Abstract Argumentation Applied to Fair Resources Allocation: A Preliminary Study

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Abstract

In this paper, we discuss the application of abstract argumentation mechanisms to resources allocation. We show how such problems can be modeled as abstract argumentation frameworks, such that specific sets of arguments corresponds to interesting solutions of the problem. By interesting solutions, here we mean *Local Envy-Free* (LEF) allocations. Envy-freeness is an important notion of fairness in resources allocation, assuming than no agent should prefer the resource allocated to another agent. We focus on LEF, a generalized form of envy-freeness, and we show that LEF allocations corresponds to some specific sets of arguments in our argument-based modeling of the problem. This work in progress paves the way to richer connections between the various models of argumentation and resources allocation problems.

Keywords

Abstract argumentation, Resources allocation, Fairness

1. Introduction

Fairness issues are important in multi-agent scenarios, including resources allocation. Among the various fairness criteria, one of them is *envy-freeness*, *i.e.* the fact that no agent is envious of another agent. A generalized version of envy-freeness is *local envy-freeness* (LEF) [1], where agents are part of a social network, and the goal is to assign each agent one object such that none of them is envious of one of her neighbours in the network. In this work, we study a transformation from LEF problems to abstract argumentation [2]. We show that there is a correspondence between LEF allocations and some specific extensions of an argumentation framework built from the LEF problem at hand. The representation of LEF problems as argumentation problems offers several advantages. First of all, there are plenty of efficient tools for computing extensions of argumentation frameworks, which is not the case with LEF problems. Then, the argumentation process can offer intuitive (and visual) explanations of why an allocation is LEF, or why there is no such allocation. Finally, the vast literature on argumentation provides tools for other fairness problems, or for identifying specific allocations, *e.g.* weighted argumentation frameworks can provide means to obtain optimal allocations (w.r.t. agents utility functions or w.r.t. the Pareto criterion).

Section 2 provides some background notions on LEF allocations and abstract argumentation.

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Section 3 discusses our transformation from resources allocation problems to abstract argumentation, and shows the relation between LEF allocations and specific extensions. Finally Section 4 concludes the paper by discussing some interesting questions for future work.

2. Background

2.1. Local Envy-Freeness

We focus on a resource allocation scenario (previously studied in [1]) where agents may know some other agents, and have preferences over the set of resources that must be allocated. The other hypotheses in our scenario are the fact that the number of agents is equal to the number of resources, and the fact that resources are indivisible goods. Formally,

Definition 1. *A* preference-based allocation problem *(PRAP) is a tuple* $\mathcal{PRAP} = \langle \mathcal{O}, \mathcal{N}, \succ, \mathcal{G} \rangle$*:*

- $\mathcal{O} = \{o_1, \dots, o_n\}$ *is the set of objects,*
- $\mathcal{N} = \{1, \dots, n\}$ *is the set of agents,*
- $\succ$ *is a set of binary relations* $\{\succ_i \mid i \in N\}$ *where* $\succ_i$ *is a linear order expressing the preferences of agent* i *over* $\mathcal{O}$*,*
- $\mathcal{G} = \langle \mathcal{N}, \mathcal{E} \rangle$ *is an undirected graph representing the social network of agents.*

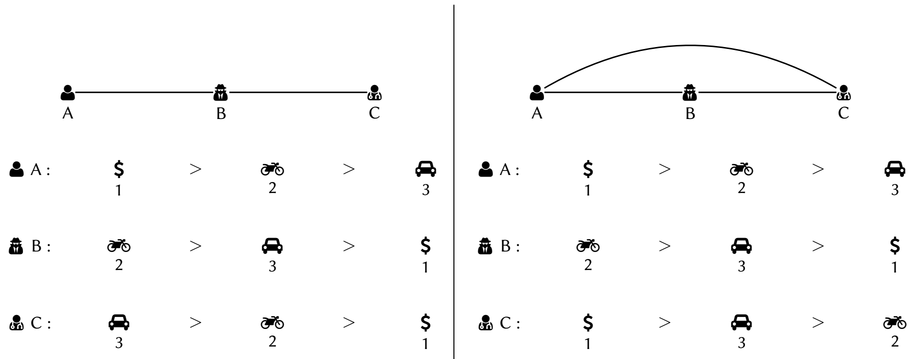
We are interested in the problem of local envy-freeness, *i.e.* whether we can assign each agent i an object o_k s.t. i does not prefer the object assigned to one of her neighbors, formally $\forall j \in \mathcal{N}$ s.t. $\{i, j\} \in \mathcal{E}$, $o_l \not\succ_i o_k$, where o_k and o_l are (respectively) the object assigned to i and the object assigned to j.

To characterize formally this concept, we represent an allocation as a set of pairs $(i, o_k) \in \mathcal{N} \times \mathcal{O}$. Given such a pair $x = (i, o_k)$, we use $\texttt{Ag}(x)$ and $\texttt{Obj}(x)$ to denote respectively the agent i and the object o_k of this pair. An allocation γ is *valid* if for each $x, y \in \gamma$, if $x \neq y$ then $\texttt{Ag}(x) \neq \texttt{Ag}(y)$ and $\texttt{Obj}(x) \neq \texttt{Obj}(y)$, *i.e.* no agent receives several objects, and no object is assigned to several agents. A valid allocation can be *partial* if $|\gamma| < |\mathcal{O}|$, and *total* if $|\gamma| = |\mathcal{O}| = |\mathcal{N}|$.

Definition 2. *Given* $\mathcal{PRAP} = \langle \mathcal{O}, \mathcal{N}, \succ, \mathcal{G} \rangle$*, an allocation is* local envy-free *(LEF) iff it is a total valid allocation such that* $\forall i, j \in \mathcal{N}$ *s.t.* $\{i, j\} \in \mathcal{E}$*, if* $(i, o_k), (j, o_l) \in \gamma$ *then* $o_l \not\succ_i o_k$*.*

Example 1. *Figure 1a describes* $\mathcal{PRAP} = \langle \mathcal{O}, \mathcal{N}, \succ, \mathcal{G} \rangle$ *where the agents are* $\mathcal{N} = \{A, B, C\}$*, the objects are* $\mathcal{O} = \{1, 2, 3\}$ *with* 1 *representing some money,* 2 *a motorbike, and* 3 *a car. The social network* $\mathcal{G}$ *is shown at the top of the Figure, and the agents preferences* $\succ$ *are given underneath. We can easily find a LEF allocation by giving each agent her preferred object.*

Now consider $\mathcal{PRAP}_2$ *given at Figure 1b, where this time all the agents know each other, and agent C's preferences are slightly modified as well. Assume there is a LEF allocation* γ*. Neither A nor C can receive the object* 1 *(because otherwise the one receiving another object would be envious of the one receving the object* 1*). Thus we must have* $(B, 1) \in \gamma$*. But in this case, both agents A and C are envious of agent* B*. So there is no LEF allocation for* $\mathcal{PRAP}_2$*.*



(a) $\mathcal{PRAP}_1$ (b) $\mathcal{PRAP}_2$

Figure 1: Social Networks and Preferences

2.2. Abstract Argumentation

Now let us recall basic notions of Dung's abstract argumentation [2].

Definition 3. *An abstract* argumentation framework *(AF) is a directed graph* $\mathcal{F} = \langle \mathcal{A}, \mathcal{R} \rangle$ *where* $\mathcal{A}$ *is the set of* arguments *and* $\mathcal{R} \subseteq \mathcal{A} \times \mathcal{A}$ *is the* attack relation.

Classical reasoning with AF uses the notion of *extensions*, *i.e.* sets of arguments that can be jointly accepted. Various semantics have been proposed to obtain the set of extensions of an AF. Formally, an extension-based semantics is a function σ such that for any AF $\mathcal{F} = \langle \mathcal{A}, \mathcal{R} \rangle$, $\sigma(\mathcal{F}) \subseteq 2^{\mathcal{A}}$. In this paper, we only need the notions of conflict-free sets and stable extensions:

Definition 4. *Given* $\mathcal{F} = \langle \mathcal{A}, \mathcal{R} \rangle$, $S \subseteq \mathcal{A}$ *is* conflict-free *(*$S \in \mathsf{cf}(\mathcal{F})$*) if* $\forall a, b \in S$, $(a, b) \notin \mathcal{R}$. *Then,* S *is a* stable extension *(*$S \in \mathsf{stb}(\mathcal{F})$*) if* $S \in \mathsf{cf}(\mathcal{F})$ *and* $\forall b \in \mathcal{A} \setminus S$, $\exists a \in S$ *s.t.* $(a, b) \in \mathcal{R}$.

Among the various generalizations of Dung's argumentation framework, we are interested in *preference-based* AFs (PAFs) [3].

Definition 5. *A* preference-based argumentation framework *is a tuple* $\mathcal{P} = \langle \mathcal{A}, \mathcal{R}, \rhd \rangle$ *where* $\langle \mathcal{A}, \mathcal{R} \rangle$ *is an AF, and* $\rhd \subseteq \mathcal{A} \times \mathcal{A}$ *is a* preference relation *over the set of arguments.*

The preference relation is only assumed to be a pre-order, *i.e.* a reflexive and transitive binary relation. The main approach for reasoning with a PAF consists in reducing it into a standard AF by combining the attacks and the preferences into a *defeat* relation. Then, the extensions of the PAF for a semantics σ are the extensions of the defeat graph under the same semantics.

Definition 6. *Given a PAF* $\mathcal{P} = \langle \mathcal{A}, \mathcal{R}, \rhd \rangle$, *we define the* defeat relation $\mathcal{D} = \{(x, y) \in \mathcal{R} \mid y \not\rhd x\}$. *Then,* $\sigma(\mathcal{P}) = \sigma(\langle \mathcal{A}, \mathcal{D} \rangle)$.

3. Translation into Abstract Argumentation

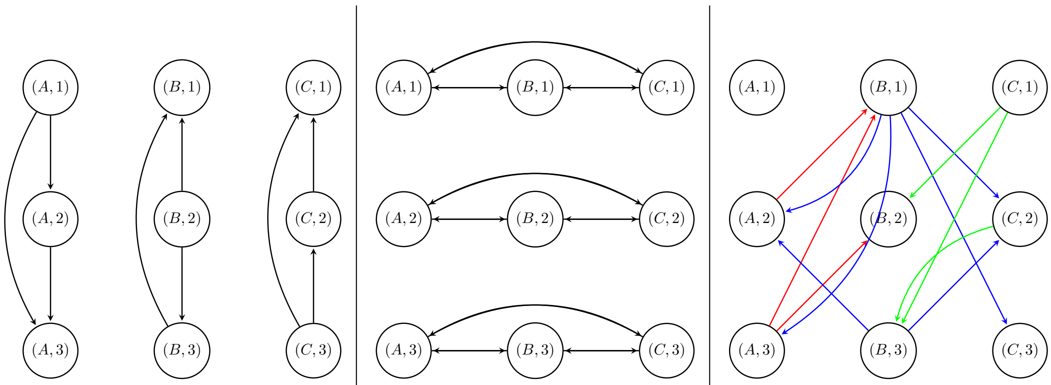
In this section we show how to transform a PRAP into a PAF, such that there is a correspondence between LEF allocations and some sets of arguments, namely conflict-free sets of size $|\mathcal{N}|$, which are guaranteed to be stable extensions as well in our case.

Definition 7. *Given $\mathcal{PRAP} = \langle \mathcal{O}, \mathcal{N}, \succ, \mathcal{G} \rangle$, we define the PAF $\mathcal{P}_{\text{LEF}} = \langle \mathcal{A}, \mathcal{R}, \triangleright \rangle$ with*

- $\mathcal{A} = \{(i, o_j) \mid i \in \mathcal{N}, o_j \in \mathcal{O}\}$ *(one argument $\simeq$ allocation of an object to an agent),*
- $\mathcal{R} = \mathcal{R}_1 \cup \mathcal{R}_2 \cup \mathcal{R}_3$*, with*
  - $\mathcal{R}_1 = \{((i, o_j), (i, o_k)) \mid i \in \mathcal{N}, o_j, o_k \in \mathcal{O}\}$ *(only one object per agent),*
  - $\mathcal{R}_2 = \{((i, o_k), (j, o_k)) \mid i, j \in \mathcal{N}, o_k \in \mathcal{O}\}$ *(only one agent per object),*
  - $\mathcal{R}_3 = \{((i, o_k), (j, o_l)) \mid i, j \in \mathcal{N}, o_k, o_l \in \mathcal{O}, \{i, j\} \in \mathcal{E}$ *and* $o_l \succ_i o_k\}$ *(envy),*
- $\triangleright = \{((i, o_k), (i, o_j)) \mid i \in \mathcal{N}, o_k \succ_i o_j\}$ *(preferences).*

Obviously, any allocation γ corresponds to a set of arguments in $\mathcal{A}$. Observe also that the defeat relation will only remove some of the attacks in $\mathcal{R}_1$, namely, for each pair of arguments $x = (i, o_j)$ and $y = (i, o_k)$, the defeat relation will contain the defeat (x, y) if $o_j \succ_i o_k$, and the defeat (y, x) if $o_k \succ_i o_j$. The other attack relations $\mathcal{R}_2$ and $\mathcal{R}_3$ are not impacted by the preferences, so they are included in the defeat relation of this PAF.

Example 2. *Let us consider again $\mathcal{PRAP}_1$ from Example 1. Figure 2 gives the defeat relation of the corresponding PAF $\mathcal{P}_{\text{LEF}}$. More precisely, Figure 2a gives the combination of $\mathcal{R}_1$ with the preferences, Figure 2b (resp. 2c) shows $\mathcal{R}_2$ (resp. $\mathcal{R}_3$). In Figure 2c, the red arrows correspond to the situations where agent A is envious (for instance, because she has received the object 2 while B has received the object 1), blue arrows are for agent B, and green arrows correspond to agent C.*



(a) $\mathcal{R}_1 + \triangleright$ (b) $\mathcal{R}_2$ (c) $\mathcal{R}_3$

Figure 2: PAF Corresponding to a PRAP

The following lemmas will help us to prove the correspondance between LEF allocations and conflict-free sets (and stable extensions) of size $|\mathcal{N}|$.

Lemma 1. *Given an allocation γ, if $\gamma \in \mathsf{cf}(\mathcal{P}_{\text{LEF}})$ then γ is valid.*

Proof. Assume γ is not valid. If $\exists x, y \in \gamma$ s.t. $\mathtt{Ag}(x) = \mathtt{Ag}(y)$, then $(x, y), (y, x) \in \mathcal{R}_1$, which implies that either (x, y) or (y, x) is in the defeat relation of $\mathcal{P}_{\text{LEF}}$, so $\gamma \notin \mathsf{cf}(\mathcal{F}_{\text{LEF}})$. Similarly, if $\exists x, y \in \gamma$ s.t. $\mathtt{Obj}(x) = \mathtt{Obj}(y)$, then $(x, y) \in \mathcal{R}_2$, which implies $\gamma \notin \mathsf{cf}(\mathcal{P}_{\text{LEF}})$. $\square$

Lemma 2. *Given a valid allocation γ, if $\gamma \notin \mathsf{cf}(\mathcal{P}_{\text{LEF}})$ then γ is not LEF.*

Proof. Assume a valid allocation γ which is not conflict-free in $\mathcal{P}_{\text{LEF}}$. By construction, since γ is valid, there are no $x, y \in \gamma$ such that $(x, y) \in \mathcal{R}_1$ or $(x, y) \in \mathcal{R}_2$, so we deduce that $\exists x, y \in \gamma$ such that $(x, y) \in \mathcal{R}_3$ which implies that γ is not LEF. □

Proposition 1. *Let γ be an allocation. γ is LEF iff $\gamma \in \mathsf{cf}(\mathcal{P}_{\text{LEF}})$ and $|\gamma| = |\mathcal{N}|$.*

Proof. First assume that γ is a LEF allocation. From Lemma 2 we deduce that γ is conflict-free. By definition, since γ is LEF then γ is total. Hence the first part of the result.

Now assume that $\gamma \in \mathsf{cf}(\mathcal{P}_{\text{LEF}})$ and $|\gamma| = |\mathcal{N}|$. From Lemma 1, we know that γ is valid. Then, since $\gamma \in \mathsf{cf}(\mathcal{P}_{\text{LEF}})$, we can guarantee that there are no $x, y \in \gamma$ such that $(x, y) \in \mathcal{R}_3$. This means that, for any $i \in \mathcal{N}$ such that $(i, o_k) \in \gamma$, there is no $j \in \mathcal{N}$ such that $(j, o_l) \in \gamma$, $\{i, j\} \in \mathcal{E}$ and $o_l \succ_i o_k$. By definition, this means that γ is LEF. □

Proposition 1 implies that LEF allocations can be easily computed thanks to a minor modification of a very classical approach for solving argumentation problems. Most of the efficient approaches for reasoning with abstract argumentation frameworks use SAT solvers. For the basic notion of conflict-freeness, it is enough to consider clauses which forbid to accept together arguments which are connected by an attack. We will use a MaxSAT version of this encoding [4], where the clauses usually corresponding to conflict-freeness will be hard clauses, and additional (unit) soft clauses will be added to ensure that the solver will return a maximal solution (in terms of cardinality). Formally,

Definition 8. *Given $\mathcal{PRAP} = \langle \mathcal{O}, \mathcal{N}, \succ, \mathcal{G} \rangle$, and $\mathcal{P}_{\text{LEF}} = \langle \mathcal{A}, \mathcal{R}, \triangleright \rangle$ the corresponding PAF. $\mathcal{D}$ denotes the defeat relation obtained from $\mathcal{R}$ and $\triangleright$. We define the following sets of hard clauses hc and soft clauses sc:*

$$hc = \{\neg x \vee \neg y \mid (x, y) \in \mathcal{D}\} \qquad sc = \{(x, 1) \mid x \in \mathcal{A}\}$$

Given the sets of clauses hc and sc, a MaxSAT solver returns a conflict-free set of $\mathcal{P}_{\text{LEF}}$ of maximal cardinality. If this solution has a cardinality equal to $|\mathcal{N}|$, then it is a LEF allocation. Otherwise, there is no LEF allocation. Another possible approach consists in adding one cardinality constraint [5] $\sum_{x \in \mathcal{A}} x = |\mathcal{N}|$ to the set of hard clauses. In this case, if a LEF allocation exists then it will be provided by a SAT solver, otherwise the solver will answer UNSAT.

Notice that such an allocation also corresponds to a stable extension of cardinality $|\mathcal{N}|$.

Corollary 1. *Let γ be an allocation. γ is LEF iff $\gamma \in \mathsf{stb}(\mathcal{P}_{\text{LEF}})$ and $|\gamma| = |\mathcal{N}|$.*

Proof. One side of the equivalence is obvious: if $\gamma \in \mathsf{stb}(\mathcal{P}_{\text{LEF}})$, then $\gamma \in \mathsf{cf}(\mathcal{P}_{\text{LEF}})$, so under the assumption that $|\gamma| = |\mathcal{N}|$, the result follows Proposition 1.

Now, assume that γ is a LEF allocation. From Proposition 1, we know that $\gamma \in \mathsf{cf}(\mathcal{P}_{\text{LEF}})$ and $|\gamma| = |\mathcal{N}|$. For a given object o_k, there is an argument $a = (i, o_k) \in \gamma$, *i.e.* the object o_k has been assigned to agent i. By definition of $\mathcal{R}_2$, a defeats all the arguments of the form (j, o_k) for $j \neq i$. Since this is true for all the objects, any argument not in γ is defeated by some argument in γ, so $\gamma \in \mathsf{stb}(\mathcal{P}_{\text{LEF}})$. □

4. Discussion

Argumentation has already shown its interest for providing explanations to other problems, like *e.g.* scheduling [6] or case-based reasoning [7], so drawing connections between argumentation and resources allocation is a natural question.

The preliminary study of this connection allows us to envision deeper relations between both frameworks. For instance, it seems possible to assign numerical values to assignments (*e.g.* the preferred object o_k of agent i receives the value n, her second preferred object receives $n-1$, etc.) in order to define a *Strength-based Argumentation Framework* (StrAF) [8] where the strength of an argument can intuitively correspond to the utility of allocating the object o_k to the agent i. Then using the semantics of StrAFs could induce interesting allocations. We plan to investigate this connection.

We are also interested in the methods allowing the explanation of arguments status in abstract argumentation (*e.g.* [9, 10]). They could allow to simply explain why the allocation of a specific object to an agent is necessary (or impossible). Also, the approach proposed by [11, 12] could provide interesting means to reduce the size of the argumentation graph, hence providing a better visual explanation of the (non-)existence of desirable allocation.

Another interesting way to go further in the study of argumentation applied to resources allocation consists in using the conflict-tolerant semantics of *Weighted Argumentation Frameworks* (WAFs) [13] in order to obtain optimal (non-LEF) allocations for instances which do not admit any LEF allocation.

These few ideas are only a small part of the possible connections between resources allocation and computational argumentation, and pave the way to a rich body of work that will allow to provide explainable solutions to fairness issues in resources allocation problem.

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