On the Conditional Preference-based Argumentation Framework

(Extended Abstract)

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Abstract

Dung's abstract Argumentation Framework (AF) has emerged as a central formalism in the area of knowledge representation and reasoning. Preferences in AF allow to represent the comparative strength of arguments in a simple yet expressive way. Preference-based AF (PAF) has been proposed to extend AF with preferences of the form a > b, whose intuitive meaning is that argument a is better than b. In this paper we discuss the recently proposed Conditional Preference-based Argumentation Framework (CPAF) [1] that extends PAF by introducing conditional preferences of the form $a > b \leftarrow body$ informally stating that a is better than b whenever the condition expressed by body is true. We discuss CPAF properties and complexity results of the well-known verification and acceptance problems under multiple-status argumentation semantics.

Keywords

Abstract Argumentation, Conditional Preferences, Computational Complexity

Introduction

Recent years have witnessed intensive formal study, development and application of Dung's abstract Argumentation Framework (AF) in various directions [2]. An AF consists of a set A of arguments and an attack relation $\Omega \subseteq A \times A$ that specifies conflicts over arguments (if argument a attacks argument b, then b is acceptable only if a is not). Thus, an AF can be viewed as a directed graph whose nodes represent arguments and edges represent attacks. The meaning of an AF is given in terms of argumentation semantics, e.g. the well-known grounded (gr), complete (co), preferred (pr), stable (st), and semistable (ss) semantics. Intuitively, an argumentation semantics tells us the sets of arguments (called σ -extensions, with $\sigma \in \{gr, co, pr, st, ss\}$) that can collectively be accepted to support a point of view in a dispute. For instance, for AF $\langle A, \Omega \rangle = \langle \{a, b\}, \{(a, b), (b, a)\} \rangle$ having two arguments, a and b, attacking each other, there are two stable extensions, $\mathtt{st}(\langle A, \Omega \rangle) = \{\{\mathtt{a}\}, \{\mathtt{b}\}\}$, and neither argument a nor b is skeptically accepted. To cope with such situations, a possible solution is to provide means for preferring one argument to another.

AF has been extended to Preference-based Argumentation Framework (PAF) where preferences stating that an argument is better than another are considered. Two main approaches have been proposed to define PAF semantics. The first approach defines the PAF semantics in terms of that of an auxiliary AF [3, 4, 5]. However, there are cases where this semantics may give counterintuitive results (see e.g. Example 3 in [1]). The problem is that preferences and attacks, in our opinion, describe different pieces of knowledge and should be considered separately. This is carried out by the second approach comparing extensions w.r.t. preferences defined over arguments [3, 4, 5].

Following this approach, the Conditional Preferencebased AF (CPAF), an extension of AF (and PAF) with a set of conditional preferences (CPs), has been recently introduced in [1]. Intuitively, the CPAF semantics prescribes as best σ -extensions (with $\sigma \in \{gr, co, pr, st, ss\}$) a subset of the σ -extensions of the underlying AF that better satisfy the conditional preferences.

As an example, consider the AF $\Lambda_1 = \langle \{ fish,$ meat, white, red}, {(fish, meat), (meat, fish), (white, red), (red, white)}, describing what a customer is going to have for lunch. (S)he will have either fish or meat, and will drink either white wine or red wine. Assume now that the customer expresses some preferences about the menus: if (s)he will have meat then would prefer to have red wine, whereas if (s)he will have fish then would prefer to have white wine. Intuitively, these preferences can be expressed by means of the following conditional preferences:

 $\texttt{red} > \texttt{white} \leftarrow \texttt{meat} \mid \texttt{white} > \texttt{red} \leftarrow \texttt{fish}.$ Λ_1 has four stable (preferred and semi-stable) extensions: $E_1 = \{ \text{fish}, \text{white} \}, E_2 = \{ \text{fish}, \text{red} \},$ $E_3 = \{ \text{meat}, \text{white} \}$ and $E_4 = \{ \text{meat}, \text{red} \}$, representing four menus. However, only E_1 and E_4 are "best" extensions according to CPs expressed by the customer.

It is worth noting that modifying the AF underlying a CPAF to capture preferences (as done e.g. in [6]) is

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not feasible in general as we have a situation where the best stable extensions are not contained in the best preferred extensions-this contradicts a well-known result for AF stating that every stable extension is a preferred extension [7]. This is also backed by our complexity analysis entailing that CPAF cannot be reduced to AF. As mentioned earlier, AF and preferences represent different pieces of knowledge, such as objective evidences and subjective beliefs, which should be clearly distinguishable. In fact, an AF represents a set of arguments and conflicts among them that leads to a set of consistent sets of arguments that can be collectively accepted (i.e. the set of extensions under a given argumentation semantics) as, for instance, the alternative menus of a restaurant. In contrast, a set of preferences delivers the best extensions, e.g. best menus according to the customer's preferences.

We assume the reader is familiar with AF and PAF semantics. We refer the interested reader to [2] for a comprehensive overview of abstract argumentation.

AF with Conditional Preferences

A conditional preference (also called preference rule) intuitively represents the fact that an argument is better than another whenever a condition expressed by a conjunction of argument literals (i.e. an argument *a* or its negation $\neg a$) is satisfied. More formally, given an AF $\langle A, \Omega \rangle$, a conditional preference (CP) is an expression of the form:

 $a_1 > a_2 \leftarrow b_1 \wedge \cdots \wedge b_m \wedge \neg c_1 \wedge \cdots \wedge \neg c_n$ where $a_1, a_2, b_1, \dots, b_m, c_1, \dots, c_n$ are distinct arguments in A and $n, m \ge 0$. $a_1 > a_2$ is said to be the *head* of the rule, whereas the conjunction of literals $b_1 \wedge \cdots \wedge b_m \wedge \neg c_1 \wedge \cdots \wedge \neg c_n$ is called *body*.

A (polynomial time verifiable) condition is imposed to avoid expressing CPs that can give counterintuitive results. That is, a set of CPs is said to be *well-formed* if there exists a function $\varphi : A \to \mathbb{N}$ such that for each CP $a > b \leftarrow body$ in the set it holds that (i) $\varphi(a) = \varphi(b)$ and (ii) $\varphi(a) \neq \varphi(c)$ for each c (or $\neg c$) occurring in *body*. Intuitively, conditions (i) and (ii) entail a form of stratification of CPs. For instance, consider a CPAF where the underlying AF has extensions {a, b} and {a, c} and the (not well-formed) preferences $c > b \leftarrow b$ and $c > b \leftarrow c$. In this situation, one would expect that {a, c} is preferred to {a, b}. However, as it will be clear after introducing the semantics of CPAF, both extensions are best-extensions. On the other hand, using the well-formed preference $c > b \leftarrow$ we obtain the expected solution.

Definition 1. A Conditional Preference-based AF (CPAF) is a triple $\langle A, \Omega, \Gamma \rangle$, where $\langle A, \Omega \rangle$ is an AF and Γ is a set of (well-formed) conditional preferences.

As an example, consider the AF $\Lambda_2 = \langle A_2, \Omega_2 \rangle$ shown in Figure 1 and the set Γ_2 consisting of the following CPs:



Figure 1: AF Λ_2 at the basis of the CPAF Δ_2 .

fish > meat \leftarrow fruit | white > red \leftarrow fish. Λ_2 has four preferred (and stable/semi-stable) extensions: $E_1 = \{ \texttt{fish}, \texttt{white}, \texttt{pie} \}, E_2 = \{ \texttt{fish}, \texttt{white}, \texttt{fruit} \}, E_3 = \{ \texttt{fish}, \texttt{red}, \texttt{fruit} \}, and E_4 = \{ \texttt{meat}, \texttt{red}, \texttt{fruit} \}$ representing possible menus. Intuitively, we expect that the best preferred extensions according to the conditional preferences in Γ_2 are E_1 and E_2 .

The meaning of a CPAF $\langle A, \Omega, \Gamma \rangle$ w.r.t. a given argumentation semantics $\sigma \in \{gr, co, pr, st, ss\}$ is given by considering the extensions that better satisfy Γ among the σ -extensions of the underlying AF $\langle A, \Omega \rangle$. This is carried out by extending the PAF comparison criteria between extensions (i.e. democratic, elitist and KTV) according to two different interpretations of the preference rules, that are *flat* and *closed* interpretations. As discussed in what follows, differently from the flat interpretation, the closed interpretation deals with the (transitive) closure of Γ .

Hereafter, we say that a (conflict-free) set of arguments E satisfies the body of a conditional preference γ (and write $E \models body(\gamma)$) iff the arguments that positively (resp. negatively) occur in the body of γ belong to E (resp. are attacked by arguments in E).

Definition 2. Given a CPAF $\langle A, \Omega, \Gamma \rangle$, for $E, F \subseteq A$ with $E \neq F$, we have that $E \succeq F$ under

• democratic (d) criterion:

if $\forall b \in F \setminus E \exists a \in E \setminus F$ and $\exists a > b \leftarrow body \in \Gamma$ such that $E \models body$ and $F \models body$;

- elitist (e) criterion:
- if $\forall a \in E \setminus F \exists b \in F \setminus E$ and $\exists a > b \leftarrow body \in \Gamma$ such that $E \models body$ and $F \models body$;
- KTV (k) criterion:

if $\forall a, b \in A \nexists a > b \leftarrow body \in \Gamma$ such that $a \in F \setminus E$, $b \in E \setminus F$, $E \models body$, and $F \models body$.

Moreover, $E \succ F$ *if* $E \succeq F$ *and* $F \not\succeq E$.

For any CPAF $\Delta = \langle A, \Omega, \Gamma \rangle$, best σ -extensions under flat interpretation and criterion $\alpha \in \{d, e, k\}$ are the extensions $E \in \sigma(\langle A, \Omega \rangle)$ such that there is no $F \in \sigma(\langle A, \Omega \rangle)$ with $F \succ E$ (under criterion α).

As an example, for the CPAF $\Delta_2 = \langle A_2, \Omega_2, \Gamma_2 \rangle$, we have that $E_2 \succ E_3$ and $E_3 \succ E_4$ under democratic, elitist and KTV criteria, whereas $E_1 \succ E_3$ and $E_2 \succ E_4$ under KTV criterion. Thus, E_1 and E_2 are the best preferred (and stable/semi-stable) extensions under any criteria.

Closed interpretation. The CPAF with flat interpretation does not generalize the PAF, in the sense that the semantics of a CPAF $\langle A, \Omega, \Gamma \rangle$ where Γ consists of unconditional preferences (i.e. preference rules with empty

Semantics	Verification	Cred. Acc.	Skept. Acc.
cod	coNP-c	Σ_2^p -C	Π_2^p -C
coe	Р	ĨP	Ĩ
co_k	coNP-c	Σ_2^p -C	Р
$\mathtt{st}_d, \mathtt{st}_e, \mathtt{st}_k$	coNP-c	$\Sigma_2^{\overline{p}}$ -C	Π^p_2 -C
\mathtt{pr}_d	coNP-c	Σ_2^p -C	Π_2^p -C
$\operatorname{pr}_{e}, \operatorname{pr}_{k}$	Π_2^p -c	Σ_2^p -h, Σ_3^p	Π_2^p -h, Π_3^p
ss_d, ss_e, ss_k	Π_2^p -C	Σ_2^p -h, Σ_3^p	Π_2^p -h, Π_3^p

Table 1

Complexity of verification and acceptance in CPAF. The results under flat and closed interpretations coincide.

body) may be not equivalent to considering a strict partial order over arguments as in PAF. In the following, we introduce a different semantics for CPAF, called *closed interpretation*, that generalizes that of PAF.

The closed interpretation assumes that Γ denotes all dependencies logically implied by it, that are elements contained in the (transitive) closure of Γ , defined as:

 $\Gamma^* = \Gamma \cup \{a_1 > a_3 \leftarrow body_1 \land body_2 \mid$

 $\{a_1 > a_2 \leftarrow body_1; a_2 > a_3 \leftarrow body_2\} \subseteq \Gamma^*\}.$ Thus, the best extensions under closed interpretation, denoted as $\sigma_{\alpha}^*(\Delta = \langle A, \Omega, \Gamma \rangle)$, are obtained by using Γ^* instead of Γ , that is $\sigma_{\alpha}^*(\langle A, \Omega, \Gamma \rangle) = \sigma_{\alpha}(\langle A, \Omega, \Gamma^* \rangle).$

It can be shown that CPAF semantics under closed interpretation extend PAF semantics, and this holds under flat interpretation if unconditional preferences representing the closure of the PAF preferences are considered (i.e. $\sigma_{\alpha}(\langle A, \Omega, \rangle) = \sigma_{\alpha}^*(\langle A, \Omega, \Gamma = \{\gamma \leftarrow | \gamma \in > \}\rangle).$

When deciding between the flat or closed interpretation, it is crucial to consider the specific context in which the user is operating and their level of familiarity with preference usage. The choice may vary depending on these factors. The closed interpretation offers a more concise representation of preferences, including (transitive) preferences that may not be immediately apparent to the user. This results in a more comprehensive consideration of preferences during the process. On the other hand, the flat interpretation gives the user direct control over the set of preferences must be explicitly provided by the user; otherwise, they will be disregarded.

Properties and Complexity

Several properties have been investigated for CPAF in [1].

A first property states that any conditional preference having an head argument occurring in the body does not play any role (under flat or closed interpretation). Note that this kind of conditional preferences is not wellformed. That is, well-formed condition avoids using useless CPs. Moreover, the satisfaction of CPs are related by subset inclusion, that is let E and F be two complete extensions of the same AF and $\gamma = a_1 > a_2 \leftarrow body$ be a CP, if $E \subseteq F$ and $E \models body$, then $F \models body$.

Several relationships arise between CPAF semantics. Irrespective of the flat or closed interpretation, best complete and grounded semantics for CPAF coincide under elitist criterion, whereas best complete and best preferred semantics coincide under the democratic criterion. Additionally, the grounded extension of the underlying AF is contained in the set of best complete extensions under KTV criterion. Analogously to what holds for AF, the existence of at least one best-stable extension ensures that best-stable and best-semi-stable extensions coincide. However, differently from AF semantics, the set of the best stable (resp. semi-stable) extensions of a CPAF is not a proper subset of the set of the best preferred extensions in general. This hold irrespective of the interpretation and preference criterion, suggesting that preferences cannot be represented in (classical) AF in general, as there are situations where the best stable extensions are not contained in the best preferred extensions.

The complexity results reported in Table 1 show that the verification and credulous/skeptical acceptance problems for CPAF are generally harder than those for AF. Verification and acceptance for CPAF are defined as for AF except that best extensions are considered instead of regular ones. That is, given a CPAF $\Delta = \langle A, \Omega, \Gamma \rangle$, under flat/closed interpretation i) the verification problem is deciding whether a set of arguments $S \subseteq A$ belongs to $\sigma_{\alpha}(\Delta)/\sigma_{\alpha}^{*}(\Delta)$; ii) the credulous (resp. skeptical) acceptance problem is deciding whether an argument $g \in A$ belongs to any (resp. every) extension in $\sigma_{\alpha}(\Delta)/\sigma_{\alpha}^{*}(\Delta)$.

Interestingly, the complexity of the verification problem for CPAF does not depend on the flat or closed interpretation. Moreover, the complexity bounds of the three considered problems for CPAF generally increases of one level in the polynomial hierarchy w.r.t that of AF and coincide with those known for PAF [8], though more general preferences can be expressed in CPAF.

Conclusion

We have discussed the CPAF framework that extends PAF with conditional preferences between arguments. In addition to exploring the connections between CPAF and rich PAF [4], as well as ranking semantics for AF [9, 10], an interesting direction for future work is investigating alternative preference criteria for comparing extensions, similar to those defined for comparing ASP models [11, 12]. Furthermore, we plan to examine conditional preferences in other argumentation frameworks (including structured ones, as done in [13]) that share a semantic relationship with AF [14, 15, 16, 17, 18, 19, 20, 21] as well as in a dynamic setting [22, 23, 24, 25, 26, 27, 28, 29, 30, 31], where objective evidence (underlying AF) and subjective beliefs (conditional preferences) may change over time.

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References

- G. Alfano, S. Greco, F. Parisi, I. Trubitsyna, Abstract argumentation framework with conditional preferences, in: Proc. of AAAI, 2023, pp. 6218– 6227.
- [2] D. Gabbay, M. Giacomin, G. R. Simari, M. Thimm (Eds.), Handbook of Formal Argumentation, volume 2, College Publications, 2021.
- [3] L. Amgoud, C. Cayrol, Inferring from inconsistency in preference-based argumentation frameworks, J. Autom. Reason. 29 (2002) 125–169.
- [4] L. Amgoud, S. Vesic, Rich preference-based argumentation frameworks, Int. J. Approx. Reason. 55 (2014) 585–606.
- [5] S. Kaci, L. W. N. van der Torre, S. Vesic, S. Villata, Preference in abstract argumentation, in: Handbook of Formal Argumentation, volume 2, College Publications, 2021.
- [6] M. Bernreiter, W. Dvorák, S. Woltran, Abstract argumentation with conditional preferences, in: Proc. of COMMA, 2022, pp. 92–103.
- [7] P. M. Dung, On the acceptability of arguments and its fundamental role in nonmonotonic reasoning, logic programming and n-person games, Artif. Intell. 77 (1995) 321–358.
- [8] G. Alfano, S. Greco, F. Parisi, I. Trubitsyna, On preferences and priority rules in abstract argumentation, in: Proceedings of the Thirty-First International Joint Conference on Artificial Intelligence, 2022, pp. 2517–2524.
- [9] E. Bonzon, J. Delobelle, S. Konieczny, N. Maudet, A comparative study of ranking-based semantics for abstract argumentation, in: Proceedings of the Thirtieth AAAI Conference on Artificial Intelligence, 2016, pp. 914–920.
- [10] J. Mailly, J. Rossit, Argument, I choose you! preferences and ranking semantics in abstract argumen-

tation, in: Proceedings of the 17th International Conference on Principles of Knowledge Representation and Reasoning, 2020, pp. 647–651.

- [11] C. Sakama, K. Inoue, Prioritized logic programming and its application to commonsense reasoning, Artif. Intell. 123 (2000).
- [12] G. Brewka, I. Niemelä, M. Truszczynski, Answer set optimization, in: Proceedings of the Eighteenth International Joint Conference on Artificial Intelligence, 2003, pp. 867–872.
- [13] P. M. Dung, P. M. Thang, T. C. Son, On structured argumentation with conditional preferences, in: Proceeding of the Thirty-Third AAAI Conference on Artificial Intelligence, 2019, pp. 2792–2800.
- [14] G. Alfano, S. Greco, F. Parisi, I. Trubitsyna, On the semantics of abstract argumentation frameworks: A logic programming approach, Theory Pract. Log. Program. 20 (2020) 703–718.
- [15] G. Alfano, S. Greco, F. Parisi, I. Trubitsyna, On acceptance conditions in abstract argumentation frameworks, Information Sciences 625 (2023) 757–779.
- [16] B. Fazzinga, S. Flesca, F. Furfaro, Revisiting the notion of extension over incomplete abstract argumentation frameworks, in: Proc. of IJCAI, 2020, pp. 1712–1718.
- [17] G. Alfano, S. Greco, F. Parisi, I. Trubitsyna, Incomplete argumentation frameworks: Properties and complexity, in: Proceedings of the Thirty-Sixth AAAI Conference on Artificial Intelligence, 2022, pp. 5451–5460.
- [18] B. Fazzinga, S. Flesca, F. Parisi, On the complexity of probabilistic abstract argumentation frameworks, ACM Trans. Comput. Log. 16 (2015) 22:1–22:39.
- [19] G. Alfano, M. Calautti, S. Greco, F. Parisi, I. Trubitsyna, Explainable acceptance in probabilistic abstract argumentation: Complexity and approximation, in: Proceedings of the 17th International Conference on Principles of Knowledge Representation and Reasoning, 2020, pp. 33–43.
- [20] G. Alfano, M. Calautti, S. Greco, F. Parisi, I. Trubitsyna, Explainable acceptance in probabilistic and incomplete abstract argumentation frameworks, Artif. Intell. (2023) 103967.
- [21] G. Alfano, S. Greco, F. Parisi, I. Trubitsyna, Argumentation frameworks with strong and weak constraints: Semantics and complexity, in: Proceedings of the Thirty-Fifth AAAI Conference on Artificial Intelligence, 2021, pp. 6175–6184.
- [22] G. Alfano, S. Greco, F. Parisi, Computing stable and preferred extensions of dynamic bipolar argumentation frameworks, in: Proc. of the 1st Workshop on Advances In Argumentation In Artificial Intelligence AI³, 2017, pp. 28–42.

- [23] G. Alfano, S. Greco, F. Parisi, A metaargumentation approach for the efficient computation of stable and preferred extensions in dynamic bipolar argumentation frameworks, Intelligenza Artificiale 12 (2018) 193–211.
- [24] G. Alfano, S. Greco, F. Parisi, G. I. Simari, G. R. Simari, An incremental approach to structured argumentation over dynamic knowledge bases, in: Proceedings of International Conference on Principles of Knowledge Representation and Reasoning (KR), 2018, pp. 78–87.
- [25] G. Alfano, S. Greco, F. Parisi, G. I. Simari, G. R. Simari, Incremental computation for structured argumentation over dynamic DeLP knowledge bases, Artif. Intell. 300 (2021) 103553.
- [26] G. Alfano, S. Greco, F. Parisi, Computing extensions of dynamic abstract argumentation frameworks with second-order attacks, in: Proc. of the 22nd International Database Engineering & Applications Symposium (IDEAS), 2018, pp. 183–192.
- [27] G. Alfano, S. Greco, F. Parisi, On scaling the enu-

meration of the preferred extensions of abstract argumentation frameworks, in: Proceedings of ACM/SI-GAPP Symposium on Applied Computing (SAC), 2019, pp. 1147–1153.

- [28] G. Alfano, S. Greco, Incremental skeptical preferred acceptance in dynamic argumentation frameworks, IEEE Intell. Syst. 36 (2021) 6–12.
- [29] G. Alfano, A. Cohen, S. Gottifredi, S. Greco, F. Parisi, G. Simari, Dynamics in abstract argumentation frameworks with recursive attack and support relations, in: Proceedings of the 24th European Conference on Artificial Intelligence (ECAI), 2020, pp. 577–584.
- [30] A. Niskanen, M. Järvisalo, Algorithms for dynamic argumentation frameworks: An incremental sat-based approach, in: Proceedings of the 24th European Conference on Artificial Intelligence (ECAI), 2020, pp. 849–856.
- [31] G. Alfano, S. Greco, F. Parisi, Incremental computation in dynamic argumentation frameworks, IEEE Intell. Syst. 36 (2021) 80–86.