Parametric Model of a Laser with External Distributed Feedback in the System of Remote Measurement of Nanovibrations

George Otto[1], Konstantin Otto[2], Hennadii Hulak[3], Volodymyr Astapenya[3], and Dmytro Tyshchenko[4]

[1]*Institute of Problems of Mathematical Machines and Systems, 42 Academician Glushkov ave., Kyiv, 03187, Ukraine*
[2]*CJSC "Altron," Lyatoshinskoho str., Kyiv, 03191, Ukraine*
[3]*Borys Grinchenko Kyiv University, 18/2 Bulvarno-Kudriavska str., Kyiv, 04053, Ukraine*
[4]*State University of Trade and Economics, 19 Kyoto str., Kyiv, 02156, Ukraine*

Abstract

The paper studies the actual problem of building an adequate model of remote measurement of the characteristics of barely noticeable vibrations (nano vibrations) of the object of observation, which may be caused by some processes inside the object or external influences on it. Solving such problems is of key importance both for engineering sciences and for ensuring the safety of information technologies. The paper proposes a model that otherwise explains the interaction of individual feedback energy components in the formation of a useful signal. This reveals the reasons and makes it possible to determine the quantitative difference in estimates of the useful signal between the models: generally accepted and proposed. The proposed model, based on the photon lifetime and the equivalent length of the composite resonator, is in good agreement with the results of practical experiments and opens up new opportunities for improving the efficiency of the Laser Feedback Interferometry (LFI) model.

Keywords

Interferometry, laser, vibrometer, nano vibration, LFI model

1. Introduction

The solution to many problems in engineering and information technology significantly depends on the success or failure of remote measurement of the characteristics of barely noticeable vibrations (nano vibrations) of the object of observation, which are due to some processes inside the object or external influences on it [1, 2].

The use of devices for measuring nano vibrations is an element of information warfare [3–4]. In particular, when designing information security systems, it is important to assess the possibility of unauthorized access to critical information owing to the artificial or natural occurrence of technical channels for information leakage [5]. Thus, in [6–8], an analysis was made of the principle of operation of laser acoustic reconnaissance systems and passive methods of protection against reading acoustic information.

The laser recorders and nano vibration meters have very high characteristics compared to other technical means of remote information acquisition. This affects the development of a threat model and the practical construction of an information security system in an enterprise [9–11].

During development and practical experiments in the field of constructing laser nano vibration meters, results were obtained [12–14], which do not agree very well with the existing theoretical model for constructing a laser vibrometer, which became the basis for a detailed analysis of this model and its refinement [15–17].

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EMAIL: ottogk@gmail.com (G. Otto); ottok@gmail.com (K. Otto); h.hulak@kubg.edu.ua (H. Hulak); v.astapenia@kubg.edu.ua (V. Astapenya); tyshchenko@knute.edu.ua (D. Tyshchenko)
ORCID: 0000-0002-2149-3516 (G. Otto); 0009-0001-2396-7336 (K. Otto); 0000-0001-9131-9233 (H. Hulak); 0000-0003-0124-216X (V. Astapenya); 0000-0002-2193-9012 (D. Tyshchenko)

2. Physical and Mathematical Principles of Building a Model

Fig. 1 shows a diagram of the proposed model of a laser vibrometer with externally distributed feedback, in which the feedback is distributed between resonators 1–3 and 1–5 which are external to the active medium.

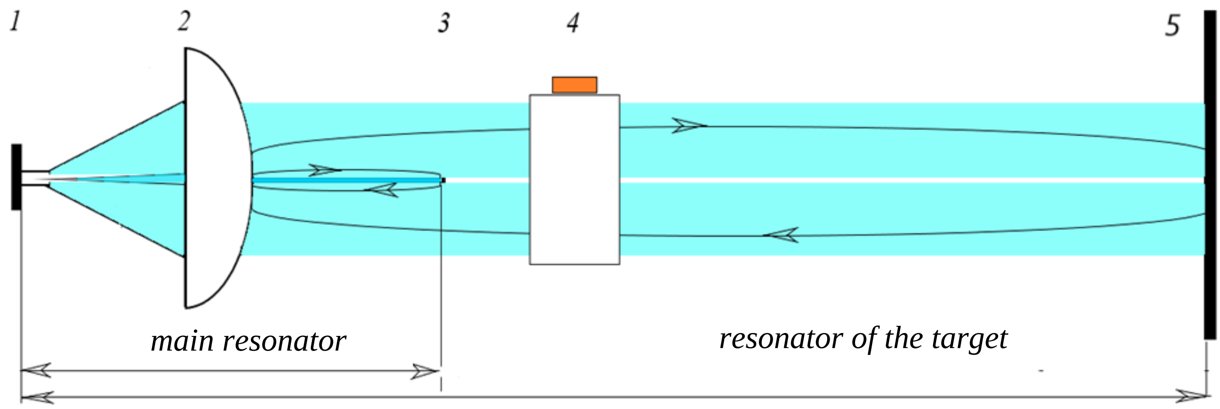

Figure. 1: Model scheme: 1 is active medium with rear mirror; 2 is a collimator; 3 is a main mirror; 4 is an acoustic-optic modulator; 5 is the surface of the object (target)

To maximize the intensity of the received radiation, a scheme with wavefront division was used, since the intensity distribution over the aperture when the beam moves in opposite directions is different. (From the laser—Gaussian radiation, with a maximum in the center, into the laser—uniform).

The ratio of the area of the main mirror—3 resonators 1–3 and the beam expanded by the collimator—2, is approximately 1⁄1000. The second resonator, 1–5, is formed by the main mirror and the surface of the object. The third resonator of the circuit, formed by mirrors 3–5, has a lower quality factor by several orders of magnitude compared to resonators 1–5, and even more so with resonators 1–3, that's why we will not consider its influence.

Further considerations are also applicable to the amplitude division scheme and are based (Fig. 1) on simple, obvious assumptions:

- A laser—conservative system
- An object surface—3rd laser mirror
- An active medium and a deaf mirror—a common part of a composite resonator.

A consequence of our assumptions is the conclusion that the feedback interferometry (Laser Feedback Interferometry—LFI) is due to the evolution of the photon lifetime in the composite resonator.

Since the lifetime is an energy-intensive parameter, the physical processes in a composite resonator are parametric. The model based on this approach will be called parametric (Parametric or P-model).

Doppler modulation is a kind of frequency modulation that has an asymmetric spectrum [18].

In the absence of a frequency shift of the probing beam relative to the reference beam, at least by an amount equal to half the useful signal spectrum width, the negative and positive parts of the spectrum overlap.

As a result, the task of restoring the law of change of any complex useful signal becomes practically unsolvable. To exclude such a situation, to shift the frequency, an Acousto-Optic Modulator (AOM)—4 was introduced into the optical scheme (Fig. 1). Modulation frequency is ω_b=30⟦·10⟧^6 Hz.

In addition, AOM performs another important function. It shifts the center of the spectrum of the useful signal much further, to the region of high frequencies, which makes the received signal relatively narrow-band and allows to filter and demodulate it with conventional radio engineering methods.

The processes that cause a change in the laser parameters are associated with a change in the current by [19, 20], which causes a change in the temperature of the laser diode, which, in turn, is due to a change in the value of the feedback from the mirror of the object—5 (Fig. 1).

In our scheme, the frequency of the feedback signal from the object, because of its narrow bandwidth, is determined not by the frequency range of the useful signal, but by the frequency of the AOM, which is chosen high enough so that the temperature of the laser active medium does not have time to track it [19]. This eliminates the inevitable (in the absence of AOM) change in the laser parameters during the reception of a relatively low-frequency useful signal and the distortions associated with their change.

Let us consider the behavior of the radiation power and the pump current caused by the change in the photon lifetime in the composite resonator, which is determined by expression (2).

A simplified photon balance equation, without considering insignificant losses due to transitions without radiation [20], has the form:

$$\frac{dS(t)}{dt} = S(t)\big(G(n) - \tau_{ph}^{-1}\big), \qquad (1)$$

where: $S(t)$ is the number of induced photons; $G(n) = \frac{P_{in}(n)}{P_{st}t}$; $P_{in}(n)$ is the power of injected electrons; P_{st} is the power of photons induced over time t; $G(n)$ is amplification per unit of time; n is the number of injected electrons; τ_{ph}^{-1} is a reciprocal lifetime of photons. $\tau_{ph} = \frac{2l}{c\ln r^{-1}}$, where l is resonator length, r is resonator reflectance, c is the speed of light.

In the stationary regime, the pump rate always exceeds the loss rate:

$$G(n) > \tau_{ph}^{-1}.$$

Let us multiply the left and right parts of (1) by the photon energy—$h\nu$. Then (1) will represent the ratio between the power of stimulated emission P_{st}, on the one hand, and pump power P_{tot} and threshold P_{th}, on the other: $P_{st} = P_{tot} - P_{th}$. The last expression can be represented as: $P_{st} = U\big(I(t) - I_{th}(t)\big)$. We have obtained the dependence of the radiation power on the pump current—the output or watt-ampere characteristic of the laser, which can otherwise be represented as:

$$P = \eta U\left(I - q\tau_p^{-1}(t)\right), \qquad (2)$$

where: P is the power of radiation (W); I is pump current (A); η is quantum efficiency; U is laser voltage (V); q is the electron charge ($A \cdot sekonds$); $\eta \cdot q \cdot \tau_p^{-1}(t) = I_{th}$ is a threshold current (A). Generation condition: $I \geq I_{th} \geq 0$—the current must exceed the threshold. Generation condition: $I \geq I_{th} \geq 0$ is the current that must exceed the threshold.

Let the laser be powered by a voltage generator ($U = const$). We give a small increment of the photon lifetime:

$$\tau_p^{-1}(t) = \tau_p^{-1}(t) + \Delta\tau_p^{-1}(t).$$

From (2) it follows that as a result, an increment can be obtained as the pump current: $I = (I + \Delta I)$ and (with the opposite sign) the radiation power:$P = (P - \Delta P)$. In the general case, we get both increments together. Let's assume the general case.

We subtract the original equation (1) from the same equation, but with increments—ΔP и ΔI:

$$\frac{dS(t)}{dt} = S(t)\big(G(n) - \tau_{ph}^{-1}\big), \qquad (3)$$

Dividing the left and right sides of (3) into (3) $-\eta U q \Delta\tau_p^{-1}(t)$, we get:

$$\frac{dS(t)}{dt} = S(t)\big(G(n) - \tau_{ph}^{-1}\big), \qquad (4)$$

Denoting:

$$\frac{\Delta P}{\eta \cdot U \cdot q \cdot \Delta\tau_p^{-1}(t)} = \delta P\big(\tau_p^{-1}\big), \frac{\Delta I}{\eta \cdot q \cdot \Delta\tau_p^{-1}(t)} = \delta I\big(\tau_p^{-1}\big),$$

we get:

$$\begin{aligned} &\frac{dS(t)}{dt} = S(t)\big(G(n) - \tau_{ph}^{-1}\big), \\ &0 \leq \delta P\big(\tau_p^{-1}\big), \delta I\big(\tau_p^{-1}\big) \leq 1. \end{aligned} \qquad (5)$$

Nothing prevents us from interpreting expressions (4) and (5) as the density of a two-point probability distribution—{ΔP, ΔI}, where the terms are the probabilities of these values. So, δP(τ_p^(-1)) is the probability that an increase in the threshold power will cause an increase in the radiation power ΔP, and δI(τ_p^(-1)) is the probability that an increase in the threshold power will cause an increase in the pump power ΔI. Indeed, each of its elements is less than 1, positive, dimensionless, their sum is equal to 1.

The change in the variables in equation (2)—radiation power ΔP and pump current ΔI with a change in the parameter τ_p (photon lifetime) occurs as a result of the evolution of the energy-intensive parameter of the operating laser, i.e. we are talking about the behavior of a laser as a non-equilibrium system in a transitional mode.

Relation (2) and the resulting probability distribution (5) do not allow us to find the desired dependence. An additional condition is required. It can be obtained by relying on one of the appropriate universal principles, such as variational ones, corresponding to the conditions of use. This possibility is provided by the principle of maximum entropy production (Maximum Entropy Production Principle—MEPP).

MEPP is formulated as follows: a non-equilibrium system, developing in natural conditions, tends to the state corresponding to the maximum entropy with the maximum possible "speed" [21]. By "natural", we mean the conditions under which there is no "targeted external influence" [22]. The latter corresponds to the first of the Assumptions we have accepted. Formulating MEPP, Jaynes [21] adopted the Shannon form of information entropy [22].

For the distribution entropy (5), the principle in the form [22] can be written as:

$$H(\delta I, \delta P) \to max. \quad (6)$$

Considering that $\delta I(\tau_p^{-1}) = 1 - \delta P(\tau_p^{-1})$, the MPP compliance condition for distribution (4) will look like this:

$$\frac{d}{d(\delta P)} H\left(\delta P(\tau_p^{-1}) + \delta I(\tau_p^{-1})\right) \equiv -\frac{d}{d(\delta P)}(\delta P \ln \delta P + (1-\delta P)\ln(1-\delta P)) = 0, \quad (7)$$

where $H(*)$ is information entropy in Shannon form.

As a result of differentiation (6), we obtain:
$\ln \delta P = \ln(1-\delta P)$, then $\delta P = \delta I = 0.5$.
Thus:

$$\Delta P = -\frac{\eta U q}{2\Delta\tau_p(t)}, \Delta I = \frac{\eta q}{2\Delta\tau_p(t)}. \quad (8)$$

This once again confirms the fact that ***the uniform distribution has the maximum entropy***.

From (8) it follows that due to the evolution of the photon lifetime—τ_p, the radiation power and the pump current undergo equal value and opposite change in sign. An increase in the value of τ_p causes a simultaneous decrease in the pump current and, equal in absolute value—an increase in the radiation power. Accordingly, a decrease in the value of τ_p will give an increase in the pump current and a proportional decrease in the radiation power.

Formulas (8) make it possible to numerically estimate $\Delta P \ u \ \Delta I$ as functions τ_p.

Previously, in (3) we specified an increment $\Delta\tau_p$. Let us establish the dependence of $\Delta\tau_p$ on the reflection coefficient of the object's mirror. Equivalent reflection coefficient of a composite resonator: $r_{ecv} = r_1 + r_2$ is here and below, r is the intensity reflection coefficient, r_1, r_2 are reflection coefficients of mirrors of partial resonators.

The distances from the common rear mirror to the mirrors r_1, r_2 are equal, respectively l_1, l_2, here and below l_1, l_2, l_{ecv}—optical lengths of resonators.

Let's denote l_{ecv} is the virtual position of the mirror r_{ecv} relative to the mirrors $r_1 \text{и}\, r_2$. It is obvious that it is located between the mirrors r_1, r_2, that is.: $l_1 \le l_{ecv} \le l_2$, $l_1 < l_2$ and its position is determined by the coefficients $r_1 and\, r_2$. Then, if $r_2 = 0$, then $l_{ecv} = l_1$, with $r_1 = 0\ l_{ecv} = l_2$. We notice that the virtual position r_{ecv} is closer to the mirror, the reflectivity of which is higher.

Such conditions will satisfy any of the many possible estimates of the average length. The mathematical expectation is one of them. Representing the reflection coefficients r_1, r_2, normalized to the sum $r_1 + r_2$ as probabilities, we obtain a two-point distribution.

Really: $r_1, r_2 \ge 0$, $0 \le \frac{r_1}{r_{ecv}}, \frac{r_2}{r_{ecv}} \le 1$, $\frac{r_1}{r_{ecv}} + \frac{r_2}{r_{ecv}} = 1$, namely, the normalized coefficients formally correspond to the definition of the probability distribution function. The mathematical expectation of the length distributed according to it is as follows:

$$M(l_i) = \sum_i p_i l_i = \frac{r_1}{r_{ecv}} l_1 + \frac{r_2}{r_{ecv}} l_2, i = 1,2.$$

In statistics, various estimates of mean values are used—harmonic, quadratic, cubic, in addition, mode, and median, but only mathematical expectation according to Cramer-Rao inequality, is optimal in the mean square sense. From this, it follows that the optimal estimate of the mean value l_{ecv} should have a minimum dispersion. Let's check this:

$$D(l_{ecv}) = \sum_i \left(l_i - M(l_i)\right)^2 \frac{r_i}{r_{ecv}} = (l_1 - l_{ecv})^2 \frac{r_1}{r_{ecv}} + (l_{ecv} - l_{ecv})^2 \frac{r_2}{r_{ecv}}.$$

We find at what l_{ecv} a minimum dispersion is achieved:

$$\frac{d\left(D(l_{ecv})\right)}{dl_{ecv}} = (-2l_1 + 2l_{ecv})\frac{r_1}{r_{ecv}} + (-2l_2 + 2l_{ecv})\frac{r_2}{r_{ecv}} =$$
$$= -2\left(l_1 \frac{r_1}{r_{ecv}} + l_2 \frac{r_2}{r_{ecv}}\right) + 2l_{ecv}\left(\frac{r_1}{r_{ecv}} + \frac{r_2}{r_{ecv}}\right) = 0$$

considering that

$\frac{r_1}{r_{ecv}} + \frac{r_2}{r_{ecv}} = 1$, $l_{ecv} = \frac{r_1}{r_{ecv}} l_1 + \frac{r_2}{r_{ecv}} l_2$.

We made sure that the accepted estimate of the equivalent length of the composite resonator (in the form of the mathematical expectation of the lengths of partial resonators) is optimal.

The above derivation of the equivalent length estimate is of a "statistical" nature.

Understanding the importance of correctly determining $the\ value\ l_{ecv}$ is the equivalent length of a composite resonator, we will give, in our opinion, a more "physical" way of determining it, which also leads to a similar result (Section 3).

Knowing the value $r_{ecv}\ and\ l_{ecv}$, let's find the value τ_{ecv} is lifetime of photons in a composite resonator. To do this, we use two equations. We obtain the first equation by assuming in equation (1) $G(t) = 0$, which is equivalent to a virtual "turn off" of gain in the generating laser. Then (1) is converted to the form:

$$\frac{dS(t)}{dt} = -\frac{1}{\tau_{ecv}} S(t). \tag{9}$$

We obtain the second equation using the definition of m is a multiple reflection coefficient:

$$S(t) = S(0) r_{ecv}^{m(t)}, \tag{10}$$

where $S(0)$ is energy in the resonator at the time of switching off the pump current; $m(t) = \frac{c \cdot t}{2 l_{ecv}}$ is the number of re-reflections in the resonator during the time t.

Let us separate the variables and integrate (9):

$$\ln S(t) = -\frac{1}{\tau_{ecv}} t + const. \tag{11}$$

Let's take a logarithm (10):

$$\ln S(t) = \ln S(0) + m(t) \ln r_{ecv}. \tag{12}$$

Then we substitute the initial conditions into the obtained equations: with $t = 0, m(t) = 0$; then: $const = \ln S(0)$. Uniting (11) and (12), we get:

$$\tau_{ecv} = \frac{2 l_{ecv}}{c \ln(r_{ecv})^{-1}} = \frac{2(\overrightarrow{r_1} l_1 + \overrightarrow{r_2} l_2)}{c(\overrightarrow{r_1} + \overrightarrow{r_2}) \ln(\overrightarrow{r_1} + \overrightarrow{r_2})^{-1}}. \tag{13}$$

In expression (13) the values $\overrightarrow{r_1}, \overrightarrow{r_2}$ are summed up. This means that the reflection coefficients: $\overrightarrow{r_1} = \sqrt{r_1} e^{i\omega t}$, $\overrightarrow{r_2} = \sqrt{r_2} e^{i(\omega t + \varphi)}$, have different phases, and interfere when added.

The result of interference τ_{ecv}:

$$\overrightarrow{\tau_{ecv}} = \tau_1 \left(1 + 2Re \frac{\sqrt{l_2}}{\sqrt{l_1}} \sqrt{\frac{r_2}{r_1}} e^{-i\varphi}\right). \tag{14}$$

Accordingly,

$$\frac{1}{\overrightarrow{\tau_{ecv}}} = \frac{1}{\tau_1} \left(1 + 2Re \sqrt{\frac{l_2 r_2}{l_1 r_1}} e^{-i\varphi}\right)^{-1} \tag{15}$$

Let us expand (15) into a power series in the problem of studying vibrations of distant objects, as a rule $\left|2 \frac{\sqrt{l_2}}{\sqrt{l_1}} \sqrt{\frac{r_2}{r_1}}\right| \ll 1$, therefore series (15) converge rapidly. This allows us to confine ourselves to the first two terms of the expansion. Otherwise, higher-order terms should be taken into account:

$$\frac{1}{\overrightarrow{\tau_{ecv}}} = \frac{1}{\tau_1} - \frac{2}{\tau_1} \frac{\sqrt{l_2}}{\sqrt{l_1}} \sqrt{\frac{r_2}{r_1}} e^{-i\varphi} + \cdots. \tag{16}$$

Let's substitute the first two terms (16) into (1):

$$\frac{dS(t)}{dt} = S(t) G(n) - \frac{1}{\tau_1} \left(1 - 2 \frac{\sqrt{l_2}}{\sqrt{l_1}} \sqrt{\frac{r_2}{r_1}} e^{-i\varphi}\right) S(t). \tag{17}$$

We have obtained the photon balance equation for P-model LFI.

Here n is the number of pump electrons per unit of time—a quantity proportional to the pump current.

In the resulting equation, the terms in the second term of the right-hand side represent the feedback radiation energy in the main resonator and the resonator of the object. Both radiations, when they enter the active medium of the laser diode, interfere:

$$\frac{dS(t)}{dt} = S(t) G(n) - S(t) \frac{1}{\tau_1} \left(1 + \frac{2 r_2 l_2}{r_1 l_1} - 4Re \frac{\sqrt{l_2}}{\sqrt{l_1}} \sqrt{\frac{r_2}{r_1}} e^{-i\varphi}\right), \tag{18}$$

where $\frac{1}{\tau_1}\left(1 + \frac{2 r_2 l_2}{r_1 l_1}\right) = \frac{1}{\overline{\overline{\tau_{ecv}}}}$—the average level of the inverse lifetime of photons in a composite resonator:

$$I_P(t) = \frac{1}{\widetilde{\tau_{ecv}}} = \frac{1}{\tau_1} 4Re \frac{\sqrt{l_2}}{\sqrt{l_1}} \sqrt{\frac{r_2}{r_1}} e^{-i\varphi} = \frac{1}{\tau_1} 4 \frac{\sqrt{l_2}}{\sqrt{l_1}} \sqrt{\frac{r_2}{r_1}} \cos \Omega t, \tag{19}$$

where: I_P is interference term, which is also a variable component of the reciprocal lifetime of a photon in a compound resonator—τ_{ecv}. Useful signal effect (8) is equal: $I_P(t) = I_P(t)/2$; $\varphi = \Omega t$ is the phase angle between the radiation reflected by the mirror of the main resonator—r_1 and radiation reflected by the surface of the object—a mirror r_2; Ω is frequency AOM.

In the formula for the interference of two quasi-coherent beams, the interference term is multiplied by the correlation coefficient. In our model (Fig. 1), this coefficient is close to one.

Not formally, this can be explained as follows: the radiation of both beams does not leave the composite resonator. It is known that a standing wave in a resonator is coherent throughout its length, which is a condition for generation. For a single-mode laser, this is due to the tendency to concentrate the energy in the highest quality mode. Thus, $\overline{\tau_{ecv}} \underset{\varphi}{\rightarrow} max$. Therefore, the average value of the equivalent lifetime in the composite resonator will be maintained at the maximum possible level.

Although $r_2 \ll r_1$, but because $l_2 \gg l_1$, the corresponding products can be comparable in values: $r_2 l_2 \cong r_1 l_1$, from which it follows that the quality factor of the target resonator may well exceed the quality factor of the main resonator, which means that the condition for the maximum quality factor of the composite resonator is the coincidence of the phase structure in both resonators—the main resonator and the target resonator $\varphi \rightarrow k\pi, k = 0,1, \dots$.

This statement is not difficult to prove using the previously applied principle of maximum entropy production (6) to the distribution of energy between partial resonators if we represent it as a function of the reflection phase difference.

In the task of analyzing the vibrations of distant objects, the greatest possible independence of the Signal-to-Noise Ratio (SNR) from an increase in the distance to the object is of decisive importance. Since the level of the device's noise does not depend on the distance to the object, SNR is determined only by the level of the useful signal—S.

Let's compare the dependence of the useful signal S on the distance to the object -l_2 between the parametric, on the one hand, conventional method, and LFI—method in the interpretation of Lang-Kobayashi [13], on the other. The useful signal, regardless of the schemes, is proportional to the interference term.

According to [13] and our scheme (Fig. 1), the radiation in the laser cavity interferes with the radiation reflected by the object:

$$\left(\frac{1}{\tau_1} E(t)e^{i\omega t} + kE(t)e^{i(\omega t+\varphi)}\right).$$

Interference term according to [2]:

$$I_{LK} = 2Re\sqrt{\frac{k}{\tau_1}}\sqrt{\frac{r_2}{r_1}}e^{i\varphi}, \qquad (20)$$

where: $k = \frac{ca}{2\eta l_1}$, $a = (1 - r_1)^2\sqrt{\frac{r_2}{r_1}}$.

We accept that: $(1 - r_1) \approx 1$, $\eta \approx 1$, $\varphi = \Omega$t. l_1 is laser resonator length, Ω is frequency AOM. ω is frequency and c is the speed of light.

Taking into account the accepted assumptions:

$$I_{LK} = \frac{\sqrt{\ln r_1^{-1}}}{\tau_1}\sqrt{\frac{r_2}{r_1}}\cos\Omega t. \qquad (21)$$

In the conventional scheme, the -th part of the radiation branched from the resonator and, reflected by the object $(1 - \kappa)$-th part: $\left(\frac{1}{\tau_1}\right)\left(\sqrt{(1-\kappa)}E(t)e^{i\omega t} + \sqrt{\kappa r_2}E(t)e^{i(\omega t+\varphi)}\right)$.

Interference term:

$$I_C = \frac{2\sqrt{\kappa - \kappa^2}}{\tau_1}\sqrt{r_2}\cos\Omega t.$$

Let us accept that $\kappa = 0.5$, because an interference term is at its maximum.

It is known [23] that the power of radiation reflected by an object falls in proportion to the square of the distance to the object. This is equivalent to a corresponding decrease in the reflection coefficient. Let's replace the constant reflection coefficient r_2 with its function from l_2:

$$r_2(l_2) = \frac{r_0}{(l_2/l_0)^2} = \frac{r_0}{l_2^2} \qquad (22)$$

Here r_0—object surface albedo, that is reflection coefficient of an object at a distance l_0, equal to the length unit l_2.

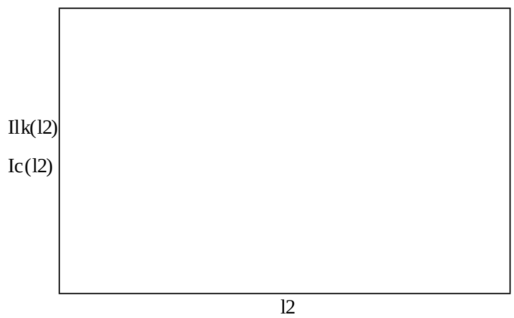

Figure 2: The ratio of useful signal levels

Fig. 2 (red lower curve) shows the ratio of the useful signal level calculated according to the P-model to the model calculated according to the Lang-Kobayashi equation:

$$\frac{I_P}{I_{LK}} = 20\log\left(\frac{4\sqrt{l_2}}{\sqrt{l_1 \ln r_1^{-1}}}\right) \quad (23)$$

The blue-colored upper curve shows the ratio of the useful signal level I_P, calculated according to the P-model, to the level Ic, calculated for the conventional model.

$$\frac{I_P}{I_C} = 20\log\frac{2\sqrt{l_2}}{\sqrt{l_1}\sqrt{(\kappa-\kappa^2)r_1}} \quad (24)$$

From Fig. 2 it can be seen that the P-model demonstrates an excess of the useful signal level at a distance of 100 m from the object by more than 32 dB and by 36 dB at a distance of 300 m about the predicted value according to the equation [13].

At the same (100 m) object distance, the excess of the P-model signal over the conventional one is already more than 40 dB and more than 45 dB at a distance of 300 m.

The conventional model as well as the LFI models, demonstrate equal sensitivity to vibration amplitude, the second sensitivity parameter. Thus, the advantage in sensitivity to reflected radiation is decisive in favor of the design of measuring instruments according to the LFI method.

3. Justification of the Estimation of the Photon Lifetime in a Compound Resonator

Let the radiation energy be at the output of the laser diode (LD). Then the feedback energy is $E_{ref} = Er = Ptr$, where P is the radiation power, t is the time of formation (filling the resonator) with feedback radiation, and r is the feedback coefficient.

Considering that in the diagram in Fig. 1, the power P without loss reaches mirror 3, which reflects only 3% of the incoming power, and 97% reaches mirror 5, we can approximately assume that all the radiated power reaches both mirrors—3 and 5. In practice, the radiated power LD is equal to the power reflected by the main mirror and the mirror of the object:

$$Pt_{ecv}r_{ecv} = Pt_1r_1 + Pt_2r_2 \quad (25)$$

Let's take into account that $r_{ecv} = r_1 + r_2$, radiation fill time of the resonator $t = \frac{l}{c}$, where l—the length of the resonator, c—speed of light. We represent (25) in the form: $P\frac{l_{ecv}}{c}r_{ecv} = P\frac{l_1}{c}r_1 + P\frac{l_2}{c}r_2$. Reducing by $\frac{P}{c}$, we get: $l_{ecv}(r_1 + r_2) = l_1r_1 + l_2r_2$, hence:

$$l_{ecv} = \frac{l_1r_1 + l_2r_2}{r_1 + r_2}. \quad (26)$$

Let's move on to amplitude reflection coefficients:

$\vec{r_1} = \sqrt{r_1}e^{i\omega t}$ is the main resonator.

$\vec{r_2} = \sqrt{r_2}e^{i(\omega t+\varphi)}$ is the resonator of the object.

Considering that $|\vec{r_2}| \ll |\vec{r_1}|$, we accept: $|\vec{r_1} + \vec{r_2}| \approx |\vec{r_1}|$.

The expression (26) is transformed into the expression:

$$l_{ecv} = l_1\left(1 + \sqrt{\frac{l_2}{l_1}}\frac{\sqrt{r_2}}{\sqrt{r_1}}\right)e^{-i\varphi}. \quad (27)$$

Let us substitute (27) into the expression for the equivalent photon lifetime:

$$\tau_{ecv} = \frac{2l_{ecv}}{\mathrm{c}\ln\sqrt{r_{ecv}^{-1}}} = \frac{2l_1\left(1 + \sqrt{\frac{l_2}{l_1}}\frac{\sqrt{r_2}}{\sqrt{r_1}}\right)e^{-i\varphi}}{\mathrm{c}\ln\sqrt{r_1^{-1}}} = \tau_{ph1}\left(1 + \sqrt{\frac{l_2}{l_1}}\frac{\sqrt{r_2}}{\sqrt{r_1}}e^{-i\varphi}\right) \quad (28)$$

We have obtained the equivalent lifetime of photons in a compound resonator in the form (18).

We are interested in the dependence of the τ_{ecv} is equivalent photon lifetime in a composite resonator on the distance between the radiation source and the measurement object. Let us replace the reflection coefficient r_2 with its dependence on l_2 according to (22):

$$\widetilde{\tau_{ecv}} = \tau_{ph1}\left(1 + Re\frac{l_0}{\sqrt{l_1l_2}}\frac{\sqrt{r_0}}{\sqrt{r_1}}e^{-i\varphi}\right) \quad (29)$$

Recall that $l_0 = 1, r_0$ is the albedo of the object surface, i.e. constant values. Thus, from (29) it follows that $\tilde{\tau}_{ecv}$ is the variable component of the equivalent lifetime of photons in a composite resonator, depending on the distance to the object like a square root.

4. Practical Testing of the Proposed Model

To solve the problem of restoring the form of oscillations of the surface of distant objects, a prototype of a laser Doppler vibrometer (hereinafter referred to as the device) based on feedback interferometry—LFI was created (Fig. 3). The vibrometer has the following characteristics: SNR > 26 db at a distance of at least 300 m from the majority of unprepared surfaces.

Sensitivity to vibration amplitude—not worse than 1 nm at a vibration frequency of 1 kHz and a bandwidth of at least 10 kHz. Radiation power—no more than 5 mW. Laser diode type—single mode, AR-coated, external cavity—320 mm. Feedback coefficient—not more than 3%.

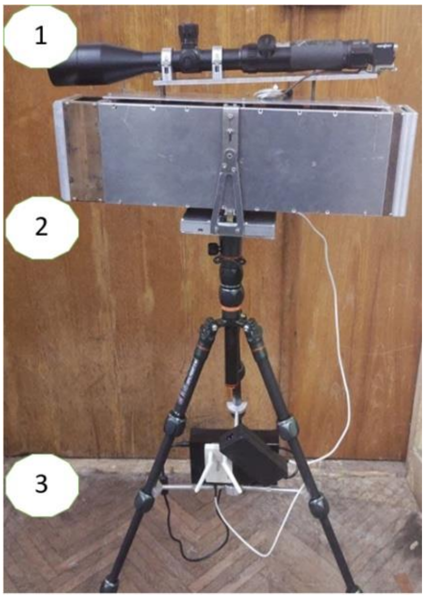

Figure 3: Appearance of the vibrometer: 1 is optic sight, 2 is vibrometer, 3 is a slewing device

The results of the device prototype demonstrated its extremely high sensitivity. So, when the measurement object was removed from the device from 10 to 300 meters, the useful signal dropped from 58 dB to 40 dB. The same high sensitivity was confirmed by the results published in [12], where interference was observed after the probing beam had passed a distance of 40km (in an optical fiber).

Such a high sensitivity cannot be explained within the framework of generally accepted models [13, 14]. Moreover, our data and the data published in [12] indicate that the achieved measurement distances exceed the calculated coherence lengths of the lasers used.

The discrepancy between the results of experiments and the previously existing theoretical estimates prompts a critical review of the generally accepted LFI model. As a result, a model is proposed that otherwise explains the interaction of individual components of the feedback energy in the formation of a useful signal. This reveals the causes and allows you to determine the quantitative difference in the estimates of the useful signal between the models: generally accepted and proposed.

5. Conclusions and Direction for Further Research

As part of the study, the following results were obtained:

- The energy equivalence of the useful signal extracted from the resonator radiation and the useful signal extracted from the pump current is shown. Quantitative estimates have been obtained.
- The concept of the equivalent length of a composite resonator, which depends on the reflection coefficient, is introduced. The optimality of the introduced estimate in the root-mean-square sense is proved, which gives grounds to consider the lifetime of photons in a composite resonator as an information parameter.

The qualitative advantage of the LFI method over the conventional one is revealed. As the distance increases, its SNR advantage increases in proportion to the square root of the distance to the object. The LFI model, represented by the Lang-Kobayashi equation, has no qualitative superiority over the conventional method. According to the Lang-Kobayashi model, as the distance to the object increases, the advantage over the conventional method in terms of SNR does not increase. The calculation of the long-range action, performed according to the P-model, shows that this is not the case. The growth of SNR about the LK model, as well as about the conventional model, is proportional to the square root of the distance to the object. This fact makes it possible to radically reconsider the limits of the range of laser measurements.

The representation of the LFI model as a parametric one allows us to consider it as a system that allows parametric resonance, which opens up new possibilities for improving the LFI efficiency.

6. References

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