Causes for Changing Profiles (Preliminary Report)*

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Abstract

User profiles are an essential Knowledge Representation tool in several areas of information technology. In a recent paper, Fermé et al. presented a formal framework for representing user profiles and profile revision operators defined through a Knowledge-Driven perspective. In this paper, we analyse the possibility of going from one given user profile to another by means of a profile revision operator. More precisely, given two profiles P and Q we present some conditions which ensure that there is a profile revision operator \odot on P and a sentence α such that $P \odot \alpha = Q$. Furthermore, considering a fixed operator \odot , we characterize the *change formulas* α which are such that $P \odot \alpha = Q$, by identifying upper and lower bounds for their sets of models. Analogous results are obtained for the case of a "system of equations" $P_i \odot_i \alpha = Q_i$ for every $i \in \{1, \dots, m\}$. Furthermore, a similar study is carried out considering profile revision operators defined on sets of profiles (which take sets of profiles to sets profiles rather than a single profile to a single profile).

1. Introduction

The study of user profiles and their dynamics over time has gained increasing attention in the field of information technology. User profiles are widely used in various areas, including recommendation systems [1, 2], adaptable user interfaces [3], personalized systems [4], and cognitive or physical rehabilitation systems [5]. In [6], a formalization of the creation, representation, and dynamics of profiles from a Knowledge-Driven perspective is proposed. The paper introduces a formal profile representation structure, using a formal language that enables clear representation of user profiles and their attributes, as well as the properties of different types of profile operations. The paper also presents several dynamic operators for profiles, including two profile revision operators, which are described below:

- (a) From one single profile to one single profile: in this case, the model represents the evolution of a single profile. This evolution is caused by an external stimulus, for example an interaction with a system, a training program, etc.
- (b) From a set of profiles to a set of profiles: these operators model the changes produced by an input in a collection of profiles and return the new profiles resulting from this process. This type of operators is particularly useful in two contexts: (i) to capture the change produced by a single event in a

population (set of profiles) (ii) to analyze changes when a single individual is represented by a set of possible profiles due to lack of information. For example, if we only know that a person's age is between 18 and 20 years old, we can represent that person using a set of three profiles: one with age 18, one with age 19, and one with age 20.

In this paper, we expand the work presented in [6] by investigating the following aspects related to the operators discussed above: (i) the possibility of always transitioning from one given user profile to another; (ii) the bounds of the models of a change formula (in terms of set inclusion) that causes a change from a given set of user profiles into another; (iii) the existence of a change formula that is solution of a given "system of equations" of the form $\Gamma_i \odot_i \mu = \Phi_i$, where Γ_i and Φ_i are the initial and final sets of profiles of an agent, and μ is the change formula, and, if such a solution exists, we determine the bounds for the set of models of that solution. Solving a system of this type could be valuable, for instance, in scenarios where one aims to train a group of agents (each represented by a profile or a set of profiles) in order to alter them towards achieving a specific objective.

The rest of the paper is organized as follows: In Section 2, we introduce the notations and recall the main background concepts that will be needed throughout this article. In Section 3 we conduct the study mentioned above. In Section 4 we briefly mention other works related to the one presented in this paper. In Section 5 we summarize the main results obtained and point out some topics for future research.

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2. Background

In this section, we present some concepts and notations that will be used throughout this article.

2.1. Formal Preliminaries

We assume that the empty disjunction is a contradiction. We shall use the symbol \perp to represent a contradiction. Given a set S, we will denote by $\mathcal{P}(S)$ the power set of S, i.e. the set of all subsets of S. Given a set A a binary relation \leq on A is:

- reflexive if and only if $\alpha \prec \alpha$ for all $\alpha \in A$;

- transitive if and only if it holds that if $\alpha \preceq \beta$ and $\beta \preceq \delta$, then $\alpha \preceq \delta$, for all $\alpha, \beta, \delta \in A$;

- antisymmetric if and only if it holds that if $\alpha \preceq \beta$ and $\beta \preceq \alpha$, then $\alpha = \beta$, for all $\alpha, \beta \in A$.

- total if and only if it holds that either $\alpha \preceq \beta$ or $\beta \preceq \alpha$ for all $\alpha, \beta \in A$.

A non-strict pre-order is a reflexive and transitive relation. A non-strict order is a non-strict pre-order which is also antisymmetric. Given a non-strict pre-order \leq on a set A, the associated strict part \prec is defined by $\alpha \prec \beta$ if and only if $\alpha \preceq \beta$ and $\beta \not\preceq \alpha$, for all $\alpha, \beta \in A$. $\alpha \simeq \beta$ will be used to denote that $\alpha \preceq \beta$ and $\beta \preceq \alpha$.

Given a set A a strict order \prec on A is an irreflexive (i.e, $\alpha \not\prec \alpha$, for all $\alpha \in A$) and transitive relation on A^{\perp} A strict order is total if and only if it holds that either $\alpha \prec \beta, \beta \prec \alpha$ or $\alpha = \beta$ for all $\alpha, \beta \in A$. Throughout the text, whenever we refer to pre-orders (respectively, orders) without mentioning whether or not they are strict, we will be referring to non-strict pre-orders (respectively, non-strict orders).

Let A be a set and Γ be a finite subset of A. Given a total strict order \prec on A, the minimum of Γ with respect to \prec is denoted by $min(\Gamma, \prec)$ and is defined as follows:

$$P = min(\Gamma, \prec) \text{ iff}$$

$$P \in \Gamma \text{ and } P \prec Q \text{ for all } Q \in \Gamma \setminus \{P\}.$$

Given a pre-order \leq on A, the set of minimal elements of Γ with respect to \preceq is denoted by $Min(\Gamma, \preceq)$ and is defined as follows:

 $Min(\Gamma, \prec) = \{P \in \Gamma : Q \not\prec P, \text{ for all } Q \in \Gamma\}.$

We note that if \leq is a total pre-order, then

$$Min(\Gamma, \preceq) = \{ P \in \Gamma : P \preceq Q, \text{ for all } Q \in \Gamma \}.$$

2.2. Profiles Definition

In this subsection, we present the formal definition of a profile and introduce appropriate language and semantics for formalizing the dynamics of profiles.

Definition 1. [6] Let $\mathbb{L} = \ll L_1, L_2, ..., L_n \gg be a tuple$ of labels. For each $i \in \{1, ..., n\}$ let D_i be a finite set associated with label L_i , that we will designate by the domain of L_i . A profile, associated with \mathbb{L} , denoted by $P_{\mathbb{L}}$ (or simply by P if the tuple of labels is clear from the context), is an element of $D_1 \times D_2 \times ... \times D_n$. The set of all profiles associated with \mathbb{L} will be denoted by $\mathbb{P}_{\mathbb{L}}$ (or simply by \mathbb{P} if the tuple of labels is clear from the context).2

Example 1. Given the tuple of labels $\mathbb{L} = \ll age$, gender, civil status, nationality \gg ; D_{age} $\{0, 1, 2, ..., 150\}, D_{gender}$ = {male, female, other}, $D_{civil \, status} =$ {single, married, divorced, widowed, other}, $D_{nationality} = \{English,$ Portuguese, Argentinian}, the following are examples of profiles: $\langle 25, male, single, English \rangle$, $\langle 45, female, married, Portuguese \rangle$.

To express properties of a profile (and therefore the possibility of changing it), we need to define a formal language:

Definition 2. [6] Given a tuple of labels $\mathbb{L} = \ll$ $L_1, L_2, ..., L_n \gg$, for each $i \in \{1, ..., n\}$, let D_i be the domain associated with the label L_i . The alphabet of symbols of the language $\mathcal{L}_{\mathbb{L}}$ (or simply \mathcal{L}) associated with \mathbb{L} that we will consider is:

- 1. $L_1, L_2, ..., L_n$ (labels);
- 2. = (symbol of equality)
- 3. (,) (punctuation symbols);
- 4. a, b, ... (elements of $\bigcup_{i=1}^{n} D_i$); 5. \perp (symbol of contradiction);
- 6. \neg , \land , \lor , \rightarrow , \leftrightarrow (symbols of connectives).

Definition 3. [6] Let $\mathbb{L} = \ll L_1, L_2, ..., L_n \gg be a tuple$ of labels. For each $i \in \{1, ..., n\}$, let D_i be the domain associated with the label L_i .

An atomic formula in $\mathcal{L}_{\mathbb{L}}$ is defined by: if L_i is a label occurring in \mathbb{L} and $a \in D_i$, then $L_i = a$ is an atomic *formula of* $\mathcal{L}_{\mathbb{L}}$ *.*

A well-formed formula (wff) of $\mathcal{L}_{\mathbb{L}}$ is defined by:

- 1. Every atomic formula of $\mathcal{L}_{\mathbb{L}}$ is a wff of $\mathcal{L}_{\mathbb{L}}$.
- 2. If A and B are wffs of $\mathcal{L}_{\mathbb{L}}$, so are $(\neg A)$, $(A \land B)$, $(A \lor B)$, $(A \to B)$ and $(A \leftrightarrow B)$.

In the following definition we recall from [6] the semantics for profile dynamics.

Definition 4. [6] Let $\mathbb{L} = \ll L_1, L_2, ..., L_k \gg be a$ set of labels. For each $i \in \{1, ..., k\}$, let D_i be the domain associated with the label L_i . A profile P = $\langle P_1, P_2, ..., P_k \rangle$ is said to satisfy a formula α , denoted by $P \models \alpha$, if it can be shown inductively to do so under the following conditions:

¹Every irreflexive and transitive relation on a set A is also antisymmetric.

²Note that $\mathbb{P}_{\mathbb{L}}$ is finite.

1. $P \models (L_i = a) \text{ iff } P_i = a;$ 2. $P \models (\neg \beta) \text{ iff } P \nvDash \beta;$ 3. $P \models (\beta \land \delta) \text{ iff } P \models \beta \text{ and } P \models \delta;$ 4. $P \models (\beta \lor \delta) \text{ iff } P \models \beta \text{ or } P \models \delta;$ 5. $P \models (\beta \to \delta) \text{ iff } P \nvDash \beta \text{ or } P \models \delta;$ 6. $P \models (\beta \leftrightarrow \delta) \text{ iff } (P \models \beta \text{ iff } P \models \delta).$

We say that P is a model of α if and only if $P \models \alpha$. The set of models of α is denoted by $\|\alpha\|$. It holds that $\|\perp\| = \emptyset$. A set of profiles Γ is said to satisfy α if and only if every profile in Γ is a model of α . We say that α is a tautology if and only if $\|\alpha\| = \mathbb{P}_{\mathbb{L}}$. We will use $\models \alpha$ to denote that α is a tautology.

The following definition introduces the notion of Γ -faithful pre-order.

Definition 5. [6] Let \mathbb{L} be a tuple of labels and Γ be a non-empty subset of $\mathbb{P}_{\mathbb{L}}$. A binary relation \preceq_{Γ} on $\mathbb{P}_{\mathbb{L}}$ is Γ -faithful if it satisfies:

- 1. If $P_i \in \Gamma$ and $P_j \in \Gamma$, then $P_i \prec_{\Gamma} P_j$ does not hold.
- 2. If $P_i \in \Gamma$ and $P_j \in \mathbb{P}_{\mathbb{L}} \setminus \Gamma$, then $P_i \prec_{\Gamma} P_j$.

If Γ is a singleton, say $\{P\}$, then we will omit the braces in the subscript of the binary relation mentioned in the above definition by writing \preceq_P instead of $\preceq_{\{P\}}$. We will also write P-faithful instead of $\{P\}$ -faithful. Note that if $\Gamma = \{P\}$, and \prec_P is a strict order on $\mathbb{P}_{\mathbb{L}}$, then the first condition of Definition 5 follows trivially, since \prec_P is irreflexive, and the second condition can be rewritten as $P \prec_P P_i$ for all $P_i \in \mathbb{P}_{\mathbb{L}} \setminus \{P\}$.

2.2.1. Model 1. From One Profile to One Profile

In this subsection we present the first model for profile dynamics. In this model, we revise a profile by a formula of the language obtaining as output a profile.

Definition 6. [6] Let $\mathbb{L} = \ll L_1, L_2, ..., L_n \gg$ be a tuple of labels and let P be a profile associated with \mathbb{L} . Let \prec_P be a P-faithful total strict order on $\mathbb{P}_{\mathbb{L}}$. The PtoP profile revision induced by \prec_P is the operator \odot_{\prec_P} such that for all sentences α :

$$P \odot_{\prec_{P}} \alpha = \begin{cases} \min(\|\alpha\|, \prec_{P}) & \text{if } \|\alpha\| \neq \emptyset \\ P & \text{otherwise} \end{cases}$$

An operator \odot is a PtoP profile revision on P if and only if there is a P-faithful total strict order on $\mathbb{P}_{\mathbb{L}}$, \prec_P , such that for all sentences α

$$P \odot \alpha = P \odot_{\prec_P} \alpha.$$

Example 2. Let $\mathbb{L} = \ll L_1, L_2 \gg and D_1 = \{a, b\}$ and $D_2 = \{1, 2, 3\}$ be the domains of, respectively, L_1 and L_2 . There are six profiles associated with \mathbb{L} . Let P be the profile $\langle a, 1 \rangle$. Consider the P-faithful total strict order \prec_P on $\mathbb{P}_{\mathbb{L}}$ defined as follows:

 $\langle a,1 \rangle \prec_P \langle b,1 \rangle \prec_P \langle a,2 \rangle \prec_P \langle b,2 \rangle \prec_P \langle a,3 \rangle \prec_P \langle b,3 \rangle.$

Let α be the sentence $L_1 = b$ and β be the sentence $L_2 = 2$. It holds that $P \odot_{\prec_P} \alpha = \langle b, 1 \rangle$ and $P \odot_{\prec_P} \beta = \langle a, 2 \rangle$.

2.2.2. Model 2. From a Set of Profiles to a Set of Profiles

We now present a model which addresses the problem of revising a set of profiles. A non-singular set of profiles of a given agent provides only partial information about that agent. For example, the following set of profiles $\{\langle 25, male, single \rangle, \langle 25, male, married \rangle\}$ expresses that the agent is a 25 years old man but we do not know if he is single or married.

Definition 7. [6] Let $\mathbb{L} = \ll L_1, L_2, ..., L_n \gg$ be a tuple of labels and let Γ be a non-empty subset of $\mathbb{P}_{\mathbb{L}}$. Let \preceq_{Γ} be a Γ -faithful binary relation on $\mathbb{P}_{\mathbb{L}}$. The \preceq_{Γ} -based SPtoSP profile revision on Γ is the operator $\odot_{\preceq_{\Gamma}} : \mathcal{L}_{\mathbb{L}} \to \mathcal{P}(\mathbb{P}_{\mathbb{L}})$ such that for all sentences α :

$$\Gamma \odot_{\preceq_{\Gamma}} \alpha = \begin{cases} Min(\|\alpha\|, \preceq_{\Gamma}) & \text{if } \|\alpha\| \neq \emptyset \\ \Gamma & \text{otherwise} \end{cases}$$

An operator \odot is a SPtoSP profile revision on Γ if and only if there exists a Γ -faithful pre-order \preceq_{Γ} on $\mathbb{P}_{\mathbb{L}}$, such that, for all sentences α , $\Gamma \odot \alpha = \Gamma \odot_{\preceq_{\Gamma}} \alpha$.

3. Profiles Dynamics

In this section we present definitions and axiomatic characterizations of some operators of profile revision. Given a profile P (respectively, a set of profiles Γ) we shall designate by *profile revision operator* (or simply by *profile revision*) on P (respectively, on Γ) a function \odot that to each sentence α associates a profile (respectively, a set of profiles), which we shall denote by $P \odot \alpha$ (respectively $\Gamma \odot \alpha$). Then, we find answers to the following questions regarding those operators:

(i) Does it hold that, for any two user profiles P_1 and P_2 (respectively, sets of profiles Γ_1 and Γ_2), there exists a profile revision operator \odot and a formula α such that $P_1 \odot \alpha = P_2$ (respectively, $\Gamma_1 \odot \alpha = \Gamma_2$)?

(ii) When P_1 and P_2 (respectively, Γ_1 and Γ_2) are such that the answer to question (i) above is positive for a certain (fixed) operator \odot , how can we obtain a *change formula* α which confirms it?

(iii) Given profiles $P_1, \ldots, P_m, Q_1, \ldots, Q_m$ (respectively, sets of profiles $\Gamma_1, \ldots, \Gamma_m, \Phi_1, \ldots, \Phi_m$), are there profile revision operators \odot_1, \ldots, \odot_m and a sentence α such that for each $i \in \{1, \ldots, m\}$, it holds that $P_i \odot_i \alpha = Q_i$ (respectively, $\Gamma_i \odot_i \alpha = \Phi_i$)?

(iv) When $P_1, \ldots, P_m, Q_1, \ldots, Q_m$ (respectively, $\Gamma_1, \ldots, \Gamma_m, \Phi_1, \ldots, \Phi_m$) are such that the answer to question (iii) above is positive for a certain (fixed) operators \odot_1, \ldots, \odot_m , how can we obtain a *change formula* α which confirms it?

3.1. Model 1. From One Profile to One Profile

The following postulates, which are based on the modified version of the AGM revision postulates and the update postulates proposed by Katsuno and Mendelzon [7, 8], will be useful towards the axiomatic characterization of the PtoP profile revision operators.

(P1) If $||\alpha|| \neq \emptyset$, then $P \odot \alpha \models \alpha$. (P2) If $||\alpha|| = \emptyset$, then $P \odot \alpha = P$. (P3) If $P \models \alpha$, then $P \odot \alpha = P$. (P4) If $||\alpha|| = ||\beta||$, then $P \odot \alpha = P \odot \beta$. (P5) If $P \odot \alpha \models \beta$ and $P \odot \beta \models \alpha$, then $P \odot \alpha = P \odot \beta$.

(P6)
$$P \odot (\alpha \lor \beta) = P \odot \alpha \text{ or } P \odot (\alpha \lor \beta) = P \odot \beta.$$

Postulate **P1** states that any consistent new information will be satisfied by the outcome. **P2** asserts that if the new information is contradictory, then no change is performed. **P3** states that if the input is already inferred from the starting profile, then no change occurs. **P4** expresses the principle of irrelevance of the syntax. Postulate **P5** states that if the outcome of revising a profile *P* by α implies β and the outcome of revising that profile by β implies α , then the outcomes of those two revisions are identical. This condition appears as (U6) in KM-update [8], as condition (C7) in [9] in a belief revision context and as a conditional logic axiom (CSO) in [10]. Finally, **P6** states that in the case of a revision by a disjunction, one of the disjoints will be preferred in the outcome.

We now present an axiomatic characterization for PtoP profile revision operators.

Observation 1. [6] Let $\mathbb{L} = \ll L_1, L_2, ..., L_n \gg be a$ tuple of labels and let P be a profile associated with \mathbb{L} . \odot is a PtoP profile revision operator on P if and only if \odot satisfies the postulates (P1) to (P4) and (P6).

The following observation establishes necessary and sufficient conditions that a change formula α must satisfy to obtain P as output of $P \odot \alpha$ where \odot is an operator, on P, that satisfies some of the postulates that characterize PtoP revision operators.

Observation 2. Let $\mathbb{L} = \ll L_1, L_2, ..., L_n \gg$ be a tuple of labels and let $P \in \mathbb{P}_{\mathbb{L}}$. Let $\odot : \mathcal{L}_{\mathbb{L}} \to \mathbb{P}_{\mathbb{L}}$ be a profile revision operator on P that satisfies (P1), (P2) and (P3). It holds that $P \odot \alpha = P$ if and only if $||\alpha|| = \emptyset$ or $P \in ||\alpha||$.

Proof. Assume that it holds that $||\alpha|| \neq \emptyset$. Hence by (P1) it follows that $P \in ||\alpha||$. The other direction follows by (P2) and (P3).

The following observation establishes a necessary condition, regarding the models of α , to ensure that $P \odot \alpha = Q$, where $P \neq Q$ and \odot is an operator, on P, that satisfies (P1), (P2) and (P3).

Observation 3. Let $\mathbb{L} = \ll L_1, L_2, ..., L_n \gg$ be a tuple of labels and let $P, Q \in \mathbb{P}_{\mathbb{L}}$ be distinct profiles. Let $\odot : \mathcal{L}_{\mathbb{L}} \to \mathbb{P}_{\mathbb{L}}$ be a profile revision operator on P that satisfies (P1), (P2) and (P3). If $P \odot \alpha = Q$, then $Q \in$ $\|\alpha\| \subseteq \mathbb{P}_{\mathbb{L}} \setminus \{P\}.$

Proof. If $\|\alpha\| = \emptyset$, then by (P2) it follows that P = Q. Contradiction. Hence $\|\alpha\| \neq \emptyset$. By (P1) it follows that $Q \in \|\alpha\|$. Let $P_i \in \|\alpha\|$. If $P_i = P$, then by (P3) it follows that Q = P. Contradiction. Hence $P_i \neq P$. Thus $\|\alpha\| \subseteq \mathbb{P}_{\mathbb{L}} \setminus \{P\}$.

The following corollary establishes a necessary condition, regarding the models of α , to ensure that α is a solution of the system of equations of the form $P_i \odot_i \mu = Q_i$, where for each $i \in \{1, \ldots, m\}$, P_i and Q_i are distinct profiles and \odot_i is an operator, on P_i , that satisfies (P1), (P2) and (P3).

Corollary 1. Let $\mathbb{L} = \ll L_1, L_2, ..., L_n \gg be a tuple of labels and let <math>\{P_1, ..., P_m\} \cup \{Q_1, ..., Q_m\} \subseteq \mathbb{P}_{\mathbb{L}}$. For each $i \in \{1, ..., m\}$ let $\odot_i : \mathcal{L}_{\mathbb{L}} \to \mathbb{P}_{\mathbb{L}}$ be profile revision operators on P_i that satisfy (P1), (P2) and (P3). If for all $i \in \{1, ..., m\}$ it holds that $P_i \odot_i \alpha = Q_i$ and $P_i \neq Q_i$, then $\{Q_1, ..., Q_m\} \subseteq ||\alpha|| \subseteq \mathbb{P}_{\mathbb{L}} \setminus \{P_1, ..., P_m\}$.

The following observation establishes lower and upper bound (in terms of set inclusion) for the models of α in order to assure the existence of a PtoP profile revision operator \odot such that $P \odot \alpha = Q$.

Observation 4. Let $\mathbb{L} = \ll L_1, L_2, ..., L_n \gg$ be a tuple of labels. Let $P, Q \in \mathbb{P}_{\mathbb{L}}$ be such that it holds that $Q \in$ $\|\alpha\| \subseteq \mathbb{P}_{\mathbb{L}} \setminus \{P\}$. Then there exists a PtoP profile revision operator \odot on P such that $P \odot \alpha = Q$.

Proof. From $Q \in ||\alpha|| \subseteq \mathbb{P}_{\mathbb{L}} \setminus \{P\}$ it follows that $P \neq Q$. Let $\mathbb{P}_{\mathbb{L}} = \{P_1, P_2, \dots, P_m\}$, be such that $P_1 = P$, $P_2 = Q$ and no repeated elements occur in $\mathbb{P}_{\mathbb{L}}$. Let \prec_P be a relation on $\mathbb{P}_{\mathbb{L}}$ such that, for all $i, j \in \{1, \dots, m\}$ it holds that $P_i \prec_P P_j$ iff i < j. Let

$$P \odot_{\prec_P} \alpha = \begin{cases} \min(\|\alpha\|, \prec_P) & \text{if } \|\alpha\| \neq \emptyset \\ P & \text{otherwise} \end{cases}$$

By definition of \prec_P it follows that: (i) \prec_P is total; (ii) \prec_P is irreflexive; (iii) \prec_P is transitive; (iv) \prec_P is *P*-faithfull. Thus \prec_P is a *P*-faithfull total strict order on $\mathbb{P}_{\mathbb{L}}$. Hence \odot_{\prec_P} is a PtoP profile revision induced by \prec_P . It remains to prove that $P \odot_{\prec_P} \alpha = Q$. By hypothesis it holds that $Q \in ||\alpha||$ and $P \notin ||\alpha||$. Thus, $min(||\alpha||, \prec_P) = Q$. Hence $P \odot_{\prec_P} \alpha = Q$.

The following corollary presents a result similar to that illustrated in Observation 4, but where a system of equations is considered (rather than a single equation).

Corollary 2. Let $\mathbb{L} = \ll L_1, L_2, ..., L_n \gg$ be a tuple of labels and let $\{P_1, ..., P_m\} \subseteq \mathbb{P}_{\mathbb{L}}$. If $\{Q_1, ..., Q_m\} \subseteq \|\alpha\| \subseteq \mathbb{P}_{\mathbb{L}} \setminus \{P_1, ..., P_m\}$. Then for all $i \in \{1, ..., m\}$ there exists a PtoP profile revision operator \odot_i on P_i such that $P_i \odot_i \alpha = Q_i$.

The following observation establishes bounds of the set of models of the change formula when the PtoP revision operator is known.

Observation 5. Let $\mathbb{L} = \ll L_1, L_2, ..., L_n \gg$ be a tuple of labels. Let $P, Q \in \mathbb{P}_{\mathbb{L}}$ be such that $P \neq Q$. Let \odot_{\prec_P} be a PtoP profile revision operator on P. It holds that:

$$P \odot_{\prec_P} \alpha = Q \text{ iff } Q \in ||\alpha|| \subseteq \Omega \cup \{Q\}.$$

Where $\Omega = \{ P_i \in \mathbb{P}_{\mathbb{L}} : Q \prec_P P_i \}.$

Proof. Let $\Omega = \{P_i \in \mathbb{P}_{\mathbb{L}} : Q \prec_P P_i\}.$ (\Rightarrow) Assume that $P \odot_{\prec_P} \alpha = Q$. It holds that $P \neq Q$ and \odot_{\prec_P} is a PtoP profile revision operator on P. Thus $\|\alpha\| \neq \emptyset$. Hence $min(\|\alpha\|, \prec_P) = Q$. Thus $Q \in \|\alpha\|.$ Let $P_i \in \|\alpha\|$. We will consider two cases: Case 1) $P_i = Q$. Thus $P_i \in \Omega \cup \{Q\}.$ Case 2) $P_i \neq Q$. Thus $P_i \in \|\alpha\|$ and $P_i \neq Q = min(\|\alpha\|, \prec_P)$. Thus $Q \prec_P P_i$. Hence $P_i \in \Omega$.

In both cases it holds that $\|\alpha\| \subseteq \Omega \cup \{Q\}$.

(⇐) Let $Q \in ||\alpha|| \subseteq \Omega \cup \{Q\}$. Hence $||\alpha|| \neq \emptyset$. We will consider two cases:

Case 1) $\|\alpha\| = \{Q\}$, then by (P1) it follows that $P \odot_{\prec_P} \alpha = Q$.

Case 2) $\|\alpha\| \neq \{Q\}$. Hence $\{Q\} \subset \|\alpha\|$. It holds that $\|\alpha\| \subseteq \Omega \cup \{Q\}$. Let $P_i \in \|\alpha\| \setminus \{Q\}$. Hence $P_i \in \Omega$. Hence $Q \prec_P P_i$. Thus $Q = min(\|\alpha\|, \prec_P)$. Therefore $P \odot_{\prec_P} \alpha = Q$.

The following corollary presents a result similar to that illustrated in Observation 5, but where a system of equations is considered.

Corollary 3. Let $\mathbb{L} = \ll L_1, L_2, ..., L_n \gg$ be a tuple of labels and let $\{P_1, ..., P_m\} \cup \{Q_1, ..., Q_m\} \subseteq \mathbb{P}_{\mathbb{L}}$ be such that $P_i \neq Q_i$ for all $i \in \{1, ..., m\}$.

For each $i \in \{1, ..., m\}$ let $\odot_{\prec_{P_i}}$ be a PtoP profile revision operator on P_i . It holds that,

$$\forall i \in \{1, \dots, m\} P_i \odot_{\prec_{P_i}} \alpha = Q_i$$

iff
$$\{Q_1, \dots, Q_m\} \subseteq \|\alpha\| \subseteq \bigcap_{k=1}^m (\Omega_k \cup \{Q_k\}).$$

Where $\Omega_k = \{P_j \in \mathbb{P}_{\mathbb{L}} : Q \prec_{P_k} P_j\}.$

Proof. (\Rightarrow) Follows from Observation 5.

 $\begin{array}{l} (\Leftarrow) \text{ Assume that } \{Q_1,\ldots,Q_m\} \subseteq \|\alpha\| \subseteq \bigcap_{k=1}^m (\Omega_k \cup \{Q_k\}), \text{ where } \Omega_k = \{P_i \in \mathbb{P}_{\mathbb{L}} : Q \prec_{P_k} P_i\}. \text{ Let } i \in \{1,\ldots,m\}. \text{ By hypothesis it yields that } \bigcirc_{\prec P_i} \text{ is a } P \text{ toP profile revision operator on } P_i \text{ and that } P_i \neq Q_i. \\ \text{Furthermore, it yields that } Q_i \in \|\alpha\| \text{ and } \|\alpha\| \subseteq \Omega_i \cup \{Q_i\}, \text{ where } \Omega_i = \{P_l \in \mathbb{P}_{\mathbb{L}} : Q \prec_{P_i} P_l\}. \\ \text{Thus, by Observation 5 it yields that } P_i \odot_{\prec P_i} \alpha = Q_i. \\ \end{array}$

3.2. Model 2. From a Set of Profiles to a Set of Profiles

In this subsection we present a study similar to the one carried in the previous subsection, but concerning profile revision operators on sets of profiles (rather than on single profiles).

The following list of postulates illustrates some of the properties of SPtoSP profile revision operators.

- **(SP1)** If $\|\alpha\| \neq \emptyset$, then $\Gamma \odot \alpha \subseteq \|\alpha\|$.
- **(SP2)** If $||\alpha|| = \emptyset$, then $\Gamma \odot \alpha = \Gamma$.
- (SP3) $\Gamma \odot \alpha \neq \emptyset$.
- (SP4) If $\|\alpha\| = \|\beta\|$, then $\Gamma \odot \alpha = \Gamma \odot \beta$.
- (SP5) If $\Gamma \cap ||\alpha|| \neq \emptyset$ then $\Gamma \odot \alpha = \Gamma \cap ||\alpha||$.
- **(SP6)** $\Gamma \odot (\alpha \lor \beta) = \Gamma \odot \alpha \text{ or } \Gamma \odot (\alpha \lor \beta) = \Gamma \odot \beta$
- or $\Gamma \odot (\alpha \lor \beta) = \Gamma \odot \alpha \cup \Gamma \odot \beta$.
- **(SP7)** $\Gamma \odot (\alpha \lor \beta) \subseteq \Gamma \odot \alpha \cup \Gamma \odot \beta.$

(SP8) If $\Gamma \odot \alpha \models \beta$ and $\Gamma \odot \beta \models \alpha$, then $\Gamma \odot \alpha = \Gamma \odot \beta$.

(SP9) $\Gamma \odot \alpha \cap \Gamma \odot \beta \subseteq \Gamma \odot (\alpha \lor \beta).$

(SP1), (SP2), (SP4) and (SP8) are the adapted versions of (P1), (P2), (P4) and (P5) respectively. (SP3) states that the outcome is always consistent. (SP5) corresponds to the AGM revision postulate vacuity and states that if there are profiles in Γ that satisfy the input sentence α , then the output of the revision of Γ by α is the set formed by those profiles. The intuition behind (SP6) is that if we wish to revise by a disjunction and there is some preference between the disjuncts, then this revision is equivalent to revising by the preferred disjunct. In the case of indifference, revising by the disjunction returns the set of profiles consisting of the union of the revision by each of the disjuncts. (SP7) states that if the revision of a set of profiles Γ by $\alpha \lor \beta$ leads to the acceptance of a profile P_k , then the same should happen when revising Γ by α or by β . (SP9) states that if the revision of a set of profiles Γ by α leads to the acceptance of a profile P_k and the revision of Γ by β also leads to the acceptance of P_k , then the same should happen when revising Γ by $\alpha \lor \beta$.

We now recall two representation theorems:

Observation 6. [6] Let $\mathbb{L} = \ll L_1, L_2, ..., L_n \gg be a$ tuple of labels, Γ be a non-empty subset of $\mathbb{P}_{\mathbb{L}}$ and \odot be a profile revision operator on Γ . The following conditions are equivalent:

- 1. \odot satisfies the postulates (SP1) to (SP5) and (SP7) to (SP9).
- 2. There exists a Γ -faithful pre-order \preceq_{Γ} on $\mathbb{P}_{\mathbb{L}}$ such that \odot is the \preceq_{Γ} -based SPtoSP profile revision on Γ .

Observation 7. [6] Let $\mathbb{L} = \ll L_1, L_2, ..., L_n \gg be a$ tuple of labels, $P \in \mathbb{P}_{\mathbb{L}}$ and $\odot : \mathcal{L}_{\mathbb{L}} \to \mathcal{P}(\mathbb{P}_{\mathbb{L}})$ be a profile revision operator on P. The following conditions are equivalent:

- 1. \odot satisfies the postulates (SP1) to (SP6).
- 2. There exists a Γ -faithful total pre-order \preceq_{Γ} on $\mathbb{P}_{\mathbb{L}}$ such that \odot is the \preceq_{Γ} -based SPtoSP profile revision on P.

The following observation states that when revising a set of user profiles Γ by a formula α , through an operator that satisfies some of the postulates introduced above, the set of user profiles stays unchanged iff α has no models or contains all the profiles in Γ .

Observation 8. Let $\mathbb{L} = \ll L_1, L_2, ..., L_n \gg$ be a tuple of labels and let Γ be a non-empty subset of $\mathbb{P}_{\mathbb{L}}$. Let $\odot : \mathcal{L}_{\mathbb{L}} \to \mathcal{P}(\mathbb{P}_{\mathbb{L}})$ be a profile revision operator on Γ that satisfies (SP1), (SP2) and (SP5). It holds that $\Gamma \odot \alpha = \Gamma$ if and only if $\|\alpha\| = \emptyset$ or $\Gamma \subseteq \|\alpha\|$.

Proof. (\Leftarrow) If $||\alpha|| = \emptyset$, then by (SP2) it follows that $\Gamma \odot \alpha = \Gamma$. Assume now that $||\alpha|| \neq \emptyset$ and $\Gamma \subseteq ||\alpha||$. Then, $\Gamma \cap ||\alpha|| = \Gamma \neq \emptyset$. Thus, by (SP5), it follows that $\Gamma \odot \alpha = \Gamma$.

(⇒) Assume that $\Gamma \odot \alpha = \Gamma$. Consider that $||\alpha|| \neq \emptyset$. Hence, by (SP1), it follows that $\Gamma \odot \alpha \subseteq ||\alpha||$. Thus $\Gamma \subseteq ||\alpha||$.

Given a set of profiles Γ , the following observation illustrates the properties that a change formula α must satisfy in order for it to hold that $\Gamma \odot \alpha = \Phi$, for a given set of profiles $\Phi \subset \Gamma$.

Observation 9. Let $\mathbb{L} = \ll L_1, L_2, ..., L_n \gg$ be a tuple of labels and let Γ be a non-empty subset of $\mathbb{P}_{\mathbb{L}}$, and $\Phi \subset \Gamma$. Let $\odot : \mathcal{L}_{\mathbb{L}} \to \mathcal{P}(\mathbb{P}_{\mathbb{L}})$ be a profile revision operator on Γ that satisfies (SP1), (SP2), (SP3) and (SP5). It holds that $\Gamma \odot \alpha = \Phi$ if and only if $\Gamma \cap ||\alpha|| = \Phi$ and $\Phi \neq \emptyset$.

Proof. (\Rightarrow) Assume that $\Gamma \odot \alpha = \Phi$. By (SP3) it holds that $\Phi \neq \emptyset$. By hypothesis it holds that $\Phi \neq \Gamma$. Thus, by (SP2), it follows that $||\alpha|| \neq \emptyset$. Hence by (SP1) it follows that $\Phi \subseteq ||\alpha||$. Thus $\Phi \cap ||\alpha|| \neq \emptyset$. Hence $\Gamma \cap ||\alpha|| \neq \emptyset$. From which it follows by (SP5) that $\Phi = \Gamma \cap ||\alpha||$. (\Leftarrow) Assume that $\Gamma \cap ||\alpha|| = \Phi$ and $\Phi \neq \emptyset$. Hence by (SP5) $\Gamma \odot \alpha = \Phi$

The following observation states the bounds (in terms of set inclusion) of the models of a change formula α in order to obtain a given set of profiles Φ as output of a *SPtoSP* revision of a set profiles Γ by α .

Observation 10. Let $\mathbb{L} = \ll L_1, L_2, ..., L_n \gg be a$ tuple of labels. Let Γ be a non-empty subset of $\mathbb{P}_{\mathbb{L}}$. Let $\odot : \mathcal{L}_{\mathbb{L}} \to \mathcal{P}(\mathbb{P}_{\mathbb{L}})$ be a profile revision operator on Γ that satisfies (SP1), (SP2) and (SP5). If $\Gamma \odot \alpha = \Phi$ and $\Phi \not\subseteq \Gamma$, then $\Phi \subseteq ||\alpha|| \subseteq \mathbb{P}_{\mathbb{L}} \setminus \Gamma$.

Proof. Assume that \odot satisfies (SP1), (SP2) and (SP5) and $\Gamma \odot \alpha = \Phi$ and $\Phi \not\subseteq \Gamma$. By (SP2) it follows that $\|\alpha\| \neq \emptyset$. Thus by (SP1) it yields that $\Phi \subseteq \|\alpha\|$. On the other hand, it holds that $\Gamma \odot \alpha \neq \Gamma \cap \|\alpha\|$. Thus, by (SP5), $\Gamma \cap \|\alpha\| = \emptyset$. Therefore, $\Phi \subseteq \|\alpha\| \subseteq \mathbb{P}_{\mathbb{L}} \setminus \Gamma$. \Box

It follows from the above observation that, if its preconditions are held, then $\|\alpha\|$ is unique whenever $\Phi = \mathbb{P}_{\mathbb{L}} \setminus \Gamma$. The following example illustrates the usefulness of the result of the above observation.

Example 3. Let $\mathbb{P}_{\mathbb{L}} = \{P_1, P_2, P_3, P_4, P_5\}$, $\Gamma = \{P_2, P_3\}$, $\Phi_1 = \{P_1, P_4\}$ and $\Phi_2 = \{P_1, P_2\}$. Hence $\mathbb{P}_{\mathbb{L}} \setminus \Gamma = \{P_1, P_4, P_5\}$. It holds that $\Phi_1 \not\subseteq \Gamma$ and $\Phi_2 \not\subseteq \Gamma$.

If $\Gamma \odot \alpha = \Phi_1$, then $\{P_1, P_4\} \subseteq ||\alpha|| \subseteq \{P_1, P_4, P_5\}$. Thus either $||\alpha|| = \{P_1, P_4\}$ or $||\alpha|| = \{P_1, P_4, P_5\}$. There is no sentence α such that $\Gamma \odot \alpha = \Phi_2$ since, although $\Phi_2 \not\subseteq \Gamma$, it cannot hold that $\{P_1, P_2\} \subseteq ||\alpha|| \subseteq \{P_1, P_4, P_5\}$.

The following three corollaries follow from Observation 10.

Corollary 4 states that if Γ and Φ are two nonempty sets of profiles such that Φ is not contained in Γ , and Φ is obtained from Γ when revising it by α through a SPtoSP profile revision operator, then Φ and Γ are disjoint sets of profiles.

Corollary 4. Let $\mathbb{L} = \ll L_1, L_2, ..., L_n \gg$ be a tuple of labels. Let Γ and Φ be non-empty subsets of $\mathbb{P}_{\mathbb{L}}$ such that $\Phi \not\subseteq \Gamma$. Let $\odot : \mathcal{L}_{\mathbb{L}} \to \mathcal{P}(\mathbb{P}_{\mathbb{L}})$ be a profile revision operator on Γ that satisfies (SP1), (SP2) and (SP5). If $\Gamma \odot \alpha = \Phi$, then $\Phi \cap \Gamma = \emptyset$.

The following corollary establishes upper and lower bounds for the set of models of formulas that are solutions of a "system of equations" of the form $\Gamma_i \odot_i \mu = \Phi_i$, where for each $i, \Phi_i \not\subseteq \Gamma_i \neq \emptyset$ and \odot_i satisfies (SP1), (SP2) and (SP5).

Corollary 5. Let $\mathbb{L} = \ll L_1, L_2, ..., L_n \gg$ be a tuple of labels. For each $i \in \{1, ..., m\}$ let Γ_i be a non-empty subset of $\mathbb{P}_{\mathbb{L}}$ and $\odot_i : \mathcal{L}_{\mathbb{L}} \to \mathcal{P}(\mathbb{P}_{\mathbb{L}})$ be a profile revision operator on Γ_i that satisfies (SP1), (SP2) and (SP5). If, for all $i \in \{1, ..., m\}$, it holds that $\Gamma_i \odot_i \alpha = \Phi_i$ and $\Phi_i \not\subseteq \Gamma_i$, then $\bigcup_{i=1}^m \Phi_i \subseteq ||\alpha|| \subseteq \mathbb{P}_{\mathbb{L}} \setminus \bigcup_{i=1}^m \Gamma_i$.

The following corollary presents a situation in which the solution of an equation of the form $\Gamma \odot \mu = \Phi$ (where the unknown of the equation is μ) is unique and a such a solution is explicitly presented.

Corollary 6. Let $\mathbb{L} = \ll L_1, L_2, ..., L_n \gg$ be a tuple of labels. Let Γ and Φ be non-empty subsets of $\mathbb{P}_{\mathbb{L}}$ such that $\Gamma \cup \Phi = \mathbb{P}_{\mathbb{L}}$ and $\Gamma \cap \Phi = \emptyset$. Let $\odot : \mathcal{L}_{\mathbb{L}} \to \mathcal{P}(\mathbb{P}_{\mathbb{L}})$ be a profile revision operator on Γ that satisfies (SP1), (SP2) and (SP5). If $\Gamma \odot \alpha = \Phi$, then $\|\alpha\| = \Phi$.

The following observation presents some conditions which ensure that, given two sets of user profiles Γ and Φ , there exists a SPtoSP revision operator \odot on Γ and a sentence α such that $\Gamma \odot \alpha = \Phi$.

Observation 11. Let $\mathbb{L} = \ll L_1, L_2, ..., L_n \gg$ be a tuple of labels. Let Γ and Φ be non-empty subsets of $\mathbb{P}_{\mathbb{L}}$ such that either $\Gamma \cap \Phi = \emptyset$ or $\Phi \subseteq \Gamma$. If $\Phi \subseteq ||\alpha|| \subseteq \Phi \cup \mathbb{P}_{\mathbb{L}} \setminus \Gamma$, then there exists a SPtoSp profile revision operator \odot on Γ such that $\Gamma \odot \alpha = \Phi$.

Proof. Let $\mathbb{L} = \ll L_1, L_2, ..., L_n \gg$ be a tuple of labels. Let Γ and Φ be non-empty subsets of $\mathbb{P}_{\mathbb{L}}$ such that it holds that either $\Gamma \cap \Phi = \emptyset$ or $\Phi \subseteq \Gamma$. Assume that $\Phi \subseteq ||\alpha|| \subseteq \Phi \cup \mathbb{P}_{\mathbb{L}} \setminus \Gamma$,

Let \odot be an operator such that (for all $\alpha \in \mathcal{L}$):

$$\Gamma \odot \alpha = \begin{cases} Min(\|\alpha\|, \prec_{\Gamma}) & \text{if } \|\alpha\| \neq \emptyset \\ \Gamma & \text{otherwise} \end{cases}$$

Where \preceq_{Γ} is such that $P_i \preceq_{\Gamma} P_j$ iff either: (i) $P_i \in \Gamma$ or (ii) $P_i \in \Phi$ and $P_j \notin \Gamma$ or (iii) $P_j \notin \Gamma \cup \Phi$. We need to prove that \preceq_{Γ} is a Γ -faithfull pre-order on $\mathbb{P}_{\mathbb{L}}$.³

We start by showing that \preceq_{Γ} is reflexive. Let $P_i \in \mathbb{P}_{\mathbb{L}}$. Case 1) $P_i \in \Gamma$. Hence $P_i \preceq_{\Gamma} P_i$.

Case 2) $P_i \notin \Gamma$.

Case 2.1) $P_i \in \Phi$. Hence $P_i \preceq_{\Gamma} P_i$.

Case 2.2) $P_i \notin \Phi$. Thus $P_i \notin \Gamma \cup \Phi$. Hence $P_i \preceq_{\Gamma} P_i$. We will now prove that \preceq_{Γ} is transitive. Let $P_i, P_j, P_k \in \mathbb{P}_{\mathbb{L}}$ be such that $P_i \preceq_{\Gamma} P_j$ and $P_j \preceq_{\Gamma} P_k$ we intend to prove that $P_i \preceq_{\Gamma} P_k$.

It follows trivially if $P_i = P_j$ or $P_j = P_k$. It also follows

that $P_i \leq_{\Gamma} P_k$ if $P_i = P_k$ (since \leq_{Γ} is reflexive, as shown above).

Assume now that $P_i \neq P_j$, $P_j \neq P_k$ and $P_i \neq P_k$. We will prove by cases:

Case 1) $P_i \in \Gamma$. Hence $P_i \preceq_{\Gamma} P_k$ by definition of \preceq_{Γ} .

Case 2) $P_i \notin \Gamma$. From $P_i \preceq_{\Gamma} P_j$ it follows by definition of \preceq_{Γ} that $P_j \notin \Gamma$. Similarly, from $P_j \preceq_{\Gamma} P_k$ it follows that $P_k \notin \Gamma$.

Case 2.1) $P_i \in \Phi$. Hence $P_i \preceq_{\Gamma} P_k$ by definition of \preceq_{Γ} . Case 2.2) $P_i \notin \Phi$. From $P_i \preceq_{\Gamma} P_j$ it follows by definition of \preceq_{Γ} that $P_j \notin \Gamma \cup \Phi$. From $P_j \preceq_{\Gamma} P_k$ it follows by definition of \preceq_{Γ} that $P_k \notin \Gamma \cup \Phi$. Thus by definition of \preceq_{Γ} it yields that $P_i \preceq_{\Gamma} P_k$.

We now show that \preceq_{Γ} is Γ -faithful.

Let $P_i \in \Gamma$ and $P_j \in \Gamma$. Hence by definition of \preceq_{Γ} it holds that $P_i \preceq_{\Gamma} P_j$. Thus $P_j \not\prec_{\Gamma} P_i$.

Let $P_i \in \Gamma$ and $P_j \notin \Gamma$. By definition of \preceq_{Γ} it follows that $P_i \preceq_{\Gamma} P_j$ and $P_j \not\preceq_{\Gamma} P_i$. Hence $P_i \prec_{\Gamma} P_j$.

By definition it follows that \odot is a SPtoSP profile revision operator on Γ .

From $\Phi \subseteq ||\alpha||$ and $\Phi \neq \emptyset$ it follows that $||\alpha|| \neq \emptyset$. We must prove that $\Gamma \odot \alpha = \Phi$, i.e., that $\Phi = Min(||\alpha||, \prec_{\Gamma})$.

Let $P_i \in ||\alpha||$. It holds that $||\alpha|| \subseteq \Phi \cup \mathbb{P}_{\mathbb{L}} \setminus \Gamma$. Hence $P_i \in \Phi \cup \mathbb{P}_{\mathbb{L}} \setminus \Gamma$. Thus $P_i \in \Phi$ or $P_i \in \mathbb{P}_{\mathbb{L}} \setminus \Gamma$. Let $P_j \in \Phi$. There are several cases to consider: Case 1) $P_i \in \Phi$.

Case 1.1) $P_i \in \Gamma$. Hence $\Phi \cap \Gamma \neq \emptyset$. Thus $\Phi \subseteq \Gamma$. Therefore $P_j \in \Gamma$. Hence, by definition of \preceq_{Γ} , it follows that $P_j \preceq_{\Gamma} P_i$.

Case 1.2) $P_i \notin \Gamma$. Hence, by definition of \preceq_{Γ} , it follows that $P_j \preceq_{\Gamma} P_i$.

Case 2) $P_i \notin \Phi$. Then $P_i \in \mathbb{P}_{\mathbb{L}} \setminus \Gamma$. Hence $P_i \notin \Gamma \cup \Phi$. Thus, by definition of \preceq_{Γ} , it follows that $P_j \preceq_{\Gamma} P_i$.

Thus in all cases it follows that $P_j \preceq_{\Gamma} P_i$. Hence $P_i \not\prec_{\Gamma} P_j$. It yields that $P_j \in ||\alpha||$. Thus $\Phi \subseteq Min(||\alpha||, \prec_{\Gamma})$. Let $P_k \in Min(||\alpha||, \prec_{\Gamma})$ and assume towards a contradiction that $P_k \notin \Phi$. Let $P_j \in \Phi \subseteq ||\alpha||$. From $P_k \in Min(||\alpha||, \prec_{\Gamma})$ it follows that $P_k \in ||\alpha|| \subseteq \Phi \cup \mathbb{P}_L \setminus \Gamma$. Thus $P_k \notin \Gamma \cup \Phi$. Therefore, $P_j \preceq_{\Gamma} P_k$. Furthermore it holds that $P_k \not\preceq_{\Gamma} P_j$. Hence $P_j \prec_{\Gamma} P_k$. From which it follows that $P_k \notin Min(||\alpha||, \prec_{\Gamma})$. Contradiction. \Box

The following example clarifies the importance of the condition "either $\Gamma \cap \Phi = \emptyset$ or $\Gamma \subseteq \Phi$ " included in the hypothesis of the statement of the previous observation.

Example 4. Let $\mathbb{P}_{\mathbb{L}} = \{P_1, P_2, P_3, P_4\}$, $\Gamma = \{P_1, P_2\}$ and $\Phi = \{P_1, P_3\}$. Hence $\Gamma \cap \Phi \neq \emptyset$ and $\Phi \not\subseteq \Gamma$. Let α be a formula such that $\|\alpha\| = \Phi$. Hence $\Phi \subseteq \|\alpha\| \subseteq \Phi \cup \mathbb{P}_{\mathbb{L}} \setminus \Gamma$. However, since $\Gamma \cap \|\alpha\| \neq \emptyset$ it follows by (SP5) that for any SPtoSP profile revision operator \odot , $\Gamma \odot \alpha = \Gamma \cap \|\alpha\| = \{P_1\} \neq \Phi$.

Given sets of profiles $\Gamma_1, \ldots, \Gamma_n, \Phi_1, \ldots, \Phi_n$, the following corollary, which follows directly from Observation

³Note that \preceq_{Γ} is total.

11, establishes the bounds, in terms of set inclusion, for the models of α in order to ensure the existence of a SPtoSP revision operator \odot_i on Γ_i such that $\Gamma_i \odot_i \alpha = \Phi_i$ for all $i \in 1, ..., n$.

Corollary 7. Let $\mathbb{L} = \ll L_1, L_2, ..., L_n \gg$ be a tuple of labels. For all $i \in \{1, ..., m\}$ let Γ_i and Φ_i be nonempty subsets of $\mathbb{P}_{\mathbb{L}}$ such that it holds that either $\Gamma_i \cap \Phi_i = \emptyset$ or $\Phi_i \subseteq \Gamma_i$.

If $\bigcup_{i=1}^{m} \Phi_i \subseteq ||\alpha|| \subseteq \bigcap_{i=1}^{m} (\Phi_i \cup \mathbb{P}_{\mathbb{L}} \setminus \Gamma_i)$, then, for all $i \in \{1, \ldots, m\}$, there exists a SPtoSp profile revision operator \odot_i on Γ_i such that $\Gamma_i \odot_i \alpha = \Phi_i$.

As seen in the Example 4, there are cases in which there are no SPtoSP profile revision operators such that $\Gamma \odot \alpha = \Phi$. However, as the following observation asserts, it is always possible, by means of a procedure of (at most) two-steps to transform a non-empty set of profiles Γ into any non-empty set of profiles $\Phi \neq \mathbb{P}_{\mathbb{L}}$.

Observation 12. Let $\mathbb{L} = \ll L_1, L_2, ..., L_n \gg be a tuple of labels, <math>\Gamma$ and Φ be non-empty subsets of $\mathbb{P}_{\mathbb{L}}$ and $\Phi \neq \mathbb{P}_{\mathbb{L}}$.⁴ It holds that either:

- there exists a formula α and a SPtoSP profile revision operator \odot such that $\Gamma \odot \alpha = \Phi$ or
- there exist formulas α_1 and α_2 and SPtoSP profile revision operators \odot_1 and \odot_2 such that ($\Gamma \odot_1$ α_1) $\odot_2 \alpha_2 = \Phi$. Note that \odot_1 is defined on Γ , therefore it is not possible to apply \odot_1 to the outcome of $\Gamma \odot_1 \alpha$ (unless $\Gamma = \Gamma \odot_1 \alpha$). Hence, in general, $\odot_1 \neq \odot_2$.

Proof. Case 1) $\Gamma \cap \Phi = \emptyset$ or $\Phi \subseteq \Gamma$. Let α be such that $\Phi \subseteq ||\alpha|| \subseteq \Phi \cup \mathbb{P}_{\mathbb{L}} \setminus \Gamma$. Then according to Observation 11 there exists a SPtoSP profile revision operator \odot such that $\Gamma \odot \alpha = \Phi$.

Case 2) $\Gamma \cap \Phi \neq \emptyset$ and $\Phi \not\subseteq \Gamma$. By hypothesis it holds that $\Phi \neq \mathbb{P}_{\mathbb{L}}, \Gamma \neq \emptyset$ and $\Phi \neq \emptyset$. We will consider two cases:

Case 2.1) $\Gamma \setminus \Phi \neq \emptyset$. Let α_1 be such that $\|\alpha_1\| = \Gamma \setminus \Phi$. It holds that $\Gamma \setminus \Phi \subseteq \Gamma$ and $\Gamma \setminus \Phi \subseteq \|\alpha_1\| \subseteq (\Gamma \setminus \Phi) \cup \mathbb{P}_{\mathbb{L}} \setminus \Gamma$. Hence by Observation 11 there exists a SPtoSP profile revision operator \odot_1 such that $\Gamma \odot_1 \alpha_1 = \Gamma \setminus \Phi$. Let α_2 be such that $\|\alpha_2\| = \Phi$. It holds that $\Gamma \setminus \Phi \cap \Phi = \emptyset$ and $\Phi \subseteq \|\alpha_2\| \subseteq \Phi \cup \mathbb{P}_{\mathbb{L}} \setminus (\Gamma \setminus \Phi)$. Hence by Observation 11 there exists a SPtoSP profile revision operator \odot_2 (on $\Gamma \odot_1 \alpha_1$) such that $(\Gamma \odot_1 \alpha_1) \odot_2 \alpha_2 = \Phi$.

Case 2.2) $\Gamma \setminus \Phi = \emptyset$. Hence $\Gamma \subseteq \Phi$. Thus $\Gamma \subset \Phi$ (since $\Phi \not\subseteq \Gamma$).

Let α_1 be such that $\|\alpha_1\| = \mathbb{P}_{\mathbb{L}} \setminus \Phi$. It holds that $\mathbb{P}_{\mathbb{L}} \setminus \Phi \neq \emptyset$, $\Gamma \cap \mathbb{P}_{\mathbb{L}} \setminus \Phi = \emptyset$ and $\mathbb{P}_{\mathbb{L}} \setminus \Phi \subseteq \|\alpha_1\| \subseteq (\mathbb{P}_{\mathbb{L}} \setminus \Phi) \cup \mathbb{P}_{\mathbb{L}} \setminus \Gamma$. Hence by Observation 11 there exists a SPtoSP profile revision operator \odot_1 such that $\Gamma \odot_1 \alpha_1 = \mathbb{P}_{\mathbb{L}} \setminus \Phi$. Let α_2 be such that $\|\alpha_2\| = \Phi$. It holds that $(\mathbb{P}_{\mathbb{L}} \setminus \Phi) \cap \Phi = \emptyset$ and $\Phi \subseteq \|\alpha_2\| \subseteq \Phi \cup \mathbb{P}_{\mathbb{L}} \setminus (\mathbb{P}_{\mathbb{L}} \setminus \Phi)$. Hence by Observation 11 there exists a SPtoSP profile revision operator \odot_2 (on $\Gamma \odot_1 \alpha_1$) such that $(\Gamma \odot_1 \alpha_1) \odot_2 \alpha_2 = \Phi$.

Given two sets of profiles Γ and Φ , and a *SPtoSP* revision operator \odot on Γ induced by a specific (known) pre-order \preceq , the following observation establishes the upper and lower bounds for the set of models of a change formula α for it to hold that $\Gamma \odot \alpha = \Phi$.

Observation 13. Let $\mathbb{L} = \ll L_1, L_2, ..., L_n \gg$ be a tuple of labels. Let Γ and Φ be two distinct non-empty subsets of $\mathbb{P}_{\mathbb{L}}$. Let \preceq_{Γ} be a Γ -faithfull pre-order on Γ such that it holds that $P_j \not\prec_{\Gamma} P_i$, for all $P_i, P_j \in \Phi$. Let $\odot_{\preceq_{\Gamma}}$ be a \preceq_{Γ} -based SPtoSP profile revision operator on Γ . It holds that:

$$\begin{split} \Gamma \odot_{\preceq_{\Gamma}} \alpha &= \Phi \\ i f f \\ \Phi &\subseteq \|\alpha\| \subseteq \Omega \cup \Phi \end{split}$$

where $\Omega = \{P_k \in \mathbb{P}_{\mathbb{L}} : P_n \prec_{\Gamma} P_k, \text{ for some } P_n \in \Phi\}.$

Proof. Let \leq_{Γ} be a Γ -faithfull pre-order on Γ . Let $\odot_{\leq_{\Gamma}}$ be a \leq_{Γ} -based SPtoSP profile revision operator on Γ . Hence, for all sentences α , it holds that:

$$\Gamma \odot_{\preceq_{\Gamma}} \alpha = \begin{cases} Min(\|\alpha\|, \preceq_{\Gamma}) & \text{if } \|\alpha\| \neq \emptyset \\ \Gamma & \text{otherwise} \end{cases}$$

 (\Leftarrow) From $\Phi \neq \emptyset$ it follows that $\|\alpha\| \neq \emptyset$. Hence, we need to prove that $\Phi = Min(\|\alpha\|, \preceq_{\Gamma})$. Let $P_i \in Min(\|\alpha\|, \preceq_{\Gamma})$. It holds that $P_i \in \|\alpha\| \subseteq \Omega \cup \Phi$. Suppose towards a contradiction that $P_i \notin \Phi$. Hence $P_i \in \Omega$. Thus there exists $P_j \in \Phi \subseteq \|\alpha\|$ such that $P_j \prec_{\Gamma} P_i$. Contradiction, since $P_i \in Min(\|\alpha\|, \preceq_{\Gamma})$. Hence $P_i \in \Phi$, from which it follows that $Min(\|\alpha\|, \preceq_{\Gamma}) \subseteq \Phi$.

Let $P_i \in \Phi \subseteq ||\alpha||$ and suppose towards a contradiction that $P_i \notin Min(||\alpha||, \preceq_{\Gamma})$. Hence there exists a $P_j \in Min(||\alpha||, \preceq_{\Gamma})$ such that $P_j \prec_{\Gamma} P_i$. Thus, by hypothesis, $P_j \notin \Phi$. Hence $P_j \in \Omega$. Thus there exists $P_k \in \Phi \subseteq ||\alpha||$ such that $P_k \prec_{\Gamma} P_j$. Contradiction since $P_j \in Min(||\alpha||, \preceq_{\Gamma})$.

 $(\Rightarrow) \text{ If } \|\alpha\| = \emptyset, \text{ then } \Gamma \odot_{\preceq_{\Gamma}} \alpha = \Phi = \Gamma. \text{ Contradiction. Hence } \|\alpha\| \neq \emptyset. \text{ Let } P_i \in \|\alpha\|. \text{ Assume that } P_i \notin \Phi. \text{ We intend to prove that } P_i \in \Omega. \text{ From } P_i \notin \Phi \text{ it follows that } P_i \notin Min(\|\alpha\|, \preceq_{\Gamma}). \text{ Hence there exists } P_j \in Min(\|\alpha\|, \preceq_{\Gamma}) = \Phi \text{ such that } P_j \prec_{\Gamma} P_i. \text{ Thus } P_i \in \Omega. \text{ On the other hand it holds that } \Phi \subseteq \|\alpha\|. \square$

The following corollary presents a result similar to that illustrated in Observation 13, but where a system of equations is considered (rather than a single equation).

 $^{{}^4\}text{If}\,\Phi=\mathbb{P}_{\mathbb{L}}$ and $\Gamma=\mathbb{P}_{\mathbb{L}}$ the result also holds.

Corollary 8. Let $\mathbb{L} = \ll L_1, L_2, ..., L_n \gg$ be a tuple of labels. For $i \in \{1, ..., m\}$ let Γ_i and Φ_i be two distinct non-empty subsets of $\mathbb{P}_{\mathbb{L}}$. For $i \in \{1, ..., m\}$ let \preceq_{Γ_i} be a Γ_i -faithfull pre-order on Γ_i such that it holds that $P_j \not\prec_{\Gamma_i} P_k$, for all $P_j, P_k \in \Phi_i$. Let $\odot_{\preceq \Gamma_i}$ be a \preceq_{Γ_i} based SPtoSP profile revision operator on Γ_i . It holds that,

$$\forall i \in \{1, \dots, m\} \prod_{i \in \Gamma_i} \alpha = \Phi_i$$

$$\inf_{i \in I} \Phi_i \subseteq \|\alpha\| \subseteq \bigcap_{i=1}^m (\Omega_i \cup \Phi_i)$$

where $\Omega_i = \{P_k \in \mathbb{P}_{\mathbb{L}} : P_n \prec_{\Gamma_i} P_k, \text{ for some } P_n \in \Phi_i\}.$

We close this section remarking that an SP-to-SP revision operator defined on a singleton is similar to a P-to-P revision operator. However, we chose to treat these operators as distinct because they have conceptual differences. In P-to-P revision, the output is always a profile, while in SP-to-SP revision, the output is a set of profiles, which may not be a singleton, even if the operator is defined on a singleton.

4. Related Works

To the best of our knowledge there are not many works that relate belief revision with user's profiles. One of those works is [11]. In that paper, the authors developed a service recommendation agent based on belief revision logic to handle the non-monotonicity problem of web service recommendation. They applied belief revision-based reasoning to determine the most suitable context for the initial service request based on the beliefs stored in the user's profile. After service request reasoning, the set of potential web services is identified and ranked. The highest-ranked services are considered to be the most desirable ones that match the user's specific interests. Another paper that establishes a connection between belief revision and user profiles is [6]. In that paper, the authors formalize the creation, representation and dynamics of profiles from a Knowledge-Driven perspective. In particular, they proposed a formal profile representation framework, based on a formal language that allows to clearly represent a user profile and its attributes. The authors introduced several operators for modeling profile's dynamics, which are based on well-known operators from the belief revision literature, and axiomatically characterized them, including the PtoP and the SPtoSP profile revision operators that have been used throughout this article.

On the other hand, the problem of identifying the change formula in a belief revision scenario is related to announcements (e.g., [12]). It is important to mention that the problem of identifying which formula causes a change is different from planning a sequence of formulas to perform a desirable change, although the two are related problems (e.g., [13, 14]). The approach most closely related to our proposal is due to Schwind *et al.* [15]. In that paper the authors considered a belief revision scenario where an announcement (a propositional formula) μ is made to a group of agents, each of these represented by a belief base. In this scenario, it is considered that μ is unknown, and that, for each agent, its previous beliefs and those obtained after the revision by μ are known. The authors characterized the set of formulas μ satisfying these requirements.

5. Conclusion and Future Work

User profiles are important tools in several areas of information technology. Given a profile, sometimes it is necessary to determine a set of tasks, or pieces of training which can transform the profile of a user into a target profile. In this paper, we provided a formal analysis of the process of changing a user profile (or a set of profiles) into a target profile (or set of profiles) by means of a profile revision operator, and we identified upper and lower bounds for the set of formulas that cause such a change. Analogous results are obtained for "systems of equations" of the form $P_i \odot_i \alpha = Q_i$ and $\Gamma_i \odot_i \alpha = \Phi_i$ (for every $i \in \{1, \ldots, m\}$). This work may have some pratical applications.

- (a) Given a set of previous and current profiles of a community, the identification of the input that caused a certain change can be useful for determining the impact of spreading fake news or for predicting the benefices of an information campaign.
- (b) In cognitive rehabilitation procedures where it is common practice to predetermine goals at the beginning of a training program.
- (c) In systems where it may be convenient to change a user's program interface in order to allow the execution of more complex interactions.

As a future work topic, we intend to model the modification of a user profile (set of profiles) into a target profile (set of profiles) through a series of minimal changes. This is useful, for example, for the application settings mentioned in items (b) and (c) above. In a cognitive rehabilitation procedure, the process that leads to rehabilitation is performed by small improvements and in the case of changes in user's program interface, these must be small, minimal or imperceptible, since abrupt changes in systems generally lead to rejections by the end-users.

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