Temporal Probabilistic Argumentation Frameworks*

Stefano Bistarelli[1,*], Victor David[1], Francesco Santini[1,*] and Carlo Taticchi[1,*]

[1] *Department of Mathematics and Computer Science -, University of Perugia, Perugia, Italy*
firstname.lastname@unipg.it

Abstract

In recent years, the notion of time has been studied in different ways in Dung-style Argumentation Frameworks. For example, time intervals of availability have been added to arguments and relations. As a result, the output of Dung semantics varies over time. In this paper, we consider the situation in which arguments hold with a certain probability distribution during a given interval. To model the uncertain character of events, we propose different notions of temporal conflict between arguments according to the type of availabilities intersection (partial, inclusive or total). Then, we refine these notions of conflict by a defeat relation, using criterion functions that evaluate an attack's significance according to the probability over time. After extending Dung's semantics with these defeat notions, we present a new temporal acceptability of arguments based on the concept of defence, allowing for finer results in time.

Keywords

Temporal Argumentation, Probabilistic Argumentation, Extended Dung's Semantics

1. Introduction

Argumentation Theory studies how conclusions can be drawn starting from a given set of facts or premises, and, in the field of Artificial Intelligence, it provides tools for modelling human-fashioned logical reasoning where the available information may be discordant. A simple yet powerful representation of conflicting information is provided by the Abstract Argumentation Frameworks [1], or AFs in short. AFs can be seen as directed graphs where the nodes are arguments and the edges represent conflict relations (called attacks) between two arguments. Since [1], many extensions have been proposed to improve the expressiveness, e.g. the addition of: a support relation [2, 3, 4, 5], a similarity relation [6, 7, 8, 9, 10], weights [11, 12, 13, 14]. Then, different so-called "semantics" have been proposed on AFs to analyse these graphs. For instance, one can derive sets of acceptable arguments, i.e. non-conflicting arguments that share specific properties. Among the set of extensions proposed in the literature, we are interested here in two types of improvement, one taking into account the notion of temporality and the second considering probabilities.

In general, the AFs considering the first notion use *time* to know when arguments or attacks are available [15, 16, 17, 18, 19]. While the works mentioned above use abstract frameworks,

CILC'23: 38th Italian Conference on Computational Logic, June 21–23, 2023, Udine, Italy

*This work has been partially supported by: INdAM - GNCS Project, CUP E53C22001930001; Project FICO, funded by Ricerca di Base 2021, University of Perugia; Project BLOCKCHAIN4FOODCHAIN, funded by Ricerca di Base 2020, University of Perugia; Project GIUSTIZIA AGILE, CUP J89J22000900005.

*This author is a member of the INdAM Research group GNCS and Consorzio CINI.

the one in [20] focuses on structured argumentation and defeasible reasoning. Then, the work in [21] associates attacks with time intervals for abstract and structured frameworks.

The second concept that interests us is the consideration of *probability* in arguments and relations; see [22] for a survey. In the literature, two main perspectives exist towards probabilistic argumentation based on constellations [23] and epistemic approaches [24]. The former approach is to consider probabilities as the possibility that an argument or a relation exists or not, which leads to the study of all possible structures (having some complexity problems [25, 26, 27]).While the latter suggests that probability denotes a degree of belief. Our study here is closer to the epistemic approach.

In this paper, we take a further step towards a more expressive AF and consider that the arguments are certain on the interval but uncertain on their occurrence. In particular, we assume only to know the probability distribution of the events associated with the arguments.

Please note that this paper is an extension of a previous short article [28], where we presented this framework of temporal and probabilistic argumentation and how to apply extension-based semantics on it (which corresponds here, up to Section 3). We add in this long version Theorem 1 linking the classical semantics of AFs with our Δ-semantics from Section 3. Then we introduce also a new way of computing the acceptability of arguments more precisely in time (Section 4). Let us now introduce our framework with the following example:

Example 1. *We want to solve a murder case for which we have the four arguments below describing the events before the victim's death:*[1]

- *argument a: witness A reports seeing a fight between the victim and another person between 1 pm and 4 pm (i.e. in the interval $\{1, \ldots, 4\}$);*
- *argument b: witness B reports to have seen the victim walking between 2 pm and 7 pm (i.e. $\{2, \ldots, 7\}$);*
- *argument c: A surveillance Camera recorded the victim walking at 3 pm (i.e. $\{3\}$);*
- *argument d: According to the Doctor, the victim died between 6 pm and 10 pm (i.e. $\{6, \ldots, 10\}$).*

The attacks between arguments a, b, c, and d are given in Figure 1, which provides a static representation of the events. The probability distribution over time for the arguments is then represented in Figure 2. In this example, we use a uniform distribution for arguments a and b, while argument d, which is more likely to occur around 8 pm, follows a normal distribution. Finally, argument c holds with probability 1 at 3 pm. Note that one can choose different probability distributions to represent various types of uncertainty.

Since events can be uncertain over time, the notion of conflict between arguments also needs to be revised. For example, two contradictory arguments, such as "the victim was fighting" and "the victim was walking", may not be in conflict if they hold at different times.

To deal with temporal and probabilistic aspects of argumentation, we first introduce Temporal Probabilistic Argumentation Frameworks (TPAFs), an extension to classical AFs, and propose a method for deriving conflict between arguments. Then, to evaluate the acceptability of arguments,

[1]Notice that, we consider events happening at a time point. Therefore, intervals are represented as sets of time points.

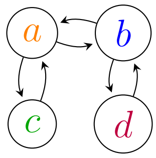

Figure 1: A framework $\mathbf{F}$ describing the events of Example 1.

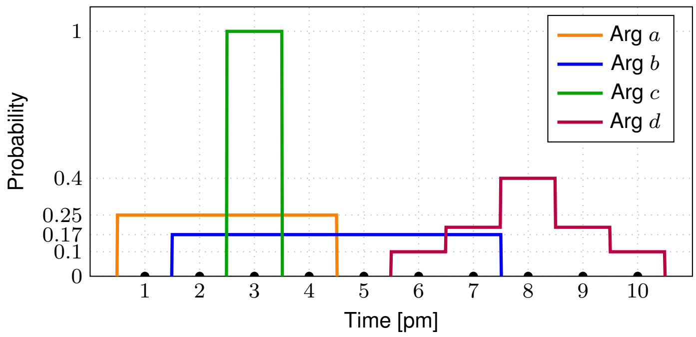

Figure 2: Probability distribution over time for the arguments in $\mathbf{F}$.

we provide a set of semantics based on the notion of defence over time. We also study the concept of minimal defence to investigate the conditions under which an argument can be accepted with respect to a time interval.

The remainder of this paper is organised as follows. Section 2 summarises the basic definitions of AF and extension-based semantics; in Section 3 we formalise TPAFs, providing conflict and defence notions and a set of semantics that take into account the probabilistic nature of event occurrences; Section 4, then, presents the idea of temporal Δ-acceptability; finally, Section 5 concludes the paper with final remarks on possible future work.

2. Preliminaries

In this section, we recall the formal definition of an Abstract Argumentation Framework and the related extension-based semantic [1].

Definition 1 (AF). *An Abstract Argumentation Framework (AF) is a pair $\langle \mathcal{A}, \mathcal{R} \rangle$ where $\mathcal{A}$ is a set of arguments, and $\mathcal{R}$ is a binary relation on $\mathcal{A}$.*

Consider two arguments a, b belonging to an AF. We denote with $(a, b) \in \mathcal{R}$ an attack from a to b; we can also say that b is *defeated* by a. In order for b to be *acceptable*, we require that every argument that defeats b is defeated in turn by some other argument of the AF.

Definition 2 (Acceptable argument). *Given an AF $\langle \mathcal{A}, \mathcal{R} \rangle$, an argument $a \in \mathcal{A}$ is acceptable with respect to $D \subseteq \mathcal{A}$ if and only if $\forall b \in \mathcal{A}$ such that b is attacking a, $\exists d \in D$ such that d is attacking b and we say that a is defended by D.*

Using the notion of defence as a criterion for distinguishing acceptable arguments in the framework, one can further refine the set of selected arguments.

Definition 3 (Extension-based semantics). *Let $\langle \mathcal{A}, \mathcal{R} \rangle$ be an AF. A set $E \subseteq \mathcal{A}$ is conflict-free if and only if $\nexists a, b \in E$ such that $(a, b) \in \mathcal{R}$. A conflict-free subset E is then* ***admissible****, if each $a \in E$ is defended by E;* ***complete****, if it is admissible and $\forall a \in \mathcal{A}$ defended by E, $a \in E$;* ***stable****, if it is admissible and attacks every argument in $\mathcal{A} \setminus E$;* ***preferred****, if it is complete and $\subseteq$-maximal;* ***grounded****, if it is complete and $\subseteq$-minimal.*

We also need the notion of time intervals for reasoning with temporal aspects of arguments. For example, in Timed AFs [15], each argument is associated with temporal intervals that express the period of time in which the argument is available.

Definition 4 (Temporal interval). *Let $\mathbf{T}$ be the discrete universe of time points. A temporal interval is a subset $I = \{t_1, t_2, \ldots, t_n\}$ of $\mathbf{T}$ where t_1 and t_n are respectively the minimum and maximum bounds of I. Moreover, $I = \{t_i\}$ denotes the instant t_i and $I = \{\}$ is not allowed.*

Definition 5 (TAFs). *A Timed Abstract Argumentation Framework (TAF) is a tuple $\langle \mathcal{A}, \mathcal{R}, Av \rangle$ where $\mathcal{A}$ is a set of arguments, $\mathcal{R}$ is a binary relation on $\mathcal{A}$, and $Av \colon \mathcal{A} \to \wp(\mathbf{T})$ is the availability function for arguments.*[2]

3. Temporal Probabilistic Argumentation Frameworks

To reason about uncertain events in time using argumentation-based tools, we must first be able to represent the probabilistic and temporal aspects of arguments in a single framework. To this end, we instantiate the generic framework proposed in [29], in which arguments are evaluated over time, and we associate each argument with the probability of its occurrence at a given time. We obtain in this way a Temporal Probabilistic Argumentation Framework (*TPAF*).

Definition 6 (TPAF). *A Temporal Probabilistic Argumentation Framework (TPAF) is a tuple $G = \langle \mathcal{A}, \mathcal{R}, \mathcal{P} \rangle$ such that: $\mathcal{A}$ is a finite set of arguments; $\mathcal{R} \subseteq \mathcal{A} \times \mathcal{A}$ is the attack relation; $\mathcal{P}^I : \mathcal{A} \to [0, 1]$ is the probability distribution of an argument over a time interval I.*

Example 2. *We use the AF $\mathbf{F}$ of Figure 1 and the probability distribution of Figure 2 to build a TPAF $\mathbf{G}$. The time points in the considered interval $I = \{1, \ldots, 10\}$ represent the hours of the day. We have that $\mathcal{P}^{\{1,\ldots,10\}}(x) = 1$ for all arguments x in $\mathbf{G}$. Furthermore, for each argument, we can obtain the probability of its occurrence at a certain instant, e.g. $\mathcal{P}^{\{8\}}(d) = 0.4$ in $\mathbf{G}$.*

Note that depending on the user's needs, for example, if the TPAF occurs over a long period of time, it is useful to be able to restrict the study of a TPAF to a specific time interval. Thus, in the rest of the article, we will specify the time interval we are working on.

When an argument has a probability of occurring equal to zero, it should not be considered in the reasoning process. Therefore, we extract, for each argument, the instants in which its probability is positive, i.e. when the argument can occur.

[2]We use $\wp(\mathbf{T})$ to indicate the powerset of $\mathbf{T}$.

Definition 7 (Positive probability over time). *Let $G = \langle \mathcal{A}, \mathcal{R}, \mathcal{P} \rangle$ be a TPAF, $a \in \mathcal{A}$ an argument, and I a time interval. We define the set of non-null probability of a in I by $\mathcal{T}^I(a) = \{t \in I | \mathcal{P}^{\{t\}}(a) > 0\}$.*

Example 3. *Consider the TPAF of Example 2. We have that $\mathcal{T}^{\{1,\ldots,4\}}(a) = \mathcal{T}^{\{1,\ldots,10\}}(a) = \{1, \ldots, 4\}$.*

Given the probability over time of arguments, the conflicts are not sure and can be interpreted in different ways according to various notions.[3] In particular, we propose three notions of conflict based on the availability of involved arguments and three criterion functions defining when the conflict is significant, i.e. it is a defeat.

Definition 8 (Temporal probabilistic conflicts). *Let $G = \langle \mathcal{A}, \mathcal{R}, \mathcal{P} \rangle$ be a TPAF, $a, b \in \mathcal{A}$ two arguments and I a time interval. We define a boolean conflict function $\mathtt{CF}^{\mathtt{I}}_{\mathtt{x}} \colon \mathcal{A} \times \mathcal{A} \to \{\top, \bot\}$, with $\mathtt{x} \in \{\mathtt{p}, \mathtt{i}, \mathtt{t}\}$ (where $\mathtt{p}$, $\mathtt{i}$ and $\mathtt{t}$ stand for partial, included and total, respectively), which determines a conflict from a to b within I when $(a, b) \in \mathcal{R}$ and:*

- *Partial conflict: $\mathtt{CF}^{\mathtt{I}}_{\mathtt{p}}(a, b) = \top$ if and only if $\mathcal{T}^I(a) \cap \mathcal{T}^I(b) \neq \emptyset$;*
- *Included conflict: $\mathtt{CF}^{\mathtt{I}}_{\mathtt{i}}(a, b) = \top$ if and only if $\mathcal{T}^I(b) \setminus \mathcal{T}^I(a) = \emptyset$;*
- *Total conflict: $\mathtt{CF}^{\mathtt{I}}_{\mathtt{t}}(a, b) = \top$ if and only if $\mathcal{T}^I(a) = \mathcal{T}^I(b)$.*

Otherwise for any $\mathtt{x} \in \{\mathtt{p}, \mathtt{i}, \mathtt{t}\}$, $\mathtt{CF}^{\mathtt{I}}_{\mathtt{x}}(a, b) = \bot$.

Example 3. (Continued) *In the following, we illustrate the different temporal probabilistic conflicts of Definition 8: $\mathtt{CF_p}^{\{1,\ldots,5\}}(a, b) = \top$ while $\mathtt{CF_p}^{\{6,7\}}(a, b) = \bot$; $\mathtt{CF_i}^{\{1,\ldots,4\}}(a, b) = \top$ while $\mathtt{CF_i}^{\{1,\ldots,4\}}(b, a) = \bot$; $\mathtt{CF}^{\{2,3,4\}}_{\mathtt{t}}(a, b) = \top$ while $\mathtt{CF}^{\{1,\ldots,4\}}_{\mathtt{t}}(a, b) = \bot$.*

Note that partial conflict and total conflict are symmetric, while the included conflict is not. Moreover, the notion of $\mathtt{CF}^{\mathtt{I}}_{\mathtt{t}}$ implies the notion of $\mathtt{CF}^{\mathtt{I}}_{\mathtt{i}}$ which implies, in turn, $\mathtt{CF}^{\mathtt{I}}_{\mathtt{p}}$.

Proposition 1 (Relation between conflicts). *Let $G = \langle \mathcal{A}, \mathcal{R}, \mathcal{P} \rangle$ be a TPAF, $a, b \in \mathcal{A}$ such that $(a, b) \in \mathcal{R}$ and I a time interval.*

- *If $\mathtt{CF}^{\mathtt{I}}_{\mathtt{t}}(a, b) = \top$ then $\mathtt{CF}^{\mathtt{I}}_{\mathtt{i}}(a, b) = \top$.*
- *If $\mathtt{CF}^{\mathtt{I}}_{\mathtt{i}}(a, b) = \top$ then $\mathtt{CF}^{\mathtt{I}}_{\mathtt{p}}(a, b) = \top$.*

The notion of conflict only considers the positive probability over time of the arguments, i.e. we only check if the probability of arguments involved in an attack is positive. We can refine the concept of conflict by using the probability values attached to arguments to establish whether a conflict is significant according to a criterion function. In addition, we use the term *defeat* to refer to a significant conflict.

Definition 9 (Criterion functions). *Let $G = \langle \mathcal{A}, \mathcal{R}, \mathcal{P} \rangle$ be a TPAF, $a, b \in \mathcal{A}$ such that $(a, b) \in \mathcal{R}$ and I a time interval. We define a boolean criterion function $\mathtt{CT}^{\mathtt{I}}_{\mathtt{x}} \colon \mathcal{A} \times \mathcal{A} \to \{\top, \bot\}$ where $\mathtt{x} \in \{\mathtt{Sg}, \mathtt{Wg}, \mathtt{A}\}$ as follows:*

[3] For example, in [30], various temporal inconsistencies are defined in the Temporal Markov Logic Networks framework.

- *Weak greater:* $\mathtt{CT}^{\mathtt{I}}_{\mathtt{Wg}}(a,b) = \top$ *if and only if* $\forall t \in I$ *such that* $\mathcal{P}^{\{t\}}(a) \times \mathcal{P}^{\{t\}}(b) > 0$, $\mathcal{P}^{\{t\}}(a) > \mathcal{P}^{\{t\}}(b)$*;*
- *Strong greater:* $\mathtt{CT}^{\mathtt{I}}_{\mathtt{Sg}}(a,b) = \top$ *if and only if* $\exists t \in I$ *such that* $\mathcal{P}^{\{t\}}(a) \times \mathcal{P}^{\{t\}}(b) > 0$ *and* $\mathcal{P}^{\{t\}}(a) > \mathcal{P}^{\{t\}}(b)$*;*
- *Aggressive:* $\mathtt{CT}^{\mathtt{I}}_{\mathtt{A}}(a,b) = \top$ *if and only if* $\exists t \in I$ *such that* $\mathcal{P}^{\{t\}}(a) \times \mathcal{P}^{\{t\}}(b) > 0$ *and* $\mathcal{P}^{\{t\}}(b) < 1$.

Otherwise $\forall \mathtt{x} \in \{\mathtt{Wg}, \mathtt{Sg}, \mathtt{A}\}$, $\mathtt{CT}^{\mathtt{I}}_{\mathtt{x}}(a,b) = \bot$.

The strong greater criterion leads to more frequently identifying a defeat, whereas the weak greater criterion will be more cautious in indicating that a conflict is significant. Note that for the aggressive criterion, there is no need to differentiate between a strong and a weak version: since we consider a probability distribution with a sum of 1 over the entire interval if there exists a non-zero probability strictly less than a 1, then there is no instant at which the probability is 1.

We show next that, as usual, the universal quantifier (weak) implies the existential (strong) one, and the greater criteria imply the aggressive criterion.

Proposition 2 (Relation between criterion functions). *Let* $G = \langle \mathcal{A}, \mathcal{R}, \mathcal{P} \rangle$ *be a TPAF,* $a, b \in \mathcal{A}$ *two arguments such that* $(a,b) \in \mathcal{R}$ *and consider a time interval* I*. We have that:*

- *If* $\mathtt{CT}^{\mathtt{I}}_{\mathtt{Wg}}(a,b) = \top$ *then* $\mathtt{CT}^{\mathtt{I}}_{\mathtt{Sg}}(a,b) = \top$*;*
- *If* $\mathtt{CT}^{\mathtt{I}}_{\mathtt{Wg}}(a,b) = \top$ *then* $\mathtt{CT}^{\mathtt{I}}_{\mathtt{A}}(a,b) = \top$*;*
- *If* $\mathtt{CT}^{\mathtt{I}}_{\mathtt{Sg}}(a,b) = \top$ *then* $\mathtt{CT}^{\mathtt{I}}_{\mathtt{A}}(a,b) = \top$.

We define a temporal probabilistic defeat function by combining a notion of conflict and a criterion function.

Definition 10 (Temporal probabilistic defeat function). *Let* $G = \langle \mathcal{A}, \mathcal{R}, \mathcal{P} \rangle$ *be a TPAF,* $a, b \in \mathcal{A}$ *two arguments, and consider a criterion function* $\mathtt{CT}$ *and a conflict function* $\mathtt{CF}$*. We define* $\Delta^{I}_{\mathtt{CT},\mathtt{CF}} : \mathcal{A} \times \mathcal{A} \to \{\top, \bot\}$ *the defeat function determining that* a *defeats* b *in the interval* I*, with respect to* $\mathtt{CT}$ *and* $\mathtt{CF}$*. In particular,* $\Delta^{I}_{\mathtt{CT},\mathtt{CF}}(a,b) = \top$*, if and only if* $\mathtt{CT}^{\mathtt{I}}(a,b) = \mathtt{CF}^{\mathtt{I}}(a,b) = \top$*. Otherwise* $\Delta^{I}_{\mathtt{CT},\mathtt{CF}}(a,b) = \bot$.

Example 3. (Continued) *We show below how different temporal probabilistic defeat functions behave according to the partial conflict of Definition 8. First, consider arguments* a *and* b *of* $\mathbf{G}$ *and the interval* $\{1, \ldots, 7\}$*. We have that* b *does not defeat* a *within* I *according to the greater criteria. In fact,* $\Delta^{\{1,\ldots,7\}}_{\mathtt{Wg},\mathtt{p}}(b,a) = \Delta^{\{1,\ldots,7\}}_{\mathtt{Sg},\mathtt{p}}(b,a) = \bot$*. If we consider the aggressive criterion, instead, we obtain* $\Delta^{\{1,\ldots,7\}}_{\mathtt{A},\mathtt{p}}(b,a) = \top$*, meaning that* b *defeats* a *in* I*.*

Then, for arguments b *and* d *in the interval* $\{1, \ldots, 10\}$ *we have* $\Delta^{\{1,\ldots,10\}}_{\mathtt{Wg},\mathtt{p}}(b,d) = \bot$ *and* $\Delta^{\{1,\ldots,10\}}_{\mathtt{Sg},\mathtt{p}}(b,d) = \Delta^{\{1,\ldots,10\}}_{\mathtt{A},\mathtt{p}}(b,d) = \top$.

For a better understanding of the impact of the defeat functions and the restriction of the time intervals, it is worthwhile to define the resulting TPAF according to these parameters.

Definition 11 (Restricted TPAF). *Let $G = \langle \mathcal{A}, \mathcal{R}, \mathcal{P} \rangle$ be a TPAF, I a time interval and Δ a temporal probabilistic defeat function (on I). We denote the restricted graph by Δ^I-$G = \langle \mathcal{A}', \mathcal{R}', \mathcal{P} \rangle$, where:*

- *the arguments are restricted to the time interval I, such that $\mathcal{A}' \subseteq \mathcal{A}$ and $\forall a \in \mathcal{A}$, $a \in \mathcal{A}'$ if and only if $\mathcal{T}^I(a) \neq \emptyset$;*
- *the attacks are restricted according to Δ such that $\mathcal{R}' \subseteq \mathcal{R}$ and $\forall (a, b) \in \mathcal{R}$, $(a, b) \in \mathcal{R}'$ if and only if $a, b \in \mathcal{A}'$ and $\Delta(a, b) = \top$.*

Let us visualise (see Figure 3) the result of a restricted TPAF according to these parameters.

Example 3. (Continued) *We can see in the following the restricted graphs Δ^I-**G**.*

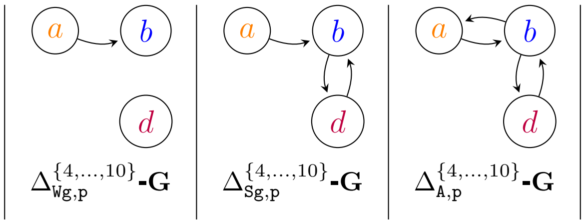

Figure 3: G between 4 and 10 according to a Δ.

The implications between the different defeat functions can be derived by analysing together the relations between conflict functions (Proposition 1) and criterion functions (Proposition 2). We show in Figure 4 the relations between all the defeat functions. In particular, we observe that the strongest (most conflicting) defeat is $\Delta^I_{\mathtt{A,p}}$ and the weakest (less conflicting) defeat is $\Delta^I_{\mathtt{Wg,t}}$.

$$\begin{array}{ccccc}
\Delta^I_{\mathtt{Wg,t}} & \longrightarrow & \Delta^I_{\mathtt{Wg,i}} & \longrightarrow & \Delta^I_{\mathtt{Wg,p}} \\
\downarrow & & \downarrow & & \downarrow \\
\Delta^I_{\mathtt{Sg,t}} & \longrightarrow & \Delta^I_{\mathtt{Sg,i}} & \longrightarrow & \Delta^I_{\mathtt{Sg,p}} \\
\downarrow & & \downarrow & & \downarrow \\
\Delta^I_{\mathtt{A,t}} & \longrightarrow & \Delta^I_{\mathtt{A,i}} & \longrightarrow & \Delta^I_{\mathtt{A,p}}
\end{array}$$

Figure 4: Relations between defeat functions, where "$\rightarrow$" reads "implies".

Until the end, we will use Δ to refer to a generic temporal probabilistic defeat function. We now extend the notion of conflict-freeness to TPAFs on the basis of a defeat function Δ.

Definition 12 (Δ-conflict-free). *Let $G = \langle \mathcal{A}, \mathcal{R}, \mathcal{P} \rangle$ be a TPAF, $S \subseteq \mathcal{A}$ a set of arguments and Δ a defeat function. S is Δ-conflict-free if and only if $\nexists a, b \in S$ such that $\Delta(a, b) = \top$.*

Example 3. (Continued) *We refer again to the TPAF of Example 2 and check if the set $S = \{a, b, c\}$ is Δ-conflict-free. We can verify that S is not $\Delta^{\{1,\dots,4\}}_{\mathtt{Sg,p}}$-conflict-free since*

$\Delta^{\{1,\dots,4\}}_{\mathtt{Sg,p}}(c,a) = \top$. *Then, S is not even $\Delta^{\{1,\dots,4\}}_{\mathtt{Sg,i}}$-conflict-free since $\Delta^{\{1,\dots,4\}}_{\mathtt{Sg,i}}(a,b) = \top$. We also observe that $\forall x,y \in \{a,b,c\}$, $\Delta^{\{1,\dots,4\}}_{\mathtt{Sg,t}}(x,y) = \bot$ and thus S is $\Delta^{\{1,\dots,4\}}_{\mathtt{Sg,t}}$-conflict-free.*

According to a Δ-conflict-free notion we define the notion of a defence of an argument against another one according to a set of arguments. This function thus returns the instants at which the argument is defended by the set against this single attacker.

Definition 13 (Δ-SingleDefence of a from b by S). *Given $G = \langle \mathcal{A}, \mathcal{R}, \mathcal{P} \rangle$ be a TPAF, I a time interval, $a, b \in \mathcal{A}$ and $S \subseteq \mathcal{A}$ be a Δ-conflict-free set of arguments within I. According to the defeat notion Δ used for the Δ-conflict-freeness, the Δ single defence of a from b with respect to S within I, is defined as follows:* $\Delta^I\text{-}\mathtt{1def}(a,b,S) =$

$$\mathcal{T}^I(a) \cap \bigcup_{c \in \{x \mid x \in S, \Delta^I(x,b)=\top\}} \mathcal{T}^I(b) \cap \mathcal{T}^I(c)$$

Example 3. (Continued) *From the TPAF* **G***, let us see what is the $\Delta^{\{2,\dots,7\}}_{\mathtt{Sg,p}}$ single defence of b from a and d with respect to the set of arguments $S = \{b, c\}$: $\Delta^{\{2,\dots,7\}}_{\mathtt{Sg,p}}\text{-}\mathtt{1def}(b,a,S) = \{3\}$ and $\Delta^{\{2,\dots,7\}}_{\mathtt{Sg,p}}\text{-}\mathtt{1def}(b,d,S) = \{6,7\}$.*

Thanks to the previous definition we can now define when an argument is Δ defended by a set of arguments in a TPAF against a single attacker. In the following, we generalise this notion of defence against a set of attackers.

Definition 14 (Δ-Defence of a with respect to S). *Let $G = \langle \mathcal{A}, \mathcal{R}, \mathcal{P} \rangle$ be a TPAF, I a time interval and S be a Δ-conflict-free set of arguments within I. The Δ-defence for a with respect to S, is defined as follows:* $\Delta^I\text{-}\mathtt{def}(a,S) =$

$$\bigcap_{b \in \{x \mid \Delta^I(x,a)=\top\}} (\mathcal{T}^I(a) \setminus \mathcal{T}^I(b)) \cup \Delta^I\text{-}\mathtt{1def}(a,b,S)$$

Note that, an argument is defended on the instants that it is not attacked or when an argument defends it on this time.

We now define Δ-admissible, Δ-complete, Δ-preferred, Δ-stable and Δ-grounded semantics for TPAFs.

Definition 15 (Δ-Semantics). *Let $G = \langle \mathcal{A}, \mathcal{R}, \mathcal{P} \rangle$ be a TPAF, I a time interval, Δ a defeat function and consider a set of arguments $E \subseteq \mathcal{A}$. We say that:*

- *E is a Δ^I-admissible extension of G within I, denoted by $E \in \Delta^I$-ad(G) if and only if for all $a \in E$ it holds that $\mathcal{T}^I(a) = \Delta^I\text{-}\mathtt{def}(a,E)$;*
- *E is a Δ^I-complete extension of G within I, denoted by $E \in \Delta^I$-co(G) if and only if E is a Δ^I-admissible extension of G and E contains all the arguments a such that $\mathcal{T}^I(a) = \Delta^I\text{-}\mathtt{def}(a,E)$;*

- *E is a Δ^I-preferred extension of G within I, denoted by $E \in \Delta^I$-pr(G) if and only if E is a $\subseteq$-maximal Δ^I-complete extension;*
- *E is a Δ^I-stable extension of G within I, denoted by $E \in \Delta^I$-st(G) if and only if E is Δ^I-admissible and for all $b \in \mathcal{A} \setminus E$, there exists $a \in E$ such that a defeats b, i.e., $\Delta^I(a,b) = \top$;*
- *E is the Δ^I-grounded extension of G within I, denoted by $E \in \Delta^I$-gr(G) if and only if E is the $\subseteq$-minimal Δ^I-complete extension.*

Example 3. (Continued)
We show in Table 1 a comparison between the different semantics with respect to $\Delta^I_{\mathtt{Wg},\mathtt{p}}$, $\Delta^I_{\mathtt{Sg},\mathtt{p}}$ and $\Delta^I_{\mathtt{A},\mathtt{p}}$. For the remaining examples in this paper, we will assume $I = \{4, \ldots, 10\}$. In this interval, each semantics returns the same sets of extensions for criteria Wg *and* Sg*. However, with the criterion* A*, argument b is able to defend itself and thus is accepted by all semantics except for the grounded one, which is empty since there are no undefeated arguments.*

$\Delta^I_{\mathtt{Wg},\mathtt{p}}$-ad$(\mathbf{G}) = \Delta^I_{\mathtt{Sg},\mathtt{p}}$-ad$(\mathbf{G})$	$\{\emptyset, \{a\}, \{d\}, \{a,d\}\}$
$\Delta^I_{\mathtt{Wg},\mathtt{p}}$-co$(\mathbf{G}) = \Delta^I_{\mathtt{Sg},\mathtt{p}}$-co$(\mathbf{G})$	$\{\{a,d\}\}$
$\Delta^I_{\mathtt{Wg},\mathtt{p}}$-pr$(\mathbf{G}) = \Delta^I_{\mathtt{Sg},\mathtt{p}}$-pr$(\mathbf{G})$	$\{\{a,d\}\}$
$\Delta^I_{\mathtt{Wg},\mathtt{p}}$-st$(\mathbf{G}) = \Delta^I_{\mathtt{Sg},\mathtt{p}}$-st$(\mathbf{G})$	$\{\{a,d\}\}$
$\Delta^I_{\mathtt{Wg},\mathtt{p}}$-gr$(\mathbf{G}) = \Delta^I_{\mathtt{Sg},\mathtt{p}}$-gr$(\mathbf{G})$	$\{\{a,d\}\}$
$\Delta^I_{\mathtt{A},\mathtt{p}}$-ad$(\mathbf{G})$	$\{\emptyset, \{a\}, \{d\}, \{a,d\}, \{b\}\}$
$\Delta^I_{\mathtt{A},\mathtt{p}}$-co$(\mathbf{G})$	$\{\emptyset, \{a,d\}, \{b\}\}$
$\Delta^I_{\mathtt{A},\mathtt{p}}$-pr$(\mathbf{G})$	$\{\{a,d\}, \{b\}\}$
$\Delta^I_{\mathtt{A},\mathtt{p}}$-st$(\mathbf{G})$	$\{\{a,d\}, \{b\}\}$
$\Delta^I_{\mathtt{A},\mathtt{p}}$-gr$(\mathbf{G})$	$\{\emptyset\}$

Table 1
Semantics on $\mathbf{G}$ over $\{4, \ldots, 10\}$.

The two following propositions show that the Δ semantics satisfy the classical properties that we have in non-temporal frameworks.

Proposition 3 (Unicity of the Δ-grounded extension). *For any $G = \langle \mathcal{A}, \mathcal{R}, \mathcal{P} \rangle$ be a TPAF, there always exists one and only one Δ-grounded extension.*

The relations between Δ-admissible, Δ-preferred, Δ-stable, and Δ-complete extensions are given below.

Proposition 4 (Relation between semantics). *Let $G = \langle \mathcal{A}, \mathcal{R}, \mathcal{P} \rangle$ be a TPAF. Then:*

1. *Let $E \subseteq \mathcal{A}$. Then, E is $\subseteq$-maximal Δ-admissible if and only if E is a $\subseteq$-maximal Δ-complete extension;*
2. *A Δ-preferred extension is also a Δ-complete extension;*

3. *A Δ-stable extension is also a Δ-preferred extension;*
4. *The Δ-grounded extension is a subset of all Δ-preferred and Δ-stable extensions.*

Let us now define the notion of sceptical acceptability according to a Δ semantics.

Definition 16 (Δ-Skeptical acceptability). *Let $G = \langle \mathcal{A}, \mathcal{R}, \mathcal{P} \rangle$ be a TPAF, I an interval, and let $\{E_1, \ldots, E_n\}$ be the set of Δ^I-extensions of G, with respect to a semantics between: admissible (ad), complete (co), preferred (pr), stable (st) and grounded (gr). An argument $a \in \mathcal{A}$, is Δ^I-skeptical acceptable under Δ^I-s, denoted by $a \in \Delta^I$-sk-s(G) where $s \in \{ad, co, pr, st, gr\}$, if and only if $\forall E \in \{E_1, \ldots, E_n\}, a \in E$.*

Example 3. (Continued) *We compare in Table 2, the different semantics using $\Delta^I_{\mathtt{Wg,p}}$, $\Delta^I_{\mathtt{Sg,p}}$ and $\Delta^I_{\mathtt{A,p}}$ on $\mathbf{G}$ between 4 and 10. As seen in Table 1, since semantics based on the* Wg *and* Sg *criteria have the same extensions, their sceptical arguments are also identical. Finally for the semantics using the criterion* A*, since each argument can defend itself (because each argument in defeat, defeats its attackers also), there is no sceptically accepted argument.*

$\Delta^I_{\mathtt{Wg,p}}$-sk-ad$(\mathbf{G}) = \Delta^I_{\mathtt{Sg,p}}$-sk-ad$(\mathbf{G})$	$\emptyset$	$\Delta^I_{\mathtt{A,p}}$-sk-ad$(\mathbf{G})$	$\emptyset$
$\Delta^I_{\mathtt{Wg,p}}$-sk-co$(\mathbf{G}) = \Delta^I_{\mathtt{Sg,p}}$-sk-co$(\mathbf{G})$	$\{a, d\}$	$\Delta^I_{\mathtt{A,p}}$-sk-co$(\mathbf{G})$	$\emptyset$
$\Delta^I_{\mathtt{Wg,p}}$-sk-pr$(\mathbf{G}) = \Delta^I_{\mathtt{Sg,p}}$-sk-pr$(\mathbf{G})$	$\{a, d\}$	$\Delta^I_{\mathtt{A,p}}$-sk-pr$(\mathbf{G})$	$\emptyset$
$\Delta^I_{\mathtt{Wg,p}}$-sk-st$(\mathbf{G}) = \Delta^I_{\mathtt{Sg,p}}$-sk-st$(\mathbf{G})$	$\{a, d\}$	$\Delta^I_{\mathtt{A,p}}$-sk-st$(\mathbf{G})$	$\emptyset$
$\Delta^I_{\mathtt{Wg,p}}$-sk-gr$(\mathbf{G}) = \Delta^I_{\mathtt{Sg,p}}$-sk-gr$(\mathbf{G})$	$\{a, d\}$	$\Delta^I_{\mathtt{A,p}}$-sk-gr$(\mathbf{G})$	$\emptyset$

Table 2
Skeptically accepted arguments on $\mathbf{G}$ over $\{4, \ldots, 10\}$.

We finally show that the Δ-semantics (Definition 15) obtain the same extension as the classical semantics (Definition 3) on the AFs coming from the restricted TPAFs according to a time interval I and a defeat function Δ.

Theorem 1 (Link between the Dung's semantics and the Δ-semantics). *Let $G = \langle \mathcal{A}, \mathcal{R}, \mathcal{P} \rangle$ be a TPAF, I a time interval and Δ a defeat function.*

- *If E is a Δ^I-admissible extension of G, then E is an admissible extension of Δ^I-G.*
- *If E is a Δ^I-complete extension of G, then E is a complete extension of Δ^I-G.*
- *If E is a Δ^I-preferred extension of G, then E is a preferred extension of Δ^I-G.*
- *If E is a Δ^I-stable extension of G, then E is a stable extension of Δ^I-G.*
- *If E is a Δ^I-grounded extension of G, then E is a grounded extension of Δ^I-G.*

Note that the previous result is not a characterisation (if and only if) since if we only look at the graph typology (as in classical AF) we can have a defence between arguments but depending on their temporality (as in TPAF), this defence may not be over the whole duration of the availability of an argument. Therefore, constructing a restricted graph and then applying classical semantics is a relaxed version of Δ-semantics. Indeed the Δ-semantics is required to accept only the totally defended arguments whereas if we apply the Dung semantics on a restricted graph, it will be enough that an argument has a defended instant to be accepted.

4. Consistent Temporal Δ-Acceptability

As we saw in the previous section, the notion of sceptically acceptable argument only identifies arguments that are acceptable in each instant of the studied interval. However, it is also interesting to know if a given argument is acceptable in some instants and defeated in others. For this purpose, we introduce a finer-grained notion of acceptability over time which extracts the instants in which an argument is defended. We first define the notion of minimal temporal Δ-acceptability which extracts the instants of an argument that are always defended according to all maximal consistent sets of arguments containing the evaluated argument.

Definition 17 (Minimal temporal Δ-acceptability). *Let $G = \langle \mathcal{A}, \mathcal{R}, \mathcal{P} \rangle$ be a TPAF, $a \in \mathcal{A}$ and I a time interval. We define the minimal temporal Δ-acceptability of a within I according to Δ by $\texttt{min-}\Delta^{\texttt{I}}\texttt{-acc}(a) = \bigcap_{S \in \texttt{maxFree-}\Delta^I(a)} \Delta^I\texttt{-def}(a, S)$, where $\texttt{maxFree-}\Delta^I(a)$ is the set of all $\subseteq$-maximal Δ^I-conflict-free set of arguments over I containing a.*

Example 3. (Continued)
The minimal temporal Δ-acceptability time of arguments in $\mathbf{G}$ is reported in Table 3. In this example we consider the $\Delta^I_{\texttt{Wg,p}}$ and $\Delta^I_{\texttt{Sg,p}}$ defeats.

$\texttt{min-}\Delta^{\texttt{I}}_{\texttt{Wg,p}}\texttt{-acc} =$ $\texttt{min-}\Delta^{\texttt{I}}_{\texttt{Sg,p}}\texttt{-acc}$	$a = \{4\}$, $b = \{5, 6, 7\}$, $c = \emptyset$, $d = \{6, \ldots, 10\}$

Table 3
Minimal temporal Δ-acceptability over 4 to 10.

As we saw in Figure 3 and Table 3, arguments b and d defeat each other (e.g. in $\Delta^I_{\texttt{Sg,p}}$-$\mathbf{G}$) altough they share some minimal temporal Δ-acceptability time. We then propose to compute an argument's consistent temporal Δ-acceptability by excluding the instants in which it is defeated.

Definition 18 (Consistent temporal Δ-acceptability). *Let $G = \langle \mathcal{A}, \mathcal{R}, \mathcal{P} \rangle$ be a TPAF, $a \in \mathcal{A}$ and I a time interval. We define the consistent temporal Δ-acceptability of a according to Δ by $\texttt{con-}\Delta^{\texttt{I}}\texttt{-acc}(a) = \texttt{min-}\Delta^{\texttt{I}}\texttt{-acc}(a) \setminus \bigcup_{b \in \mathcal{A} \textit{ such that } \Delta^I(b,a) = \top} \texttt{min-}\Delta^{\texttt{I}}\texttt{-acc}(b)$.*

Example 3. (Continued)
Figures 5 and 6 illustrate the consistent temporal Δ-acceptability time of arguments in $\mathbf{G}$ within the interval $\{4, \ldots, 10\}$.
We can notice that in contrast to the results of the semantics and, in particular, for the sceptically accepted arguments, we have here different results between the criteria Wg *and* Sg*. According to the* Wg *criterion, arguments a and d are never defeated and thus sceptically accepted. On the other hand, for the* Sg *criterion, argument a is never defeated, and a defends d; thus, they are sceptically accepted.*
It is interesting to note that a defending d differs from d not being defeated. Indeed, if we look at the instants 6 and 7, in Figure 6 these times are not consistent temporal Δ-acceptable for

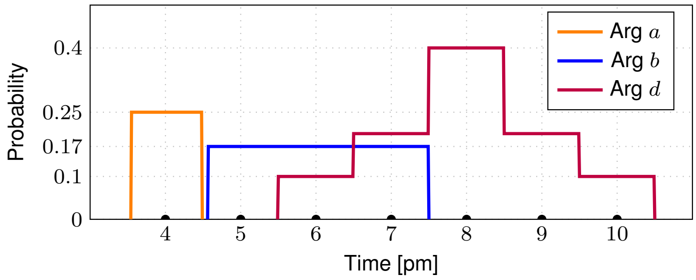

Figure 5: Consistent temporal $\Delta^I_{\mathtt{Wg},\mathtt{p}}$-acceptability.

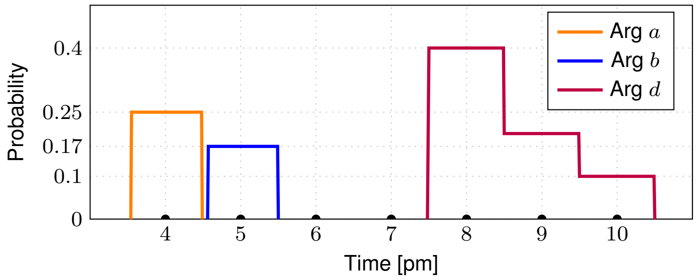

Figure 6: Consistent temporal $\Delta^I_{\mathtt{Sg},\mathtt{p}}$-acceptability.

d because a does not defeat b in $\{6, 7\}$, hence d is defeated in $\{6, 7\}$; which is not the case in Figure 5 where d is not defeated.

Moreover, it is also important to note that thanks to this notion, even if an argument does not have all its time acceptable (as for sceptically accepted arguments), we can extract the subsets of times that are consistent temporal Δ-acceptable, as for the argument b at time 5.

Finally, as can be seen in Figure 5, with the criterion function Wg, it is possible that arguments which attack each other (e.g. b and d where $(b, d), (d, b) \in \mathcal{R}$) are not considered defeated. Therefore, it is up to the user to determine when these arguments are acceptable in overlapping time intervals. We now show that when using the criteria functions Sg and A with the conflict function p, it is, however, not possible to have arguments that attack each other and have consistent temporal Δ-acceptability times in common when they have a different probability distribution.

Proposition 5 (Consistency with the attack relation). *Let $G = \langle \mathcal{A}, \mathcal{R}, \mathcal{P} \rangle$ be a TPAF, $a, b \in \mathcal{A}$ two arguments such that $(a, b), (b, a) \in \mathcal{R}$ and I a time interval. If $\exists t \in I$ such that $\mathcal{P}^{\{t\}}(a) \neq \mathcal{P}^{\{t\}}(b)$, $\mathcal{P}^{\{t\}}(a) > 0$, and $\mathcal{P}^{\{t\}}(b) > 0$; and $\mathtt{x} \in \{\mathtt{Sg}, \mathtt{A}\}$, then $\mathtt{con}\text{-}\Delta^{\mathtt{I}}_{\mathtt{x},\mathtt{p}}\text{-}\mathtt{acc}(a) \cap \mathtt{con}\text{-}\Delta^{\mathtt{I}}_{\mathtt{x},\mathtt{p}}\text{-}\mathtt{acc}(b) = \emptyset$.*

5. Conclusion

The ability to model and reason with probability on events occurrence is crucial for addressing real-world argumentation problems. The framework we propose captures the temporal probabilistic nature of arguments and provides a tool for drawing conclusions starting from a set of conflicting facts/events for which the placement in time is uncertain. The probability associated with timed arguments is subject to interpretations which vary according to context and use case. For this reason, we propose different criteria for establishing whether arguments are in conflict and if the conflict is significant enough to represent a defeat. Based on this graph restriction process (as shown in Figure 3), we can then apply different semantics to calculate the acceptability of the arguments, for instance, those of Dung (Section 3) or news more adapted to the notion of time, as the consistent temporal acceptability which allows assessing the acceptability of arguments over finer time scales (Section 4).

In the future, we plan to carry on this work by examining other aspects of argumentation that relate uncertainty to the notion of time. The current proposal considers events lasting only a single instant (e.g. "the victim died between 6 pm and 10 pm"). A natural extension to that is allowing events with a duration in time (e.g. "the victim has been walking between 2 pm and 7 pm"). In this case, we could use a probability measure to express the likelihood of an event taking place over a time interval. Finally, in addition to uncertainty about the time when a given argument is valid, we may also consider probability associated with arguments and attacks (i.e. they are uncertain), as in Probabilistic Argumentation Frameworks [23]. Consequently, other criterion functions could be introduced for evaluating conflicts based on topological uncertainty.

References

[1] P. M. Dung, On the acceptability of arguments and its fundamental role in nonmonotonic reasoning, logic programming and n-person games, Artificial Intelligence (1995) 321–357.

[2] C. Cayrol, M.-C. Lagasquie-Schiex, On the acceptability of arguments in bipolar argumentation frameworks, in: ECSQARU'8, Springer, 2005, pp. 378–389.

[3] G. Boella, D. M. Gabbay, L. van der Torre, S. Villata, Support in abstract argumentation, in: COMMA'10, 2010, pp. 40–51.

[4] A. Cohen, S. Gottifredi, A. J. García, G. R. Simari, A survey of different approaches to support in argumentation systems, The Knowledge Engineering Review 29 (2014) 513–550.

[5] C. Cayrol, M.-C. Lagasquie-Schiex, An axiomatic approach to support in argumentation, in: Theory and Applications of Formal Argumentation: Third International Workshop, TAFA, Springer, 2015, pp. 74–91.

[6] L. Amgoud, V. David, Measuring similarity between logical arguments, in: Proceedings of the Sixteenth International Conference on Principles of Knowledge Representation and Reasoning KR, 2018, pp. 98–107.

[7] L. Amgoud, V. David, D. Doder, Similarity measures between arguments revisited, in: ECSQARU'15, Springer, 2019, pp. 3–13.

[8] L. Amgoud, V. David, An adjustment function for dealing with similarities, in: Computational Models of Argument (COMMA), IOS Press, 2020, pp. 79–90.

[9] L. Amgoud, V. David, Similarity measures based on compiled arguments, in: ECSQARU, Springer, 2021, pp. 32–44.

[10] L. Amgoud, V. David, A General Setting for Gradual Semantics Dealing with Similarity, in: 35th AAAI Conference en Artificial Intelligence, AAAI Press, 2021, pp. 6185–6192.

[11] P. Dunne, A. Hunter, P. McBurney, S. Parsons, M. Wooldridge, Weighted argument systems: Basic definitions, algorithms, and complexity results, Artificial Intelligence (2011) 457–486.
[12] S. Coste-Marquis, S. Konieczny, P. Marquis, M. A. Ouali, Selecting extensions in weighted argumentation frameworks., COMMA 12 (2012) 342–349.
[13] L. Amgoud, J. Ben-Naim, D. Doder, S. Vesic, Acceptability semantics for weighted argumentation frameworks, in: IJCAI'2017, 2017, pp. 56–62.
[14] S. Bistarelli, F. Rossi, F. Santini, A novel weighted defence and its relaxation in abstract argumentation, Int. J. Approx. Reason. 92 (2018) 66–86.
[15] M. L. Cobo, D. C. Martínez, G. R. Simari, On admissibility in timed abstract argumentation frameworks, in: ECAI 2010 - 19th European Conference on Artificial Intelligence, volume 215 of *FAIA*, IOS Press, 2010, pp. 1007–1008.
[16] W. Zhang, Y. Liang, A temporal based multilateral argumentation dialogue framework, in: ICCASM, 2012, pp. 1244–1247.
[17] M. C. Budán, M. J. G. Lucero, C. I. Chesñevar, G. R. Simari, An approach to argumentation considering attacks through time., SUM 7520 (2012) 99–112.
[18] M. C. Budán, M. L. Cobo, D. C. Martínez, G. R. Simari, Bipolarity in temporal argumentation frameworks, International Journal of Approximate Reasoning 84 (2017) 1–22.
[19] J. Zhu, A Temporal Argumentation Framework with Sets of Attacking Arguments, Master thesis (sup. Bruno Yun), University of Aberdeen, 2020.
[20] J. C. Augusto, G. R. Simari, Temporal defeasible reasoning, Knowledge and information systems 3 (2001) 287–318.
[21] M. C. Budán, M. J. G. Lucero, C. I. Chesñevar, G. R. Simari, Modeling time and valuation in structured argumentation frameworks, Inf. Sci. 290 (2015) 22–44.
[22] A. Hunter, S. Polberg, N. Potyka, T. Rienstra, M. Thimm, Probabilistic argumentation: A survey, in: D. Gabbay, M. Giacomin, G. Simari (Eds.), Handbook on Formal Argumentation, College Publications, 2021, pp. 397–441.
[23] H. Li, N. Oren, T. J. Norman, Probabilistic argumentation frameworks, in: Theorie and Applications of Formal Argumentation: First International Workshop, TAFA 2011, volume 7132 of *LNCS*, Springer, 2011, pp. 1–16.
[24] M. Thimm, A probabilistic semantics for abstract argumentation, in: ECAI - 20th European Conference on Artificial Intelligence, volume 242 of *FAIA*, IOS Press, 2012, pp. 750–755.
[25] P. Dondio, Toward a computational analysis of probabilistic argumentation frameworks, Cybernetics and systems 45 (2014) 254–278.
[26] B. Fazzinga, S. Flesca, F. Furfaro, Probabilistic bipolar abstract argumentation frameworks: complexity results., in: IJCAI, 2018, pp. 1803–1809.
[27] S. Bistarelli, V. David, F. Santini, C. Taticchi, Computing grounded semantics of uncontroversial acyclic constellation probabilistic argumentation in linear time, Workshop: Advances in Argumentation in Artificial Intelligence (AI3) (2022).
[28] S. Bistarelli, V. David, F. Santini, C. Taticchi, Towards a temporal probabilistic argumentation framework, Special Track: Uncertain Reasoning, of FLAIRS'36 (2023).
[29] M. C. Budán, M. G. Lucero, C. Chesnevar, G. R. Simari, Modeling time and valuation in structured argumentation frameworks, Information Sciences 290 (2015) 22–44.
[30] V. David, R. Fournier-S'niehotta, N. Travers, Parameterisation of reasoning on temporal markov logic networks, 2022. URL: https://arxiv.org/abs/2211.16414.