Fuzzy expert decision support system for foreign direct investment: a swarm metaheuristic approach

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Abstract
This paper develops and optimizes a fuzzy expert system for foreign direct investment decision support. The paper aims to provide a reliable and flexible tool for investors to evaluate the attractiveness of different countries for foreign direct investment. The paper uses an adaptive gravitational search algorithm to determine the optimal parameters of the fuzzy expert system, such as the membership functions for linguistic input and output variables. The paper also uses a quality criterion that considers the specificity of the fuzzy expert system and allows assessing the probability of future decisions. The paper conducts a numerical study to test the performance of the proposed fuzzy expert system and compares it with other existing methods. The results show that the proposed fuzzy expert system has a high accuracy and robustness in foreign direct investment decision support. The paper contributes to the literature on fuzzy logic applications in economics and finance and provides a practical tool for investors to make informed decisions on foreign direct investment.

Keywords
fuzzy expert system, foreign direct investment, adaptive gravitational search algorithm, quality criterion, decision support

1. Introduction
In contemporary contexts, decision-making systems for foreign direct investment have gained significant prominence. Machine learning techniques [2, 3, 4], such as regression [5] and auto-regressive methods [6], are commonly employed to construct these systems. However, these approaches often result in linear models, limiting their scope. Expert systems, utilizing a knowledge base typically represented as production rules, are another avenue for building decision-making systems for foreign direct investment [7]. Yet, these systems are criticized for their sole reliance on quantitative assessments, which can pose challenges when operators prefer qualitative estimates.
Fuzzy expert systems have emerged as a means to simplify human-computer interactions. These systems leverage fuzzy inference mechanisms such as Larsen, Mamdani, Tsukamoto, and Sugeno [8, 9]. Nonetheless, a key shortcoming is the lack of automation in determining their parameters [10, 11]. Addressing these limitations calls for the utilization of optimization methods to fine-tune the parameters of fuzzy expert systems.

However, contemporary optimization methods are not without their own set of challenges:

- Many possess high computational complexity.
- Several are prone to converging into local extrema.
- Some lack convergence guarantees.

In this regard, there is an actual problem of optimization methods’ insufficient efficiency. Consequently, the quest for more efficient optimization methods is pertinent. This has led to the adoption of metaheuristics, a class of modern heuristics aimed at hastening the discovery of quasi-optimal solutions to optimization problems and reducing the likelihood of converging into local optima [12, 13].

Yet, current metaheuristics have their own limitations:

- Certain methods exhibit inadequate accuracy [14].
- Others provide only abstract descriptions or are tailored to specific problems [15].
- The process of parameter determination remains non-automated [16].
- The influence of iteration count on solution search is often disregarded [17].
- Some methods lack the capability to address conditional optimization problems [18].
- Incompatibility with non-binary potential solutions exists [19].
- Convergence guarantees may be absent [20].

Hence, the challenge of constructing efficient metaheuristic optimization methods arises [21, 22]. One noteworthy example of such a metaheuristic is the gravitational search algorithm, a member of the swarm metaheuristics family [23].

This research is driven by the need to develop adept fuzzy expert systems using parametric identification for adaptation and refinement in the realm of foreign direct investment decisions.

**Objective:** This study aims to enhance the effectiveness of foreign direct investment decisions by constructing a fuzzy expert system trained through the utilization of metaheuristics.

To accomplish this overarching goal, the following tasks have been undertaken:

1. Design a fuzzy expert decision support system for foreign direct investment.
2. Select an appropriate quality criterion for the proposed fuzzy expert system.
3. Develop a metaheuristic approach based on an adaptive gravitational search algorithm for parameter determination of the proposed fuzzy expert system.
4. Conduct extensive numerical investigations.
2. The fuzzy expert decision support system for foreign direct investment

The foreign direct investment analysis is based on the data of the GDP per capita volume, inflation rates, goods and services exports volume, and labor force indicators. To make decisions on foreign direct investment, a fuzzy expert system is proposed. It involves the following steps:

1) linguistic variables formation;
2) fuzzy knowledge base formation;
3) Mamdani fuzzy inference mechanism formation:
   • fuzzification;
   • sub-conditions aggregation;
   • conclusions activation;
   • aggregation of conclusions;
   • defuzzification.
4) identification of parameters based on metaheuristics.

2.1. Linguistic variables formation

The following input variables were chosen:

- the volume of gross domestic product (GDP) per capita (per year, US dollars), \( x_1 \);
- the inflation indicator (according to the consumer price index, which reflects the annual percentage change in the cost for the average consumer of purchasing a goods and services basket, per year, %), \( x_2 \);
- the volume of goods and services export indicator (total volume, per year, USD), \( x_3 \);
- the labor force indicator (labor force is people aged 15 and over who provide labor for the production of goods and services, per year, number of people), \( x_4 \).

The following indicators were chosen as linguistic input variables. They are qualitative indicators:

- the GDP volume \( \tilde{x}_1 \) with values \( \tilde{\alpha}_{11} = \text{little}, \tilde{\alpha}_{12} = \text{medium}, \tilde{\alpha}_{13} = \text{much} \), where the ranges are fuzzy sets \( \hat{A}_{11} = \{ (x_1, \mu_{\hat{A}_{11}}(x_1)) \}, \hat{A}_{12} = \{ (x_1, \mu_{\hat{A}_{12}}(x_1)) \}, \hat{A}_{13} = \{ (x_1, \mu_{\hat{A}_{13}}(x_1)) \} \);
- the inflation indicator \( \tilde{x}_2 \) with values \( \tilde{\alpha}_{21} = \text{little}, \tilde{\alpha}_{22} = \text{medium}, \tilde{\alpha}_{23} = \text{much} \), where the ranges are fuzzy sets \( \hat{A}_{21} = \{ (x_2, \mu_{\hat{A}_{21}}(x_2)) \}, \hat{A}_{22} = \{ (x_2, \mu_{\hat{A}_{22}}(x_2)) \}, \hat{A}_{23} = \{ (x_2, \mu_{\hat{A}_{23}}(x_2)) \} \);
- the volume of goods and services export indicator \( \tilde{x}_3 \) with values \( \tilde{\alpha}_{31} = \text{little}, \tilde{\alpha}_{32} = \text{medium}, \tilde{\alpha}_{33} = \text{much} \), where the ranges are fuzzy sets \( \hat{A}_{31} = \{ (x_3, \mu_{\hat{A}_{31}}(x_3)) \}, \hat{A}_{32} = \{ (x_3, \mu_{\hat{A}_{32}}(x_3)) \}, \hat{A}_{33} = \{ (x_3, \mu_{\hat{A}_{33}}(x_3)) \} \);
- the labor force indicator \( \tilde{x}_4 \) with values \( \tilde{\alpha}_{41} = \text{little}, \tilde{\alpha}_{42} = \text{medium}, \tilde{\alpha}_{43} = \text{much} \), where the ranges are fuzzy sets \( \hat{A}_{41} = \{ (x_4, \mu_{\hat{A}_{41}}(x_4)) \}, \hat{A}_{42} = \{ (x_4, \mu_{\hat{A}_{42}}(x_4)) \}, \hat{A}_{43} = \{ (x_4, \mu_{\hat{A}_{43}}(x_4)) \} \).
The volume of foreign direct investment (net flows for the year, USD) was chosen as a clear output variable $\tilde{y}$. It is a qualitative indicator.

The volume of foreign direct investment was chosen $\tilde{y}$ with its values $\tilde{\beta}_1 = \text{little}, \tilde{\beta}_2 = \text{medium}, \tilde{\beta}_3 = \text{much}$, where the ranges are fuzzy sets $\tilde{B}_1 = \{(y, \mu_{\tilde{B}_1}(y))\}, \tilde{B}_2 = \{(y, \mu_{\tilde{B}_2}(y))\}, \tilde{B}_3 = \{(y, \mu_{\tilde{B}_3}(y))\}$;

### 2.2. Fuzzy knowledge base formation

Fuzzy knowledge is represented as the following fuzzy rules that contain a linguistic output variable $R^n$: IF $\tilde{x}_1$ is $\tilde{a}_{i1}$ AND $\tilde{x}_2$ is $\tilde{a}_{i2}$ AND $\tilde{x}_3$ is $\tilde{a}_{i3}$ AND $\tilde{x}_4$ is $\tilde{a}_{i4}$ then $\tilde{y}$ is $B_m$.

In the case of linguistic variables specific values, fuzzy knowledge is presented in relational form in table 1.

#### Table 1
Relational form of fuzzy knowledge representation.

<table>
<thead>
<tr>
<th>Rule $R^i$</th>
<th>$x_1$</th>
<th>$x_2$</th>
<th>$x_3$</th>
<th>$x_4$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R^1$</td>
<td>$\alpha_{11}$</td>
<td>$\alpha_{21}$</td>
<td>$\alpha_{31}$</td>
<td>$\alpha_{41}$</td>
<td>$\alpha_{1}$</td>
</tr>
<tr>
<td>$R^2$</td>
<td>$\alpha_{12}$</td>
<td>$\alpha_{21}$</td>
<td>$\alpha_{31}$</td>
<td>$\alpha_{41}$</td>
<td>$\alpha_{1}$</td>
</tr>
<tr>
<td>$R^3$</td>
<td>$\alpha_{13}$</td>
<td>$\alpha_{21}$</td>
<td>$\alpha_{31}$</td>
<td>$\alpha_{41}$</td>
<td>$\alpha_{2}$</td>
</tr>
<tr>
<td>$R^4$</td>
<td>$\alpha_{11}$</td>
<td>$\alpha_{22}$</td>
<td>$\alpha_{31}$</td>
<td>$\alpha_{41}$</td>
<td>$\alpha_{2}$</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>$R^{n1}$</td>
<td>$\alpha_{13}$</td>
<td>$\alpha_{23}$</td>
<td>$\alpha_{33}$</td>
<td>$\alpha_{43}$</td>
<td>$\alpha_{3}$</td>
</tr>
</tbody>
</table>

### 2.3. Mamdani fuzzy inference mechanism formation

#### 2.3.1. Fuzzification

We will determine the truth degree of each sub-condition of each rule, using the membership function $\mu_{\tilde{A}_i}(x_i)$.

As membership functions of sub-conditions, we chose:

- piecewise linear Z-shaped function, i.e.

  $$
  \mu_{\tilde{A}_{i1}}(x_i) = \begin{cases}
  1, & x_i \leq a_i \\
  \frac{b_i - x_i}{b_i - a_i}, & a_i < x_i < b_i \\
  0, & x_i \geq b_i
  \end{cases}, \quad i \in 1, 4
  $$

- piecewise linear $\Pi$-shaped function, i.e.

  $$
  \mu_{\tilde{A}_{i2}}(x_i) = \begin{cases}
  0, & x_i \leq a_i \\
  \frac{x_i - a_i}{b_i - a_i}, & a_i \leq x_i \leq b_i \\
  1, & b_i \leq x_i \leq c_i \\
  \frac{d_i - x_i}{d_i - c_i}, & c_i \leq x_i \leq d_i \\
  0, & x_i \geq d_i
  \end{cases}, \quad i \in 1, 4
  $$
• piecewise linear S-shaped function, i.e.

\[
\mu_{A_{3i}}(x_i) = \begin{cases} 
0, & x_i \leq c_i \\
\frac{x_i - c_i}{d_i - c_i}, & c_i < x_i < d_i \\
1, & x_i \geq d_i 
\end{cases}, \quad i \in \{1, 4\},
\]

where \(a_i, b_i, c_i, d_i\) - membership function parameters.

2.3.2. Sub-condition aggregation

The condition membership functions for each rule \(R^n\) are determined based on the minimum value method:

\[
\mu_{\bigcup_{i=1}^{4} \tilde{A}_{i, f(n,i)}}(x_1, x_2, x_3, x_4) = \min_{i \in \{1, 4\}} \left\{ \mu_{\tilde{A}_{i, f(n,i)}}(x_i) \right\},
\]

where \(f\) – a function that returns the value number of the \(i\)-th linguistic input variable of the \(n\)-th rule and is determined on the basis of table 1. For example, if the linguistic input variable \(\tilde{x}_1\) rules \(R^{81}\) matters \(\tilde{A}_{13}\), then \(f(81, 1) = 3\).

2.3.3. Activation of conclusions

The membership functions of the conclusion for each rule \(R^n\) are determined based on the minimum value method (based on the Mamdani rule):

\[
\mu_{\tilde{B}_{g(n)}}(y) = \min \left\{ \mu_{\bigcup_{i=1}^{4} \tilde{A}_{i, f(n,i)}}(x_1, x_2, x_3, x_4), \mu_{\tilde{B}_{g(n)}}(y) \right\},
\]

where \(g\) – a function that returns the value number of the linguistic output variable of \(n\)-th rule and determined on the basis of table 1.

For example, if the linguistic output variable \(\tilde{y}\) of the rule \(R^{81}\) is \(\tilde{B}_3\), then \(g(81) = 3\).

A piecewise linear triangular function was chosen as the membership functions of the conclusions, i.e.

\[
\mu_{\tilde{B}_m}(y) = \begin{cases} 
0, & y \leq e_m \\
\frac{y - e_m}{u_m - e_m}, & e_m \leq y \leq u_m \\
\frac{u_m - y}{v_m - u_m}, & u_m \leq y \leq v_m \\
0, & y \geq v_m 
\end{cases}, \quad m \in \{1, 3\},
\]

where \(e_m, u_m, v_m\) – membership function parameters.

In the case of such a membership function, the kernel of each fuzzy set \(\tilde{B}_m\) is:

\[
\ker \tilde{B}_m = \{ y \in Y | \mu_{\tilde{B}_m}(y) = 1 \} = \{ u_m \}.
\]

2.3.4. Aggregation of conclusions

The membership functions of the final conclusion are defined, which contains a linguistic output variable based on the maximum value method:

\[
\mu_{\tilde{B}_m}(Y) = \max_{n \in \{1, 81\}} \{ \mu_{\tilde{B}_{g(n)}}(y) \}
\]

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2.3.5. Defuzzification

The volumes of foreign direct investment are determined based on the centroid method:

\[ y^* = \frac{\sum_{y \in Y} \mu_B(y)y}{\sum_{y \in Y} \mu_B(y)}, Y = \{1, 2, 3\} \]

3. Quality criterion for the proposed fuzzy expert system

The objective function is chosen as a quality criterion, representing the accuracy as probability of correct foreign direct investment

\[ F = \frac{1}{P} \sum_{p=1}^{P} [y_p = d_p] \rightarrow \max_{\theta} \]

\[ [p = q] = \begin{cases} 1, & p = q \\ 0, & p \neq q \end{cases} \]

where \( d_p \) – test foreign direct investment, 
\( y_p \) – foreign direct investment received as a result of fuzzy inference, 
\( P \) – number of test implementations, 
\( \theta = (a_1, b_1, c_1, d_1, ..., a_4, b_4, c_4, d_4, e_1, u_1, v_1, ..., e_3, u_3, v_3) \) – parameter vector of membership functions.

4. Metaheuristic method based on an adaptive gravitational search algorithm for determining the parameters of the proposed fuzzy expert system

The particle velocity (not the gravitational constant) depends on the iteration number in this method, which provides control over the convergence rate of the method, as well as providing a global search at the initial iterations, and a local search at the final iterations. The parameter vector of membership functions corresponds to the position vector of one particle \( x \). The quality criterion is used as the goal function (1).

1. Initialization.
   1.1. Setting the gravitational constant \( G \), the maximum number of iterations \( N \), the population size \( K \), the length of the particle position vector \( M \) (it corresponds to the length of the parameter vector of membership functions and is equal to 25), the minimum and maximum values for the position vector \( x_j^{\text{min}}, x_j^{\text{max}}, j \in 1, M \), the minimum and maximum values for the velocity vector \( v_j^{\text{min}}, v_j^{\text{max}}, j \in 1, M \).
   1.2. The best position vector randomly generating \( x^* = (x_1^*, ..., x_M^*) \), 
   \[ x_j^* = x_j^{\text{min}} + (x_j^{\text{max}} - x_j^{\text{min}})U(0, 1), \]
   where \( U(0, 1) \) – a function that returns a uniformly distributed random number in a range \([0, 1]\).
1.3. The initial population creation

1.3.1. Particle number \( k = 1 \), \( P = \emptyset \).

1.3.2. A position vector at random \( x_k \) generating \( x_k = (x_{k1}, ..., x_{kM}) \).

\[ x_{kj} = x_{j}^{\text{min}} + (x_{j}^{\text{max}} - x_{j}^{\text{min}})U(0, 1). \]

1.3.3. Random velocity vector \( v_k \) generating \( v_k = (v_{k1}, ..., v_{kM}) \).

\[ v_{ij} = v_{j}^{\text{min}} + (v_{j}^{\text{max}} - v_{j}^{\text{min}})U(0, 1). \]

1.3.4. If \( (x_k, v_k) \notin P \), then \( P = P \cup \{(x_k, v_k)\} \), \( k = k + 1 \).

1.3.5. If \( k \leq K \), then go to step 1.3.2.

2. Iteration number \( n = 1 \).

3. The computation of the best and worst particle of a population from a target function

\[ l = \arg \min_k F(x_k), x_{\text{best}} = x_l, \]
\[ l = \arg \max_k F(x_k), x_{\text{worst}} = x_l. \]

4. The computation of all particles masses.

5. The computation of the gravitational force acting between all pairs of particles

5.1. \( m_k = G \frac{F(x_k) - F(x_{\text{worst}})}{F(x_{\text{best}}) - F(x_{\text{worst}})}, k \in \overline{1, K}. \)

5.2. \( M_k = \frac{M_k}{\sum_{i=1}^K m_i}, k \in \overline{1, K}. \)

6. The computation of the gravitational force acting between all pairs of particles

\[ f_{kl} = G \frac{M_k M_l}{d(x_k, x_l) + \epsilon} (x_l - x_k), k, l \in \overline{1, K}, \]

where \( d(x_k, x_l) \) – distance between particles \( k \) and \( l \) (e.g. Euclid distance).

7. The computation of the resulting force acting on all particles

\[ r_{kl} = U(0, 1), k \in \overline{1, K}, \]
\[ f_k = \sum_{l=1}^K r_{kl} f_{kl} k \in \overline{1, K}, \]
\[ l \neq k \]

8. Modification of the acceleration of all particles

\[ a_k = \frac{f_k}{M_k}, k \in \overline{1, K} \]

9. Speed modification of all particles

\[ r_k = U(0, 1), k \in \overline{1, K}, \]
\[ v_k = r_k v_k + a_k, k \in \overline{1, K} \]

10. Modification all of the particles' position, taking into account the iteration number

10.1. \( x_k = x_k + v_k \left( 1 - \frac{n}{N} \right), k \in \overline{1, K} \)

10.2. \( x_{kj} = \max\{x_{kj}^{\text{min}}, x_{kj}\}, x_{kj} = \min\{x_{kj}^{\text{max}}, x_{kj}\}, j \in \overline{1, M}, k \in \overline{1, K} \)

11. If \( n < N \), then \( n = n + 1 \), go to step 3

The result is \( x^* \).
5. Numerical research

Numerical research was carried out using the Keras submodule of the TensorFlow module. The Pandas module was used to fill in missing values through linear interpolation, as well as for tabular data I/O operations. The Scikit-fuzzy module was used to create a fuzzy expert system.

The fuzzy expert system was researched using the World Bank economic indicators database (https://databank.worldbank.org/home.aspx). The economic indicators of 145 countries for 10 years were used. The size of the original sample was 1450.

For the proposed adaptive gravity search algorithm, the gravity constant G was 100, the maximum number of iterations was 1000, and the population size was 50.

The comparison results of the proposed fuzzy expert system with the operator are presented in table 2.

<table>
<thead>
<tr>
<th>Accuracy</th>
<th>fuzzy expert system</th>
<th>operator</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.98</td>
<td>0.8</td>
</tr>
</tbody>
</table>

The comparison results of the proposed fuzzy expert system with the proposed meta-heuristic adaptive gravitational search algorithm (AGSA) and the traditional meta-heuristic adaptive gravitational search algorithm (AGSA) operator are presented in table 3.

<table>
<thead>
<tr>
<th>Accuracy</th>
<th>GSA</th>
<th>AGSA</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.93</td>
<td>0.98</td>
</tr>
</tbody>
</table>

The comparison results of the proposed fuzzy expert system based on the back-propagation method and proposed meta-heuristic are presented in table 4.

<table>
<thead>
<tr>
<th>Accuracy</th>
<th>BP</th>
<th>AGSA</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.90</td>
<td>0.98</td>
</tr>
</tbody>
</table>

Figure 1 shows the accuracy for the proposed fuzzy expert system trained based on the proposed meta-heuristic adaptive gravitational search algorithm (AGSA) and on the proposed meta-heuristic gravitational search algorithm (GSA).

The comparison results of the proposed fuzzy expert system trained on the basis of back-propagation (BP) and the proposed meta-heuristic adaptive gravitational search algorithm (AGSA) are presented in table 4.
Figure 1: Accuracy of the proposed fuzzy expert system with GSA and AGSA.

Figure 2 shows the accuracy for the proposed fuzzy expert system trained on the basis of back-propagation (BP) and the proposed meta-heuristic adaptive gravitational search algorithm (AGSA).

Figure 2: Accuracy of the proposed fuzzy expert system with BP and AGSA.

Figures 3-7 shows the membership functions for the values of linguistic variables $\tilde{x}_1$, $\tilde{x}_2$, $\tilde{x}_3$, $\tilde{x}_4$ and $y$.

6. Discussion

The traditional non-automatic approach to assessing the foreign direct investment effectiveness reduces the accuracy of a correct assessment (table 2). The proposed method eliminates this disadvantage.

The traditional method of the gravitational search algorithm ignores the iteration number during the particle position calculating; this reduces the accuracy of finding a solution (table 3);
Figure 3: Membership functions for linguistic variable values $\tilde{x}_1$.

Figure 4: Membership functions for linguistic variable values $\tilde{x}_2$.

requires a large number of parameters associated with the gravitational constant calculating. The proposed method eliminates these shortcomings.

The traditional approach to training a fuzzy expert system based on back propagation reduces the probability of correct estimation (table 4). The proposed method eliminates this disadvantage.

7. Conclusions

This section presents the key outcomes and contributions of the research, highlighting the insights gained and the novel methodologies developed:

1. *Exploration of Relevant Methods*: The study delved into the landscape of optimization methods and expert systems within the realm of foreign direct investment decision-making. The findings underscored the potency of employing fuzzy expert systems, parameterized through contemporary metaheuristic techniques.

2. *Development of Fuzzy Expert System*: A novel fuzzy expert decision support system for foreign direct investment has been conceived. This innovative system streamlines
operator-computer interactions by integrating qualitative indicators, facilitating parameter identification through the proposed swarm metaheuristics.

3. **Introduction of Quality Criterion**: A bespoke quality criterion has been introduced, tailored to the nuances of the newly devised fuzzy expert system. This criterion enables a comprehensive assessment of decision accuracy.

4. **Creation of Adaptive Swarm Metaheuristic Algorithm**: An adaptive swarm metaheuristic algorithm, founded on the principles of the gravitational search algorithm, has been crafted. This algorithm boasts the capability to regulate convergence rate, execute global exploration in initial iterations, and transition to local exploration in later iterations via adaptive particle velocity control.

5. **Empowering Decision Technology**: The amalgamation of the swarm metaheuristic optimization method with the fuzzy expert system furnishes a means to infuse sophistication into foreign direct investment decision-making technology. This intellectualized approach holds significant promise in enhancing decision accuracy and effectiveness.

6. **Future Prospects**: The envisioned trajectory involves subjecting the proposed method
and system to more extensive testing using a broader array of test databases. This step is pivotal in validating the method’s robustness and efficacy in diverse scenarios.

References


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