = v \ldots, v \geq 1, \) represent factorial moments of the order of several requests that are in the rearrangement buffer.

Differentiating (12) and (13) by \( z \) \( v \) times and then considering \( z = 1, \) will be obtained:

\[ (\lambda + \mu_{3-i}) \nu_{i1v} = \mu_i \nu_{i2,v-1} + \mu_i \nu_{i2,v}, \quad i = 1,2, v \geq 1, \]

\[ (\lambda u(R - k) + \mu) \nu_{kiv} = u(R - k)\mu z \nu_{k+1,i} + \nu_{k+1,i,v-1} + \lambda \nu_{k-1,i}, \quad k = 2, R, \quad i = 1,2, v \geq 1. \]

At fixed values, \( i = 1,2 \) \( \text{ra} v = 1,2 \ldots (14) \) and (15) is a system of equations concerning the unknowns \( \nu_{kiv}, \quad k = \overline{1, R}, i = 1,2 \) with a non-degenerate matrix of coefficients.

The solution of this system of equations is determined by the following theorem.

\[ \nu_{kiv} = \lambda \alpha_k \nu_{k-1,i} + v \sum_{j=1}^{R-k} \mu_i \nu_{k+i,i,v-1} + \nu_{k+1,i,v-1}, \quad k = \overline{1, R}, \quad i = 1,2. \]

\[ \alpha_{Ri} = \frac{1}{\mu}, \quad \alpha_{ji} = (\lambda + \mu - \lambda \mu_{i+1,i})^{-1}, \quad j = 2, r + 1, \]

\[ \alpha_{il} = (\lambda + \mu_{3-i} - \lambda \mu_{2,i})^{-1}, \quad i = 1,2. \]

The validity of the theorem can easily be shown by substituting ratios (16) of equations (14) and (15), as a result of which these equations turn to identity.

To control the calculations according to formulas (16) and (17), the following relations can be useful, which result from equations (14) and (15) by summing them \( k = 1,2 \ldots R. \)

\[ \nu_{i,v} = \frac{\nu_i}{\mu_{3-i}} \left[ \nu_{i,v-1} - \nu_{i-1,v-1} \right], \quad i = 1,2, v = 1,2. \]

In particular, for \( v = 1 \) the following will be obtained:

\[ \nu_{i,1} = \frac{\nu_i}{\mu_{3-i}} \sum_{k=2}^{R} p_{ki}, \quad i = 1,2. \]

In conclusion, it is necessary to focus on the connection of factorial moments of the request’s number in the rearrangement buffer with the initial moments of the rearrangement delay in stationary mode.

Let \( w_{iv} \) be the initial moment of order \( v \) of the rearrangement time, taking into account state \( j, \) determining the orderliness of the node. For analyzed network segment:

\[ w_{iv} = \frac{\nu_i}{\lambda \mu^{3-i}} \sum_{k=2}^{R} p_{ki}, \quad i = 1,2, v = 1,2, \ldots \]
where $\lambda_D = \lambda(1 - p_R)$—the intensity of servicing flow of requests.

\[
\lambda_D w_{i,v} = \frac{1}{\mu^{v-1}} \left[ \mu_{i,v} + \sum_{j=1}^{v-1} \left( \frac{\mu_i}{\mu_{i,j}} \right)^j \right] \]

The proof of the theorem is based on relations (18) and (20).

It follows from Theorem 2 that when $v = 1$, the average value of the reordering time and the number of applications in the reordering buffer $w_1$ and $v_1$ related by the ratio:

\[
\lambda_D w_1 = v_1. \tag{22}
\]

It is worth noting that the ratio is an analog of Little's well-known formula and has an obvious physical interpretation.

The algorithm for calculating the characteristics of the analyzed network segment was implemented [9].

Figure 2: Dependence of the average reordering time $w_1$ on requests in the reordering buffer

In Figs. 2 and 3 the dependences of the average rearrangement time are shown $w_1$ and the mean and standard deviation of the number of requests in the reorder buffer $v_1$ and $\sigma_v$ from system load $\rho$ at the value of the storage volume $r = 10$.

The calculations show that $\rho$ the values of $w_1, v_1$ and $\sigma_v$ also increase.

Figure 3: Dependence of the mean and standard deviation of the number of requests in the reorder buffer $v_1$ and $\sigma_v$ from system load $\rho$

Theorem 2. For a network segment $M | M | 2 | r$ taking into account the rearrangement, the following ratios take place:

\[
\lambda_D w_{i,v} = \frac{1}{\mu^{v-1}} \left[ \mu_{i,v} + \sum_{j=1}^{v-1} \left( \frac{\mu_i}{\mu_{i,j}} \right)^j \right], \quad i = 1, 2, v = 1, 2, \ldots \tag{21}
\]

With a sufficiently large load on the system, the values of these indicators stabilize, which is quite understandable from physical considerations [10].

Table 1

<table>
<thead>
<tr>
<th>$r$</th>
<th>$t_1$</th>
<th>$t_2$</th>
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<td>5</td>
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<td>0.86</td>
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<td>20</td>
<td>0.99</td>
<td>15.19</td>
</tr>
<tr>
<td>25</td>
<td>1.16</td>
<td>23.71</td>
</tr>
<tr>
<td>30</td>
<td>1.41</td>
<td>48.05</td>
</tr>
</tbody>
</table>

As noted, the approach used in the work allows us to calculate the factorial moments of the number of requests in the reordering buffer more efficiently from the point of view of resource consumption. This conclusion confirms the results shown in Table 1.

Figure 4: Request processing time on a fast node and slow node

5. Conclusions

A sensor network is built with predefined and described functions. However, if it is necessary to scale a network segment, it is necessary to define conditions for the redistribution of requests between nodes to ensure security.

The reconfiguration of an information transmission system between nodes and relaying of messages is based on the construction of optimal routes for the transmission of messages, as well as an introduction of efficiency criteria and minimizing the loss of data arrays.
6. References


