Preferential Temporal Description Logics with Typicality and Weighted Knowledge Bases

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Abstract

In this paper we define an extension of a temporal description logic with a typicality operator, to allow for defeasible reasoning in a preferential temporal description logic. We show that a preferential extension of $LTL_{\mathcal{ALC}}$ with typicality can be polynomially encoded into $LTL_{\mathcal{ALC}}$, and the approach allows borrowing some decidability and complexity results. We consider as well a multi-preferential temporal semantic for temporal weighted knowledge bases with typicality.

1. Introduction

Preferential extensions of Description Logics (DLs) allow reasoning with exceptions through the identification of *prototypical properties* of individuals or classes of individuals. *Defeasible inclusions* are allowed in the knowledge base, to model typical, defeasible, non-strict properties of individuals. Their semantics extends DL semantics with a preference relation among domain individuals, along the lines of the preferential semantics introduced by Kraus, Lehmann and Magidor [1, 2] (KLM for short). Preferential extensions and rational extensions of the description logic $\mathcal{ALC}$ [3] have been studied [4, 5], and several different closure constructions have been developed [6, 7, 8, 9, 10, 11], inspired by Lehmann and Magidor's rational closure [2] and Lehmann's lexicographic closure [12]. More recently, *multi-preferential* extensions of DLs have been developed, by allowing multiple preference relations with respect to different concepts [13, 14, 15], as the semantic for ranked and for weighted knowledge bases with typicality.

Temporal extensions of Description Logics are very well-studied in DLs literature, see the survey on temporal DLs and their complexity and decidability [16]. While preferential extensions of LTL with defeasible temporal operators have been recently studied [17, 18, 19] to enrich temporal formalisms with non-monotonic reasoning features, preferential extensions (and, more specifically, *typicality based* extensions) of temporal DLs have not been considered so far, up to

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our knowledge.

To fill this gap, in this paper we develop a preferential extension of Temporal DLs, based on the approach proposed in [5] to define a description logic with typicality. More specifically, we build over a temporal extension of $\mathcal{ALC}$, $LTL_{\mathcal{ALC}}$ [16], based on Linear Time Temporal Logic (LTL), and develop its extension with typicality.

Generalizing the approach in [5], we define a preferential temporal description logic $LTL_{\mathcal{ALC}}$ with typicality, $LTL^{\mathbf{T}}_{\mathcal{ALC}}$, by adding to the language a typicality operator $\mathbf{T}$ that selects the most typical instances of a concept C. The resulting temporal DL with typicality allows for representing temporal properties of concepts which admit exceptions, e.g., for instance that, normally, professors teach at least a course until the end of the semester, although exceptions are permitted.

We show that the preferential extension of $LTL_{\mathcal{ALC}}$ with typicality can be polynomially encoded into $LTL_{\mathcal{ALC}}$, and this approach allows borrowing decidability and complexity results from $LTL_{\mathcal{ALC}}$. We also consider a multi-preferential extension of $LTL_{\mathcal{ALC}}$, by allowing a concept-wise preferential semantics where different preferences are associated to different concepts. The encoding also applies to this case. We discuss possible extensions of the closure constructions for weighted knowledge bases [14, 20] to the temporal case. It allows for a finer grained representation of the plausibility of prototypical properties of a concept, including temporal properties, by assigning weights to the different typicality properties.

2. The Description Logic $\mathcal{ALC}$

In this section we recall the syntax and semantics of the description logic $\mathcal{ALC}$ [3] and of its temporal extension $LTL_{\mathcal{ALC}}$ [16].

2.1. $\mathcal{ALC}$

Let N_C be a set of concept names, N_R a set of role names and N_I a set of individual names. The set of $\mathcal{ALC}$ *concepts* (or, simply, concepts) can be defined inductively as follows:

- $A \in N_C$, $\top$ and $\bot$ are concepts;
- if C and D are concepts, and $r \in N_R$, then $C \sqcap D$, $C \sqcup D$, $\neg C$, $\forall r.C$, $\exists r.C$ are concepts.

An $\mathcal{ALC}$ knowledge base (KB) K is a pair $(\mathcal{T}, \mathcal{A})$, where $\mathcal{T}$ is a TBox and $\mathcal{A}$ is an ABox. The TBox $\mathcal{T}$ is a set of concept inclusions (or subsumptions) $C \sqsubseteq D$, where C, D are concepts. The ABox $\mathcal{A}$ is a set of assertions of the form $C(a)$ and $r(a, b)$ where C is a concept, a and b are individual names in N_I and r a role name in N_R.

An $\mathcal{ALC}$ *interpretation* is defined as a pair $I = \langle \Delta, \cdot^I \rangle$ where: Δ is a domain — a set whose elements are denoted by $x, y, z, \ldots$, and $\cdot^I$ is an extension function that maps each concept name $C \in N_C$ to a set $C^I \subseteq \Delta$, each role name $r \in N_R$ to a binary relation $r^I \subseteq \Delta \times \Delta$, and each individual name $a \in N_I$ to an element $a^I \in \Delta$. It is extended to complex concepts:

$\top^I = \Delta, \qquad \bot^I = \emptyset, \qquad (\neg C)^I = \Delta \backslash C^I,$

$(\exists r.C)^I = \{x \in \Delta \mid \exists y.(x, y) \in r^I \text{ and } y \in C^I\}, \qquad (C \sqcap D)^I = C^I \cap D^I,$

$(\forall r.C)^I = \{x \in \Delta \mid \forall y.(x, y) \in r^I \Rightarrow y \in C^I\}, \qquad (C \sqcup D)^I = C^I \cup D^I.$

The notions of satisfiability of a KB in an interpretation and entailment are defined as follows:

Definition 1 (Satisfiability and entailment). *Given an $\mathcal{ALC}$ interpretation $I = \langle \Delta, \cdot^I \rangle$:*
- I satisfies an inclusion $C \sqsubseteq D$ if $C^I \subseteq D^I$;
- I satisfies an assertion $C(a)$ (resp., $r(a,b)$) if $a^I \in C^I$ (resp., $(a^I, b^I) \in r^I$).
Given a KB $K = (\mathcal{T}, \mathcal{A})$, an interpretation I satisfies $\mathcal{T}$ (resp. $\mathcal{A}$) if I satisfies all inclusions in $\mathcal{T}$ (resp. all assertions in $\mathcal{A}$); I is a model *of K if I satisfies $\mathcal{T}$ and $\mathcal{A}$.*

A concept inclusion $F = C \sqsubseteq D$ (resp., an assertion $C(a)$, $r(a,b)$), is entailed by K, written $K \models F$, if for all models $I = \langle \Delta, \cdot^I \rangle$ of K, I satisfies F.

Given a knowledge base K, the *subsumption* problem is the problem of deciding whether an inclusion $C \sqsubseteq D$ is entailed by K. The *satisfiability* problem is the problem of deciding whether a knowlwdge base K has a model. The *concept satisfiability* problem is the problem of deciding, for a concept C, whether C is consistent with K (i.e., whether there exists a model I of K, such that $C^I \neq \emptyset$).

3. The Temporal Description Logic $LTL_{\mathcal{ALC}}$

The concepts of the temporal description logic $LTL_{\mathcal{ALC}}$ can be formed from standard constructors using the temporal operators $\bigcirc$ (next), $\mathcal{U}$ (until), $\Diamond$ (eventually) and $\Box$ (always) of linear time temporal logic (LTL). The set of temporally extended concepts is as follows:

$$C ::= A \mid \top \mid \bot \mid C \sqcap D \mid C \sqcup D \mid \neg C \mid \forall r.C \mid \exists r.C \mid \bigcirc C \mid C \mathcal{U} D \mid \Diamond C \mid \Box C$$

where $A \in N_C$, and C and D are temporally extended concepts.

A *temporal interpretation* for $LTL_{\mathcal{ALC}}$ is a pair $\mathcal{I} = (\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}})$, where $\Delta^{\mathcal{I}}$ is a nonempty domain; $\cdot^{\mathcal{I}}$ is an extension function that maps each concept name $C \in N_C$ to a set $C^{\mathcal{I}} \subseteq \mathbb{N} \times \Delta^{\mathcal{I}}$, each role name $r \in N_R$ to a relation $r^{\mathcal{I}} \subseteq \mathbb{N} \times \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$, and each individual name $a \in N_I$ to an element $a^{\mathcal{I}} \in \Delta^{\mathcal{I}}$. Following [16] we assume individual names to be *rigid*, i.e., having the same interpretation at any time point. In a pair $(n, d) \in \mathbb{N} \times \Delta^{\mathcal{I}}$, n represents a time point and d a domain element; $(n, d) \in C^{\mathcal{I}}$ means that d is an instance of concept C at time point n, and similarly for $(n, d_1, d_2) \in r^{\mathcal{I}}$. Function $\cdot^{\mathcal{I}}$ is extended to complex concepts as follows:

$$\top^{\mathcal{I}} = \mathbb{N} \times \Delta^{\mathcal{I}} \qquad \bot^{\mathcal{I}} = \emptyset \qquad (\neg C)^{\mathcal{I}} = (\mathbb{N} \times \Delta^{\mathcal{I}}) \backslash C^{\mathcal{I}}$$
$$(C \sqcap D)^{\mathcal{I}} = C^{\mathcal{I}} \cap D^{\mathcal{I}} \qquad (C \sqcup D)^{\mathcal{I}} = C^{\mathcal{I}} \cup D^{\mathcal{I}}$$
$$(\exists r.C)^{\mathcal{I}} = \{(n, x) \in \mathbb{N} \times \Delta^{\mathcal{I}} \mid \exists y.(n, x, y) \in r^{\mathcal{I}} \text{ and } (n, y) \in C^{\mathcal{I}}\}$$
$$(\forall r.C)^{\mathcal{I}} = \{(n, x) \in \mathbb{N} \times \Delta^{\mathcal{I}} \mid \forall y.(n, x, y) \in r^{\mathcal{I}} \Rightarrow (n, y) \in C^{\mathcal{I}}\}$$
$$(\bigcirc C)^{\mathcal{I}} = \{(n, x) \in \mathbb{N} \times \Delta^{\mathcal{I}} \mid (n+1, x) \in C^{\mathcal{I}}\}$$
$$(\Diamond C)^{\mathcal{I}} = \{(n, x) \in \mathbb{N} \times \Delta^{\mathcal{I}} \mid \exists m \geq n \text{ such that } (m, x) \in C^{\mathcal{I}}\}$$
$$(\Box C)^{\mathcal{I}} = \{(n, x) \in \mathbb{N} \times \Delta^{\mathcal{I}} \mid \forall m \geq n, \ \ (m, x) \in C^{\mathcal{I}}\}$$
$$(C \mathcal{U} D)^{\mathcal{I}} = \{(n, x) \in \mathbb{N} \times \Delta^{\mathcal{I}} \mid \exists m \geq n \text{ s.t. } (m, x) \in D^{\mathcal{I}} \text{ and } (k, x) \in C^{\mathcal{I}}, \forall k \ (n \leq k < m)\}$$

While the definition above assumes a *constant domain* (i.e., that the domain elements are the same at all time points), in the following we will also consider the case with *expanding domains*, when there is a sequence of increasing domains $\Delta_0^{\mathcal{I}} \subseteq \Delta_1^{\mathcal{I}} \subseteq \ldots$, one for each time point.

Let a TBox $\mathcal{T}$ be a set of concept inclusions $C \sqsubseteq D$, where C, D are temporally extended concepts, as above. It has been proven that concept satisfiability in $LTL_{\mathcal{ALC}}$ w.r.t. TBoxes is EXPTIME-complete, both with expanding domains [21] and with constant domains [16].

The complexity of other cases and, specifically, the cases of temporal ABoxes [22] and temporal TBoxes (which allow temporal operators over concept inclusions), have as well been studied in the literature, and we refer to [16] for a discussion of the result and algorithms for satisfiability checking.

In the next section we develop a preferential extension of $LTL_{\mathcal{ALC}}$. For simplicity, we focus on the case of non-temporal ABox and TBox, i.e., with the TBox containing a set of concept inclusions $C \sqsubseteq D$, where C, D are temporally extended concepts, but without temporal operator applied to the concept inclusions themselves.

4. $LTL^{\mathbf{T}}_{\mathcal{ALC}}$: A Preferential Extension of $LTL_{\mathcal{ALC}}$ with Typicality

In this section we define an extension of the temporal description logic $LTL_{\mathcal{ALC}}$ allowing typicality concepts of the form $\mathbf{T}(C)$, where C is a $LTL_{\mathcal{ALC}}$ concept. The instances of $\mathbf{T}(C)$ are intended to be the *typical* instances of a concept C. Following [5], we call $\mathbf{T}$ a *typicality operator*. The concept $\mathbf{T}(C)$ can be used on the left hand side of concept inclusions to express defeasible properties of a concept C of the form $\mathbf{T}(C) \sqsubseteq D$, meaning that the *typical* instances of concept C are also instances of concept D (normally, C's are D's). We can therefore distinguish between properties that hold for all instances of C, expressed by *strict inclusions* ($C \sqsubseteq D$), and those that only hold for the typical instances of C, expressed by *typicality* or *defeasible inclusions* ($\mathbf{T}(C) \sqsubseteq D$).

Unlike [5, 9], where a typicality operator was introduced for $\mathcal{ALC}$, here we do not require that the typicality operator only occurs on the left hand side of concept inclusions, and this choice is in agreement with [23, 24]. As usual, we assume that the typicality operator $\mathbf{T}$ cannot be nested. *Extended concepts* can be built by combining the concept constructors in $LTL_{\mathcal{ALC}}$ with the typicality operator. They can freely occur in concept inclusions, such as, for instance, in the following ones:

$$\mathbf{T}(\mathit{Professor}) \sqsubseteq (\exists \mathit{teaches}.\mathit{Course})\,\mathcal{U}\,\mathit{Semester\_End}$$
$$\exists \mathit{lives\_in}.\mathit{Town} \sqcap \mathit{Young} \sqsubseteq \mathbf{T}(\Diamond \exists \mathit{granted}.\mathit{Loan})$$

The first inclusion means that normally professors teach at least a course until the end of the semester (but exceptions are allowed). The second one means that persons living in town and being young are typical in the set of individuals eventually being granted a loan.

We define a preferential extension, $LTL^{\mathbf{T}}_{\mathcal{ALC}}$, of $LTL_{\mathcal{ALC}}$. As for the preferential extension of the logic $\mathcal{ALC}$ [5], we define the semantics of $LTL^{\mathbf{T}}_{\mathcal{ALC}}$ in terms of preferential models, extending ordinary models of $LTL^{\mathbf{T}}_{\mathcal{ALC}}$ with a *preference relation* $<$ on the domain, whose intuitive meaning is to compare the "typicality" of domain elements, that is to say, $x < y$ means that *domain element x is more typical than y*. The typical instances of an (extended) concept C (the instances of

$\mathbf{T}(C)$) are the instances x of C that are minimal with respect to the preference relation $<$ (i.e., no other instances of C are preferred to x).

In the following, we will consider a collection of preference relations $<^n$, one for each time point n. They will be defined as the projections of a relation $<$ over the single time points.

Definition 2 (Preferential temporal interpretations for $LTL^{\mathbf{T}}_{\mathcal{ALC}}$)**.** *An $LTL^{\mathbf{T}}_{\mathcal{ALC}}$ interpretation is a structure $\mathcal{M} = (\Delta^{\mathcal{I}}, <, \cdot^{\mathcal{I}})$ where:*

- *$(\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}})$ is a temporal interpretation as for $LTL_{\mathcal{ALC}}$, as introduced in Section 3, but the interpretation function $\cdot^{\mathcal{I}}$ is extended to typicality concepts (see below);*
- *the relation $< \subseteq \mathbb{N} \times \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$ associates to each time point n a preference $<^n$ over the domain $\Delta^{\mathcal{I}}$ such that, for all $n \in \mathbb{N}$, $<^n = \{(a,b) \mid (n,a,b) \in <\}$ and relation $<^n$ is an irreflexive, transitive and well-founded relation over $\Delta^{\mathcal{I}}$;*
- *the interpretation of typicality concepts $\mathbf{T}(C)$ is defined as follows:*

$$(\mathbf{T}(C))^{\mathcal{I}} = \{(n,d) \mid d \in Min_{<^n}(C^{\mathcal{I}}_n), \textit{ for } n \in \mathbb{N}\}$$

where $C^{\mathcal{I}}_n = \{d \mid (n,d) \in C^{\mathcal{I}}\}$ are the instances of C at time point n, and $Min_{<^n}(S) = \{u : u \in S \text{ and } \nexists z \in S \text{ s.t. } z <^n u\}$.

Furthermore, we say that relation $<^n$ is well-founded *if, for all $S \subseteq \Delta^{\mathcal{I}}$, for all $x \in S$, either $x \in Min_{<^n}(S)$ or $\exists y \in Min_{<^n}(S)$ such that $y <^n x$.*

For each timepoint n, relation $<^n$ has the properties of preference relation in KLM preferential interpretations [1, 2]. When *modularity* also holds for $<^n$ (i.e., for all $x, y, z \in \Delta^{\mathcal{I}}$, $x <^n y$ implies ($x <^n z$ or $z <^n y$)), $<^n$ has the properties of preference relation in rational (or ranked) KLM interpretations [2]. In the following, however, we will not restrict to modular relations $<^n$.

The relation $<$ can be regarded as a function associating to each time point n a preference relation $<^n$ over $\Delta^{\mathcal{I}}$, i.e., $<^n \subseteq \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$. At each time point n, the typicality concept $\mathbf{T}(C)$ is interpreted as the set of maximally preferred C-elements, according to the preference relation $<^n$ for time point n.

As for the temporal language $LTL_{\mathcal{ALC}}$ [16], although in this section we have used a constant domain $\Delta^{\mathcal{I}}$ in a preferential temporal interpretation, expanding domains could have been considered as well, by letting a domain $\Delta^{\mathcal{I}}_n$, for each time point n.

The notions of satisfiability and model of a knowledge base can be easily extended to $LTL^{\mathbf{T}}_{\mathcal{ALC}}$ with non-temporal ABox and TBox. As $\mathcal{A}$ is a non-temporal ABox, the assertions in $\mathcal{A}$ are evaluated at time point 0. On the other hand, all inclusions in the (non-temporal) TBox $\mathcal{T}$ have to be satisfied at all time points.

Definition 3 (Satisfiability in $LTL^{\mathbf{T}}_{\mathcal{ALC}}$)**.** *Given an $LTL^{\mathbf{T}}_{\mathcal{ALC}}$ interpretation $\mathcal{M} = \langle \Delta^{\mathcal{I}}, <, \cdot^{\mathcal{I}} \rangle$, $\mathcal{M}$ satisfies a concept inclusion $C \sqsubseteq D$ iff $C^{\mathcal{I}} \subseteq D^{\mathcal{I}}$; $\mathcal{M}$ satisfies an assertion $C(a)$ (resp., $r(a,b)$) iff $(0, a^{\mathcal{I}}) \in C^{\mathcal{I}}$ (resp., $(0, a^{\mathcal{I}}, b^{\mathcal{I}}) \in r^{\mathcal{I}}$).*
Given an $LTL^{\mathbf{T}}_{\mathcal{ALC}}$ knowledge base $K = (\mathcal{T}, \mathcal{A})$, the interpretation $\mathcal{M}$ is a model *of K if $\mathcal{M}$ satisfies all concept inclusions in $\mathcal{T}$ and all assertions in $\mathcal{A}$. An $LTL^{\mathbf{T}}_{\mathcal{ALC}}$ knowledge base $K = (\mathcal{T}, \mathcal{A})$ is* satisfiable *in $LTL^{\mathbf{T}}_{\mathcal{ALC}}$ if a model $\mathcal{M} = \langle \Delta^{\mathcal{I}}, <, \cdot^{\mathcal{I}} \rangle$ of K exists.*

The fact that each irreflexive and transitive relation $<^n$ on Δ is well-founded guarantees that, for any $<^n$, there are no infinite descending chains of elements of $\Delta^{\mathcal{I}}$.

At any time point n there is a possibly different relation $<^n$, which allows to identify the typical instances of a concept C at any time point n. As observed in [5] for $\mathcal{ALC}$ with typicality, the meaning of $\mathbf{T}$ can be split into two parts: for any element $x \in \Delta^{\mathcal{I}}$, $x \in (\mathbf{T}(C))^I$ when (i) $x \in C^I$, and (ii) there is no $y \in C^I$ such that $y < x$ (note that, for $\mathcal{ALC}$ with typicality, there is a single preference relation $<$ on the domain $\Delta^{\mathcal{I}}$). In order to isolate the second part of the meaning of $\mathbf{T}$, one can introduce a Gödel-Löb modality (for which we use the symbol $\Box_<$, while $\Box$ is used for the temporal operator *always*), and interpret the preference relation $<$ as the inverse of the accessibility relation of this modality. Well-foundedness of $<$ ensures that typical elements of C^I exist whenever $C^I \neq \emptyset$, by avoiding infinitely descending chains of elements. The interpretation of $\Box_<$ in $\mathcal{M}$ is as follows: $(\Box_< C)^I = \{x \in \Delta^{\mathcal{I}} \mid$ for every $y \in \Delta^{\mathcal{I}}$, if $y < x$ then $y \in C^I\}$. For the case of $\mathcal{ALC}$ with typicality, it has been proven that x is a typical instance of C if and only if it is an instance of C and $\Box_< \neg C$, that is: given an interpretation $\mathcal{M}$, a concept C and an element $x \in \Delta$, $x \in (\mathbf{T}(C))^I$ iff $x \in (C \sqcap \Box_< \neg C)^I$ [5].

This modal interpretation of the typicality operator $\mathbf{T}$ in terms of a Gödel-Löb modality $\Box_<$ has been used to define an encoding of $\mathcal{SROIQ}^P\mathbf{T}$ into $\mathcal{SROIQ}$ [23] as well as for encoding a preferential extension of $\mathcal{SHIQ}$ into $\mathcal{SHIQ}$, by introducing a new role $P_<$ in the DL language to represent the preference relation. In the next section, we will extend this encoding to the temporal case for $\mathcal{ALC}$.

5. Encoding of $LTL^{\mathbf{T}}_{\mathcal{ALC}}$ in $LTL_{\mathcal{ALC}}$

In this section we show that reasoning in $LTL^{\mathbf{T}}_{\mathcal{ALC}}$ can be reduced polynomially to reasoning in $LTL_{\mathcal{ALC}}$. The idea, as reported above, is to define an encoding of the typicality concept in the temporal description logic, by interpreting $\mathbf{T}(C)$ as a formula $A \sqcap \Box_< \neg A$, where the accessibility relation of the modality $\Box_<$ is the inverse of the preference relation.

The interpretation of $\mathbf{T}(C)$ at a time point n is to be evaluated based on the preference relation $<$ at time point n, i.e., based on $<^n$. We represent the preference relation $<$ in a preferential temporal interpretation $\mathcal{M}$ (see Definition 2) by introducing a new role $P_<$ in the language. Also, we represent a concept $\mathbf{T}(C)$ with the concept $C \sqcap \Box_{\neg C}$, where $\Box_{\neg C}$ is a *new concept name* which is intended to capture the meaning of formula $\Box_< \neg C$ (dropping the $<$ to make notation lighter). Finally, we will introduce additional concept inclusion axioms to capture the interplay between role $P_<$ and the new concepts $\Box_{\neg C}$, as well as to enforce the properties of the preference relations $<^n$.

Let $K = (\mathcal{T}, \mathcal{A})$ be a $LTL^{\mathbf{T}}_{\mathcal{ALC}}$ knowledge base and let N_C, N_R, N_I be the set of concept names, role names and individual names in the language of K. We define the encoding $K' = (\mathcal{T}', \mathcal{A}')$ of K in $LTL_{\mathcal{ALC}}$ over the concept names, role names and individual names in N'_C, N'_R, N'_I, as follows.

The language of K' contains all the individual names, concept names and role names in the language of K (i.e., $N_C \subseteq N'_C$, $N_R \subseteq N'_R$, $N_I \subseteq N'_I$). For each $\mathbf{T}(A)$ occurring in K (where A is any, possibly complex, temporally extended concept), we introduce in N'_C a new atomic concept $\Box_{\neg A}$ and, for each inclusion $C \sqsubseteq D \in \mathcal{T}$, we introduce in $\mathcal{T}'$ the inclusion

$C' \sqsubseteq D'$, where C' and D' are obtained from C and D, respectively, by replacing the occurrence of any concept $\mathbf{T}(A)$ with the concept $A \sqcap \Box_{\neg A}$. Note that concept $\Box_{\neg A}$ may have a different interpretation at each time point.

As mentioned above, to capture the properties of the $\Box_<$ modality, a new role name $P_<$ is introduced to represent the relation $<$ in preferential models, and the following concept inclusion axioms are introduced in $\mathcal{T}'$, for all concepts A such that $\mathbf{T}(A)$ occurs in $\mathcal{T}$:

$$\Box_{\neg A} \sqsubseteq \forall P_<.(\neg A \sqcap \Box_{\neg A}) \quad (1)$$

$$\neg\Box_{\neg A} \sqsubseteq \exists P_<.(A \sqcap \Box_{\neg A}) \quad (2)$$

The first inclusion accounts for the transitivity of the preference relations $<^n$. The second inclusion accounts for the smoothness (see [2]) of the preference relations $<^n$, i.e., the fact that if an element is not a typical A element at a time point n, then there must be a typical A element preferred to it according to $<^n$. The property holds for a well-founded relation $<^n$.

We also define ABox $\mathcal{A}'$ by replacing each occurrence of the concept $\mathbf{T}(A)$ in any individual assertions $C(d)$ in $\mathcal{A}$, with the concept $A \sqcap \Box_{\neg A}$, and by including in $\mathcal{A}'$ all the resulting assertions. All the assertions of the form $R(a, b) \in \mathcal{A}$ are included unaltered in $\mathcal{A}'$.

Proposition 1. *For a temporal knowledge base $K = (\mathcal{T}, \mathcal{A})$ in $LTL^{\mathbf{T}}_{\mathcal{ALC}}$, let K' be the encoding of K in $LTL_{\mathcal{ALC}}$. It holds that K is satisfiable in $LTL^{\mathbf{T}}_{\mathcal{ALC}}$ iff K' is satisfiable in $LTL_{\mathcal{ALC}}$.*

As it is clear, the encoding above is polynomial in the size of the knowledge base K and, more precisely, if $|K|$ is the size of K, the size of K' is $O(|K|)$.

As a consequence of Proposition 1, the decidability and complexity results that have been proven to hold for the temporal description logic $LTL_{\mathcal{ALC}}$ also extend to the preferential temporal description logic $LTL^{\mathbf{T}}_{\mathcal{ALC}}$. Note that our encoding does not depend on assumptions on constant domains, and it works as well for expanding domains.

In particular, for non-temporal TBoxes $\mathcal{T}$, that is, a set of concept inclusions $C \sqsubseteq D$, where C, D are $LTL^{\mathbf{T}}_{\mathcal{ALC}}$ concepts, the following holds as a consequence of the encoding above and of the results for $LTL_{\mathcal{ALC}}$ with expanding domains and with constant domains [21, 16].

Corollary 1. *Concept satisfiability in $LTL^{\mathbf{T}}_{\mathcal{ALC}}$ w.r.t. TBoxes is* EXPTIME*-complete, both with expanding domains and with constant domains.*

Note that this encoding which exploits $\mathcal{ALC}$ constructs can as well be adopted for more expressive logics, although for expressive DLs alternative encodings might be viable. The preferential extension $LTL^{\mathbf{T}}_{\mathcal{ALC}}$ and its encoding in $LTL_{\mathcal{ALC}}$ can as well be considered for knowledge bases with temporal TBoxes and temporal ABoxes, with minor modifications of the proof of Proposition 1. While we leave the detailed treatment of these cases for future work, in the next sections, we move to consider a *multi-preferential semantics* for temporal $\mathcal{ALC}$ with typicality, as well as possible *closure constructions* for these extension.

6. A Multi-preferential Temporal Extension of $\mathcal{ALC}$

Following [13, 14, 20], we can consider a multi-preferential extension of temporal $\mathcal{ALC}$ with typicality $LTL^{\mathbf{T}}_{\mathcal{ALC}}$. Let us call it $LTL^{\mathbf{T},m}_{\mathcal{ALC}}$, by associating a preference relation $<_{C_i}$ with each concept C_i in a set of distinguished concepts $\mathcal{C} = \{C_1, \ldots, C_k\}$. The underlying idea is that the distinguished concepts C_i represent the aspects with respect to which domain individuals are compared. For instance, Tom may be more typical than Bob as a student ($tom <_S bob$), but less typical as an employed student ($bob <_{ES} tom$).

In the temporal case, this means that, at each time point n, there are different preference relations $<^n_{C_1}, \ldots, <^n_{C_k}$ one for each $C_i \in \mathcal{C}$. Let us assume, for the moment, that the typicality operator only applies to the distinguished concepts C_i. The notion of multi-preferential temporal interpretation for $LTL^{\mathbf{T},m}_{\mathcal{ALC}}$ is defined as follows:

Definition 4 (Multi-preferential temporal interpretations for $LTL^{\mathbf{T},m}_{\mathcal{ALC}}$). *Let $\mathcal{C} = \{C_1, \ldots, C_k\}$ be a set of distinguished concepts. An $LTL^{\mathbf{T},m}_{\mathcal{ALC}}$ interpretation over $\mathcal{C}$ is a structure $\mathcal{M} = (\Delta^{\mathcal{I}}, <_{C_1}, \ldots, <_{C_k}, \cdot^{\mathcal{I}})$ where:*

- *$\Delta^{\mathcal{I}}$ is a nonempty domain;*
- *for each $C_i \in \mathcal{C}$, relation $<_{C_i} \subseteq \mathbb{N} \times \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$ associates to each time point n a preference $<^n_{C_i}$ over the domain $\Delta^{\mathcal{I}}$ such that, for all $n \in \mathbb{N}$, $<^n_{C_i} = \{(a, b) \mid (n, a, b) \in <_{C_i}\}$ and relation $<^n_{C_i}$ is an irreflexive, transitive and well-founded relation over $\Delta^{\mathcal{I}}$;*
- *The interpretation function $\cdot^{\mathcal{I}}$, introduced in Section 3 for temporally extended goals, is extended to typicality concepts $\mathbf{T}(C_i)$ as follows:*

$$(\mathbf{T}(C_i))^{\mathcal{I}} = \{(n, d) \mid d \in Min_{<^n_{C_i}}(C^{\mathcal{I}}_{i,n}), \textit{ for } n \in \mathbb{N}\}$$

where $C^{\mathcal{I}}_{i,n} = \{d \mid (n, d) \in C^{\mathcal{I}}_i\}$ are the instances of C_i at time point n.

Let us define an $LTL^{\mathbf{T},m}_{\mathcal{ALC}}$ knowledge base $K = \langle \mathcal{T}, \mathcal{A} \rangle$ as an $LTL^{\mathbf{T}}_{\mathcal{ALC}}$ knowledge base in which only typicality concepts of the form $\mathbf{T}(C_i)$ may occur. An encoding in $LTL_{\mathcal{ALC}}$ of the different preference relations associated to concepts can be defined in a similar way as for the single preference relation $<$, but requires to introduce a new role $P_{<_{C_i}}$, for each distinguished concept $C_i \in \mathcal{C}$, as well as a new concept name $\Box_{\neg C_i}$ to encode a typicality concept $\mathbf{T}(C_i)$ occurring in the TBox with concept $C_i \sqcap \Box_{\neg C_i}$ in $LTL_{\mathcal{ALC}}$. The two axioms, (1) and (2) need as well to be introduced for all $C_i \in \mathcal{C}$.

The result that concept satisfiability in $LTL^{\mathbf{T}}_{\mathcal{ALC}}$ w.r.t. TBoxes is EXPTIME-complete also extends to the multi-preferential temporal $\mathcal{ALC}$ under the $LTL^{\mathbf{T},m}_{\mathcal{ALC}}$ semantics.

6.1. Global Preference

Note that, given the preferences $<^n_{C_i}$ for the distinguished concepts, one can interpret $\mathbf{T}(C_i)$ at time point n as the set of minimal C_i elements w.r.t. $<^n_{C_i}$. However, to provide an interpretation of the typicality concept $\mathbf{T}(C)$ for an arbitrary C (such as, for instance, $\mathbf{T}(Employee \sqcap Student)$), one would need to define a preference relation with respect to C or, in alternative, a notion of global preference relation. Many notions of preference combination have been considered and studied in the literature [25, 26]. Following [13], a notion of global preference $<$ can, for instance, be defined by exploiting a modified *Pareto* combination of the preference relations $<_{C_1}, \ldots, <_{C_k}$, which takes into account the *specificity relation* $\succ$ among concepts, e.g., that concept $PhDStudent$ is more specific than concept $Student$ ($PhDStudent \succ Student$), and its properties override the properties of $Student$, when conflicting. The global preference relation $<^n$ at time point n can be defined from $<^n_{C_1}, \ldots, <^n_{C_k}$ as follows:

$$x <^n y \text{ iff } \; (i)\; x <^n_{C_i} y, \text{ for some } C_i \in \mathcal{C}, \text{ and}$$
$$(ii)\; \text{ for all } C_j \in \mathcal{C},\; x \leq^n_{C_j} y \text{ or } \exists C_h (C_h \succ C_j \text{ and } x <^n_{C_h} y).$$

We interpret $\mathbf{T}(C)$, for an arbitrary concept C, at time point n, as the set of minimal C-elements with respect to $<^n$, i.e., $(\mathbf{T}(C))^{\mathcal{I}} = \{(n,d) \mid d \in Min_{<^n}(C^{\mathcal{I}}_n), \text{ for } n \in \mathbb{N}\}$, where $C^{\mathcal{I}}_n = \{d \mid (n,d) \in C^{\mathcal{I}}\}$ are the instances of concept C at time point n. This leads to the definition of a *concept-wise multi-preferential temporal interpretation* (cw^m-interpretation) for $LTL^{\mathbf{T},m}_{\mathcal{ALC}}$ as a a tuple $\mathcal{M} = \langle \Delta, <_{C_1}, \ldots, <_{C_k}, <, \cdot^I \rangle$, where $\langle \Delta, <_{C_1}, \ldots, <_{C_k}, \cdot^I \rangle$ is an $LTL^{\mathbf{T},m}_{\mathcal{ALC}}$ multi-preferential temporal interpretation (see Definition 4), and $< \subseteq \mathbb{N} \times \Delta \times \Delta$ is a *global preference relation*, defined from the relations $<^n \subseteq \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$ as follows: $(n,a,b) \in <$ iff $(a,b) \in <^n$.

7. Temporal Weighted KBs and a Closure Construction

Preferential logics provide a rather weak notion non-monotonic inference and can be used as the basis for stronger notions of entailment, based on canonical minimal models and closure constructions [2, 12]. Similar constructions have been developed for preferential DLs [6, 9, 10, 13]. In this section we show that these constructions can be extended to the preferential temporal description logic considered above. In the following, we define a notion of *temporal weighted* $LTL^{\mathbf{T},m}_{\mathcal{ALC}}$ *knowledge base* by allowing for *weighted defeasible inclusions* for the distinguished concepts, as done in [14, 20] for the (non-temporal) DLs with typicality. The idea is to allow for the definition of prototypical properties of a class, with (positive or negative) weights representing the degree of plausibility/implausibility of each property, including as well the temporal dimension.

A *temporal weighted* $LTL^{\mathbf{T},m}_{\mathcal{ALC}}$ *knowledge base* K *over* $\mathcal{C}$ is a tuple $\langle \mathcal{T}, \mathcal{T}_{C_1}, \ldots, \mathcal{T}_{C_k}, \mathcal{A} \rangle$, where $\mathcal{T}$ is a TBox in $LTL^{\mathbf{T},m}_{\mathcal{ALC}}$, $\mathcal{A}$ is an ABox in $LTL^{\mathbf{T},m}_{\mathcal{ALC}}$ and, for each $C_i \in \mathcal{C}$, $\mathcal{T}_{C_i}$ is a set of weighted defeasible inclusions, of the form $\mathbf{T}(C_i) \sqsubseteq D_h$, having weight w^i_h, a real number.

Consider, for instance, the weighted knowledge base $K = \langle \mathcal{T}_{strict}, \mathcal{T}_{Emp}, \mathcal{T}_{Student}, \mathcal{A} \rangle$, over the set of distinguished concepts $\mathcal{C} = \{Emp, Student, Professor\}$, with empty ABox, and with $\mathcal{T}$ containing the inclusions:

$Emp \sqsubseteq Adult \qquad Adult \sqsubseteq \exists has\_SSN.\top \qquad PhdStudent \sqsubseteq Student$

$\mathbf{T}(Professor) \sqsubseteq (\exists Teaches.Course) \mathcal{U} Semester\_End$

where, for instance, $\mathcal{T}_{Emp}$ contains the weighted defeasible inclusions:

(d_1) $\mathbf{T}(Emp) \sqsubseteq Young$, - 50 $\qquad$ (d_2) $\mathbf{T}(Emp) \sqsubseteq \exists has\_boss.Emp$, 100

(d_3) $\mathbf{T}(Emp) \sqsubseteq \exists has\_classes.\top$, -70;

and $\mathcal{T}_{Student}$ contains the defeasible inclusions:

(d_4) $\mathbf{T}(Student) \sqsubseteq Young$, 90 $\qquad$ (d_5) $\mathbf{T}(Student) \sqsubseteq \exists has\_classes.\top$, 80

(d_6) $\mathbf{T}(Student) \sqsubseteq \exists hasScholarship.\top$, -30

(d_7) $\mathbf{T}(Student) \sqsubseteq \Diamond(Promoted \sqcup Rejected)$, 100

The meaning is that, while an employee normally has a boss, he is not likely to be young or have classes. Furthermore, between the two defeasible inclusions (d_1) and (d_3), the second one is considered to be less plausible than the first one. Negative weights represent penalties. Given two employees Tom and Bob such that, at time point n, Tom is not young, has no boss and has classes, while Bob is not young, has a boss and has no classes, considering the weights above, we will regard Bob as being more typical than Tom as an employee at n.

Given a *temporal interpretation* $\mathcal{I} = (\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}})$ for $LTL_{\mathcal{ALC}}$, we say that $x \in \Delta^{\mathcal{I}}$ *satisfies* $\mathbf{T}(C_i) \sqsubseteq D$ *in* $\mathcal{I}$ *at time point* n, if $(n,x) \notin C_i^{\mathcal{I}}$ or $(n,x) \in D^{\mathcal{I}}$ (otherwise x *violates* $\mathbf{T}(C_i) \sqsubseteq D$ in $\mathcal{I}$ at time point n). For a concept $C_i \in \mathcal{C}$ and a domain element $x \in \Delta^{\mathcal{I}}$, the *weight* $W_{i,n}^{\mathcal{I}}(x)$ *of* x *w.r.t.* C_i *in* $\mathcal{I}$ *at time point* n, is defined considering the inclusions $(\mathbf{T}(C_i) \sqsubseteq D_h\ , w_h^i) \in \mathcal{T}_{C_i}$, as follows:

$$W_{i,n}^{\mathcal{I}}(x) = \begin{cases} \sum_{h:(n,x)\in D_h^{\mathcal{I}}} w_h^i & \text{if } x \in C_{i,n}^{\mathcal{I}} \\ -\infty & \text{otherwise} \end{cases} \tag{3}$$

where $-\infty$ is added at the bottom of real values. Informally, given an interpretation $\mathcal{I}$, for $(n,x) \in C_i^{\mathcal{I}}$, the weight $W_{i,n}^{\mathcal{I}}(x)$ of x wrt C_i at time point n is the sum of the weights of all defeasible inclusions for C_i satisfied by x in $\mathcal{I}$ at time point n. The more plausible are the satisfied inclusions, the higher is the weight of x. The lowest weight, $-\infty$, is given to all domain elements which are not instances of C_i at time point n.

Based on this notion of weight of a domain element wrt a concept, a preference relation $<_{C_i}^n$ can be built from a given interpretation $\mathcal{I}$ and a weighted knowledge base K. At time point n, an element x is preferred to element y wrt C_i if the sum of the weights of the defaults in $\mathcal{T}_{C_i}$ satisfied by x at n is higher than the sum of the weights of defaults in $\mathcal{T}_{C_i}$ satisfied by y at n: for $x, y \in \Delta$,

$$x <_{C_i}^n y \quad \text{iff} \quad W_{i,n}^{\mathcal{I}}(x) > W_{i,n}^{\mathcal{I}}(y) \tag{4}$$

Note that $<_{C_j}^n$ is a strict modular and well-founded partial order, and all C_i-elements are preferred wrt $<_{C_i}$ to the domain elements which are not instances of C_i. The higher is the weight of an element wrt C_i (at n) the more preferred is the element w.r.t. C_i at time point n. In the example above, $W_{i,n}^{\mathcal{I}}(bob) = 30 > W_{i,n}^{\mathcal{I}}(tom) = -70$ (for $C_i = Emp$) and, hence, $bob <_{Emp} tom$, i.e., Bob is more typical than Tom as an employee.

Let us define a *concept-wise multi-preferential temporal semantics* (cw^m temporal semantics) for a weighted knowledge base.

Definition 5. *A* concept-wise multi-preferential temporal model *(cw^m-model) of a weighted* $LTL_{\mathcal{ALC}}^{\mathbf{T},m}$ *knowledge base* $K = \langle \mathcal{T}, \mathcal{T}_{C_1}, \ldots, \mathcal{T}_{C_k}, \mathcal{A} \rangle$ *over* $\mathcal{C}$ *is a concept-wise multi-preferential interpretation* $\mathcal{M} = \langle \Delta^{\mathcal{I}}, <_{C_1}, \ldots, <_{C_k}, <, \cdot^{\mathcal{I}} \rangle$*, such that: for all* $j = 1, \ldots, k$,

$$<_{C_j} = \{(n,x,y) : n \in \mathbb{N} \textit{ and } x <_{C_j}^n y\},$$

where each $<_{C_j}^n$ *is defined from* $\mathcal{T}_{C_j}$ *and* $\langle \Delta^{\mathcal{I}}, \cdot^{\mathcal{I}} \rangle$*, according to condition (4);* $<$ *is the resulting global preference relation, as defined in Section 6.1; and* $\langle \Delta^{\mathcal{I}}, <, \cdot^{\mathcal{I}} \rangle$ *satisfies* $\mathcal{T}$ *and* $\mathcal{A}$ *according to satisfiability in Definition 3.*

Based on the notion of cw^m-model of a KB, the notions of *concept-wise entailment* (or cw^m-entailment) and *canonical cw^m-entailment* can be defined in a natural way for weigthed KBs in $LTL_{\mathcal{ALC}}^{\mathbf{T},m}$, as in the non-temporal case [20].

Let us restrict consideration to *canonical* models, i.e., models which are large enough to contain all the relevant domain elements (see [13]). Let $Conc_K$ be the set of all non-temporal concepts C occurring in K plus their complements $\neg C$.

Definition 6. *Given a ranked knowledge base $K = \langle \mathcal{T}, \mathcal{T}_{C_1}, \ldots, \mathcal{T}_{C_k}, \mathcal{A} \rangle$ a model $\mathcal{M} = \langle \Delta^{\mathcal{I}}, <_{C_1}, \ldots, <_{C_k}, <, \cdot^{\mathcal{I}} \rangle$ of K is* canonical *for K if, for any set of concepts $\{D_1, \ldots, D_m\} \subseteq Conc_K$ such that $D_1 \sqcap \ldots \sqcap D_m$ is satisfiable with respect to $\langle \mathcal{T}, \mathcal{A} \rangle$, it holds that for all time points n, there exists a domain element $x \in \Delta^{\mathcal{I}}_n$ such that $(n, x) \in D_i^{\mathcal{I}}$ for all $i = 1, \ldots, m$.*

The idea is that, in a canonical model for K, any conjunction of concepts occurring in K, or their complements, when consistent with the TBox $\mathcal{T}$ and the ABox $\mathcal{A}$ of K, must have some instance in the domain at each time point n. Existence of canonical interpretations has been proven in the non-temporal case for knowledge bases which are consistent under the preferential (or ranked) semantics for typicality [9]. A similar construction can be developed for the temporal case, exploiting the fact that, in the case we have considered (that of KBs with non-temporal TBoxes and non-temporal ABoxes), the interaction between the temporal component and the DL component of the temporal DL is rather limited (see [16]).

Definition 7 (cw^m-entailment [14])**.** *An inclusion $\mathbf{T}(C) \sqsubseteq D$ is cw^m-entailed from a weighted knowledge base K if it is satisfied in all canonical cw^m-models $\mathcal{M}$ of K.*

The study of the properties of this semantic, such as the KLM properties [2], which have been studied for description logics with typicality in the non-temporal case, will be considered for future work, as well as the development of alternative semantic constructions.

8. Conclusions

In this paper we have developed a preferential temporal description logics with typicality $LTL^{\mathbf{T}}_{\mathcal{ALC}}$. The monotonic logic $LTL^{\mathbf{T}}_{\mathcal{ALC}}$ can be further extended to define a semantics for weighted knowledge bases, by introducing multiple preferences. The paper discusses these extensions, showing that the concept-wise multi-preferential semantic in [13] adapts smoothly to the temporal case.

On a different route, a preferential LTL with defeasible temporal operators has been studied in [18, 19]. The decidability of meaningful fragments of the logic has been proven, and tableaux based proof methods for such fragments have been developed [17, 19]. Instead, our approach does not consider defeasible temporal operators (nor preferences over time points), but it combines standard LTL operators with the typicality operator in a temporal $\mathcal{ALC}$ (where preferences are over the domain elements).

A different approach for combining defeasibility in temporal DL formalism has been proposed in [27], by combining a temporal action logic [28] for reasoning about actions (whose semantics is based on a notion of temporal answer set) and an $\mathcal{EL}^{\perp}$ ontology. The approach provides a polynomial encoding of an action theory extended with an $\mathcal{EL}^{\perp}$ knowledge base in normal form, into the language of the temporal action logic. The temporal action logic studied in [28] is based on an extension of LTL, called Dynamic Linear Time Temporal Logic (DLTL) introduced in [29], which allows for complex actions. The proof methods for this action logic are based on ASP encodings of bounded model checking [28, 30], and can then be exploited for reasoning about actions in an extended action theory. Defeasibility in [27], and in the related work on reasoning about actions in Description Logics [31, 32, 33] (often not based on temporal logics), is concerned with the non-monotonicity of the frame problem and, in the literature, different solutions are explored. Our paper, instead, aims at representing temporal properties of concepts which admit exceptions, through a notion of typicality, and is not specifically intended for reasoning about actions.

The encoding of $LTL^{\mathbf{T}}_{\mathcal{ALC}}$ into $LTL_{\mathcal{ALC}}$ provides decidability and complexity results for the monotonic logic $LTL^{\mathbf{T}}_{\mathcal{ALC}}$ for free. For the multi-preferential case, proof methods for defeasible temporal reasoning with weighted knowledge bases have to be investigated, possibly for fragments of $LTL^{\mathbf{T},m}_{\mathcal{ALC}}$. This will be subject of future work.

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