Constructions of Hypergraphical Regular Representations via k-uniform Hypergraphs of Certain Groups of Order Greater than 32

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Abstract

The article explores the problem of hypergraphical regular representation of finite group G via k-uniform hypergraph. A hypergraphical regular representation preserves the group's structure within the hypergraph. The paper builds upon previous research and investigates hypergraphical regular representations of certain groups of orders greater than 32. The paper presents an algorithm based on methods by Mihálová and Erskine and Tuite. The algorithm is implemented in the computational system *GAP* to find hypergraphical regular representations for certain groups. The results include a table with groups for which hypergraphical regular representations via k-uniform hypergraphs were obtained.

Keywords

Hypergraphical Regular Representation, Groups, k-uniform hypergraph, Dual hypergraphs, GAP

1. Introduction

A hypergraph is a generalisation of a graph where a hyperedge can connect more than two vertices. We are focusing on the problem of hypergraphical regular representation of groups. We seek to find hypergraphs that faithfully represent the structure of a group by preserving its symmetries.

The problem of hypergraphical regular representation was formulated as an extension of the graphical regular representation problem. Given a graph Γ, an automorphism of Γ is a permutation ϕ of the vertex set, such that the vertices u and v form an edge if and only if the vertices $\phi(u)$ and $\phi(v)$ also form an edge. The set of all automorphisms of Γ together with the operation of composition forms the automorphism group $Aut(\Gamma)$. *The graphical regular representation (GRR)* of a group G is a graph Γ whose automorphism group is the group G in its regular action. The search for representations of groups started with Kőnig in 1936 [1]. He posed a question: *Does there exist a graph Γ for a given group G such that $Aut(\Gamma) \cong G$?* Two years later, Frucht showed that for every finite group G, there exist infinitely many non-isomorphic connected graphs Γ such that $Aut(\Gamma)$ is isomorphic to G. However, the automorphism groups of these graphs are not necessarily regular. Multiple researchers worked on the problem of graphical regular representation over the years. Godsil [2] summed up the previous findings and published the complete list of groups not admitting a graphical regular representation: abelian groups with an exponent greater than 2, generalised dicyclic groups, a group isomorphic to one of 13 groups whose order is not greater than 32 ($\mathbb{Z}_2^2$, $\mathbb{Z}_2^3$, $\mathbb{Z}_2^4$, $\mathbb{D}_3$, $\mathbb{D}_4$, $\mathbb{D}_5$, $\mathbb{A}_4$, $\mathbb{Q} \times \mathbb{Z}_3$, $\mathbb{Q} \times \mathbb{Z}_4$, $\langle a, b, c \mid a^2 = b^2 = c^2 = 1, abc = bca = cab\rangle$, $\langle a, b \mid a^8 = b^2 = 1, b^{-1}ab = a^5\rangle$, $\langle a, b, c \mid a^3 = b^3 = c^2 = 1, ab = ba, (ac)^2 = (bc)^2 = 1\rangle$, $\langle a, b, c \mid a^3 = b^3 = c^3 = 1, ac = ca, bc = cb, b^{-1}ab = ac\rangle$). Later, Babai[3] raised and solved the question concerning the regular representation of a group via directed graphs. He published a complete list of groups not admitting the digraphical regular representation: $\mathbb{Z}_2^2, \mathbb{Z}_2^3, \mathbb{Z}_2^4, \mathbb{Z}_3^2$ and $\mathbb{Q}_8$. An overview of different types of regular representations was presented by Spiga [4].

We build upon the previous research and investigate the problem of hypergraphical regular representation for certain groups of orders greater than 32. Section 2 covers the necessary concepts, including definitions of hypergraphs, hypergraphical regular representation and dual structures. In Section 3, we present an overview of the previous research and results in hypergraphical regular representation. In Section 4, we describe the methods by Mihálová [5] and Erskine and Tuite [6] that we used to obtain the hypergraphical regular representation of certain groups of order greater than 32. Section 5 contains our algorithm, which was implemented in computational system *GAP*. We describe particular commands in more detail. In Section 6, we present our results with a table containing groups for which we found a hypergraphical regular representation via k-uniform hypergraph.

2. Preliminaries

Our work is concerned with combinatorial structures - hypergraphs. *A hypergraph* $\Gamma = (V(\Gamma), \mathscr{E}(\Gamma))$ is an

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ordered pair of the vertex set $V(\Gamma)$ and the hyperedge set $\mathscr{E}(\Gamma)$. In certain contexts, hyperedges may be referred to as *blocks*. Each hyperedge $E \in \mathscr{E}(\Gamma)$ is a nonempty subset of the vertex set $V(\Gamma)$, i.e. $1 \leq |E| \leq |V(\Gamma)|$. Generally, hyperedges are blocks of various sizes. Our focus is on k-uniform hypergraphs, which are hypergraphs with hyperedges of the same size k, i.e. $\forall E \in \mathscr{E}(\Gamma) : |E| = k$. Note that a 2-uniform hypergraph is also an undirected graph. Our method requires working with hypergraphs whose vertices are in the same number of hyperedges. *The degree $d(u)$ of a vertex u in a hypergraph* Γ is the number of hyperedges which contain the vertex u. A hypergraph is *d-regular* if all vertices have the same degree d, i. e. $\forall u \in V(\Gamma) : d(u) = d$. *An automorphism of a hypergraph* Γ is a bijection $\phi : V(\Gamma) \mapsto V(\Gamma)$ such that vertices $v_1, v_2, ..., v_n \in V(\Gamma)$ form a hyperedge if and only if vertices $\phi(v_1), \phi(v_2), ..., \phi(v_n) \in \mathscr{E}(\Gamma)$ also form a hyperedge. *The automorphism group* $Aut(\Gamma)$ of a hypergraph Γ is formed by the set of all automorphisms of the hypergraph with the operation of composition. Since we are dealing with groups and hypergraphs, defining a Cayley hypergraph is in place. Over the years, several definitions of Cayley hypergraphs have been formulated. The definitions differ in their approach to creating hyperedges and in the minimal possible size of hyperedges. We choose the definition by Jajcay and Jajcayova in [7] because it best suits our problem. Let $\mathscr{P}(G)$ be the powerset of the elements of G, G_L be the left regular action of G and X be $\bigcup_{i=1}^{s} B_i^{G_L}$, where $B_i \in \mathscr{P}(G)$ for $1 \leq i \leq |\mathscr{P}(G)|$. *A Cayley hypergraph $HCay(G, X)$* is a hypergraph Γ with the elements of G as the vertex set and the elements of X as the hyperedge set. For k-uniform $HCay(G, X)$ it holds that $B_i \in \mathscr{P}_k(G)$.

We can define the regular representation of hypergraphs based on previous definitions. *The hypergraphical regular representation (HRR)* of a group G is a Cayley hypergraph $HCay(G, X)$ where for every two vertices $u, v \in V(HCay(G, X))$, there exists exactly one automorphism ϕ such that $\phi(u) = v$ from the automorphism group of the hypergraph $Aut(HCay(G, X))$. This means that the automorphism group $Aut(HCay(G, X))$ acts regularly on the set of vertices of $V(HCay(G, X))$. Several researchers have studied the existence of hypergraphical regular representations for various groups and types of hypergraphs.

Since we use groups in our research, concepts from group theory and their connection to hypergraphs are needed. Given a group G and a set A, *the group action of G on A* is a map $\cdot : G \times A \mapsto A$ (denoted as $g \cdot a$, $\forall g \in G, a \in A$) with the following properties: $\forall g_1, g_2 \in G, \forall a \in A : g_1 \cdot (g_2 \cdot a) = (g_1 \cdot g_2) \cdot a$ and $\forall a \in A : 1_G \cdot a = a$. *An orbit* of an element $a \in A$ under the action of a group G on a set A is denoted as $G \cdot a$. In particular for hyperedges, *an orbit of a hyperedge $E \in \mathscr{E}(\Gamma)$* is the set of all hyperedges in $\mathscr{E}(\Gamma)$ that are equivalent to E by the elements of group G acting on $\mathscr{E}(\Gamma)$. It can be defined formally as $G \cdot E = \{g \cdot E | g \in G, E \in \mathscr{E}(\Gamma)\}$, where G is the group acting on $\mathscr{E}(\Gamma)$ and $\cdot$ is the induced action of G on $\mathscr{E}(\Gamma)$. The intersection of different orbits is empty.

An incidence structure of a hypergraph Γ is an ordered pair $\mathscr{I} = (V(\mathscr{I}), E(\mathscr{I}))$. The vertex set $V(\mathscr{I})$ is a partition into two disjoint sets: $V_1 = V(\Gamma)$ and $V_2 = \mathscr{E}(\Gamma)$. The edge set satisfies $E(\mathscr{I}) \subseteq V_1 \times V_2$. Vertices $v \in V_1$ and $E \in V_2$ are incident iff $v \in E$, i.e. $E(\mathscr{I}) = \{(v, E) | v \in E, E \in \mathscr{E}(\Gamma)\}$. The incidence structure of a hypergraph Γ preserves the symmetries of Γ. Based on [8], the automorphism group of the incidence structure of a hypergraph Γ is isomorphic to the automorphism group of Γ (Fig. 1).

In our method, we are using the concept of dual incidence structure. *A dual incidence structure* $\mathscr{I}^* = (V(\mathscr{I}^*), E(\mathscr{I}^*))$ of the incidence structure $\mathscr{I}$ is an ordered pair of the vertex set $V(\mathscr{I}^*)$ and the edge set $E(\mathscr{I}^*)$. The partitions in the vertex set are swapped compared to the partitions in the incidence structure. Given a hypergraph Γ, the partitions of the dual incidence structure are $V_1 = \mathscr{E}(\Gamma)$ and $V_2 = V(\Gamma)$ (Fig. 2).

We use the knowledge of complementary hypergraphs. Let Γ be a k-uniform hypergraph. By Γ^C, we denote a k-uniform hypergraph defined as an ordered pair $(V(\Gamma), \mathscr{P}_k(V(\Gamma)) \setminus \mathscr{E}(\Gamma))$, i.e. the hyperedges of Γ^C are complements of the hyperedges of Γ. By H^C, we denote a $(V(\Gamma) - k)$-uniform hypergraph, where $\mathscr{E}(H^C)$ are complements of $\mathscr{E}(\Gamma)$ of size $V(\Gamma) - k$.

3. Previous research and similar questions

Foldes and Singhi [9] were the first to investigate the problem of hypergraphical regular representation. They proved the existence of a hypergraphical regular representation via a 3-uniform hypergraph for every finite group of odd order greater than or equal to 5^7. In the same year, Foldes [10] proved that cyclic groups $\mathbb{Z}_n$ (except $n = 3, 4, 5$) admit regular representation using a 3-uniform hypergraph. Collaboratively, Foldes and Singhi [11] established a polynomial lower bound $p(k)$ for the order of the group that admits a hypergraphical regular representation via k-uniform hypergraph for $k \geq 3$. For every finite group G such that $p(k) \leq |G|$, there exists a k-uniform hypergraph which is a hypergraphical regular representation of G. The lower bound for $k = 3$ is $p(3) > 2^6$ and for $k \geq 4$ is $p(k) > 4k + 2$. They suggested that for $k = 3$, the lower bound should be improved to a linear polynomial of the form $k + c$, where c is a constant. Jajcay [12] studied the problem of hypergraphical regular representation for hypergraphs whose hyperedges are not necessarily regular. He improved the

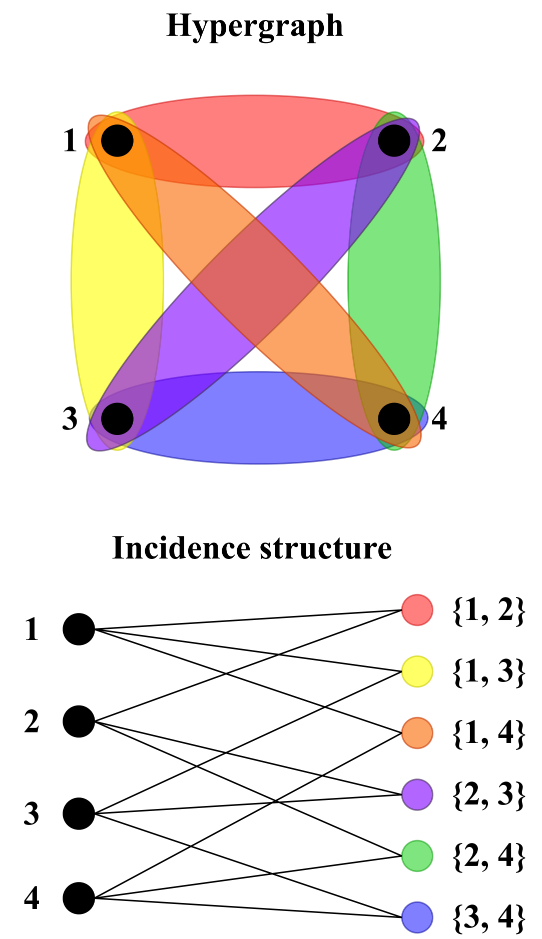

Figure 1: For illustration, the hypergraph Γ (which is a simple graph) has the set of vertices $V(\Gamma) = \{1, 2, 3, 4\}$, the set of hyperedges $\mathscr{E}(\Gamma) = \{\{1, 2\}, \{1, 3\}, \{1, 4\}, \{2, 3\}, \{2, 4\}, \{3, 4\}\}$ and $Aut(\Gamma) = \mathbb{S}_4$.
The incidence structure $\mathscr{I}$ of the hypergraph Γ is an undirected graph with the vertex set $V_1 = V(\Gamma)$ and $V_2 = \mathscr{E}(\Gamma)$ and the edge set $E(\mathscr{I}) = \{(1, \{1, 2\}), (1, \{1, 3\}), (1, \{1, 4\}), (2, \{1, 2\}), (2, \{2, 3\}), (2, \{2, 4\}), (3, \{1, 3\}), (3, \{2, 3\}), (3, \{3, 4\}), (4, \{1, 4\}), (4, \{2, 4\}), (4, \{3, 4\})$ and $Aut(\mathscr{I}) = \mathbb{S}_4$.
The automorphism groups of $\mathscr{I}$ and Γ are isomorphic.

lower bound $p(k) \geq 6$ for hypergraphs with varying sizes of hyperedges. Also, he showed the non-existence of a hypergraphical regular representation for four finite groups $\mathbb{Z}_3, \mathbb{Z}_4, \mathbb{Z}_4, \mathbb{Z}_2^2$. It supports the findings in [10]. Nonetheless, his solution heavily depends on hypergraphs with hyperedges of different sizes. In this case, criteria for admitting a regular representation are less restrictive than for k-uniform hypergraphs. Consequently, a group that does not admit a hypergraphical regular representation via a non-uniform hypergraph does not admit a hypergraphical regular representation via a k-uniform hypergraph. Furthermore, Jajcay and Jajcayova [13] listed the groups without a hypergraphical regular representation by 3-uniform hypergraphs. The list includes the groups mentioned in [12], as well as some other groups: $\mathbb{Z}_3, \mathbb{Q}_8, \mathbb{Z}_2^3, \mathbb{Z}_4^3, \mathbb{Z}_5^3, \mathbb{D}_5 \times \mathbb{Z}_5$. Their colleague Martin Mačaj verified computationally that there exists a hypergraphical regular representation through 3-uniform hypergraph for groups of order $6 \leq |G| \leq 32$. Recently, we computationally verified [5] a generalised conjecture by Jajcayová [14]. We stated which groups admit or do not admit a hypergraphical regular representation via k-uniform hypergraph for groups of orders less than or equal to 32 and the whole spectrum of $3 \leq k \leq |G|$.

4. Overview of methods

We aim to find hypergraphical regular representations for groups of order greater than 32. We already found hypergraphical representations of groups of order less than 33 in [5]. However, exhaustive search, the main method in [5], is challenging for groups of greater order. We decided to merge the method from [5] with the method by Erskine and Tuite in [6] to obtain hypergraphical regular representation for groups of greater order. We briefly describe these methods.

The computational method in [5] was based on theoretical proofs in [13]. We went through all groups of orders less than 33. For each group, we computed the permutation of the right multiplication for each element within the group. Based on the permutations, we obtained orbits of k-uniform hyperedges. As a next step, we went through all combinations of orbits. We constructed the hypergraph and asked for its automorphism group. If the order of the automorphism group was equal to the order of the group, we confirmed the existence of the hypergraphical regular representation for that group.

Erskine and Tuite [6] used their method to obtain new record graphs. *A record graph* is a graph with the smallest *known* number of vertices for a given girth (size of the smallest cycle in the graph) and degree of vertices. We will adopt their method to create a d-regular k-uniform hypergraph Γ and construct an incidence structure $\mathscr{I}$ from Γ (Fig. 1). Subsequently, we obtain dual incidence structure $\mathscr{I}^*$ from which we can obtain a k-regular d-uniform hypergraph known as the *dual hypergraph* Γ^* (Fig. 2). The dual hypergraph Γ^* has the same automorphism group as the original hypergraph Γ. However, the order of the dual hypergraph is $\frac{d}{k}|V(\Gamma)|$. The order of Γ^* can be smaller, equal or greater than the order of Γ depending on values of d and k.

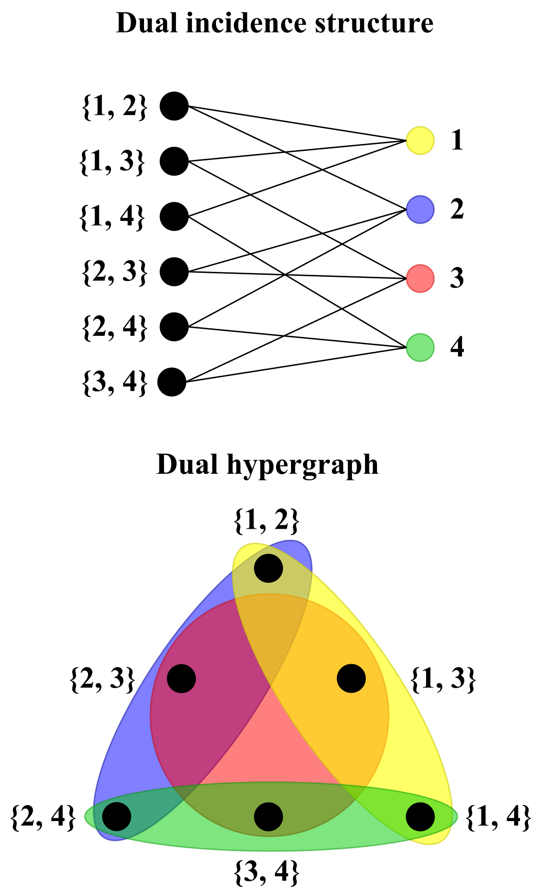

Figure 2: Dual incidence structure and dual hypergraph of the hypergraph in Fig. 1

By combining these methods, we state a proposition (Proposition 1) for solving the problem of hypergraphical regular representation of groups of order greater than 32. Based on the proposition and the relationship between the order of Γ and Γ^*, we can construct a hypergraphical regular representation of a group with greater order from a group of a smaller order.

Proposition 1. *Let Γ be a d-regular k-uniform hypergraph with property $|\mathscr{E}(\Gamma)| = |Aut(\Gamma)|$. By constructing a dual hypergraph Γ^*, we can obtain a hypergraphical regular representation of a group G via k-regular d-uniform hypergraph, where $G \cong Aut(\Gamma) \cong Aut(\Gamma^*)$ and $|V(\Gamma^*)| = |\mathscr{E}(\Gamma)| = |Aut(\Gamma^*)|$.*

5. Algorithm

We created an algorithm from methods in Section 4 and implemented it in the system for discrete computational algebra - *GAP* [15]. It is a free and open-source system with its own programming language and importable packages with multiple implemented functions. We used two packages: *DESIGN* and *GRAPE*. The algorithm was implemented in *GAP* version *4.12.2*. The *GAP* system does not have a specific structure for a hypergraph. Closest to the hypergraph structure is a block design in *DESIGN* package. The *DESIGN* package is for constructing, classifying, partitioning and studying block designs. By definition, the block design is very similar to our incidence structure of a hypergraph. The use of the package is conditioned by the previous import of *GRAPE* package. The *GRAPE* package is designed for computations, constructions and analysis of graphs with relations to groups.

In our algorithm (Algorithm 1), we go through all groups. For each group, we obtain the right multiplication permutations of the group elements stored in the variable `permutations`. We get orbits of hyperedges from permutations in the variable `orbits`. A detailed explanation of these commands can be found in [5]. We compute the number of all possible k-uniform hyperedges and store it in the variable `numAllEdges`. We use this variable to compute the maximal possible degree of vertices in a k-uniform hypergraph saved in the variable `maxDegree`. Both variables `numAllEdges` and `maxDegree` are needed for recognising a complement hypergraph Γ^C later.

We construct hypergraphs from combinations of orbits. By *c-combination of orbits*, we denote a combination of c orbits. Based on [12], we know that we need to construct hypergraphs from c-combinations of orbits in the range $1 \leq c \leq \lfloor \frac{|orbits|}{2} \rfloor$. Hypergraphs constructed from c-combinations of orbits in the range $\lfloor \frac{orbits}{2} \rfloor + 1 \leq c \leq |orbits|$ are complements of the hypergraphs above. We compute the upper bound for combinations of orbits and store it in the variable bound. In a for-cycle, we go through all values of c in the range $1 \leq c \leq$ bound. For every value c, we compute all c-combinations of orbits. For each combination of orbits, we extract the particular edges to the `edges` variable. We construct a hypergraph Γ with a block design structure in *DESIGN* package. If the hypergraph is regular, we have a d-regular k-uniform hypergraph, which is important for the dual hypergraph method by [6]. Then, we compute the automorphism group of Γ. We are interested only in hypergraphs with $Aut(\Gamma) > 32$, as we already know which groups of order less than 33 admit hypergraphical regular representation [5]. If $Aut(\Gamma) > 32$, we have three options. First, if $|Aut(\Gamma)| = |\mathscr{E}(\Gamma)|$, we have found a hypergraphical regular representation via k-regular d-uniform hypergraph for group G, where $G \cong Aut(\Gamma)$.

Second, if $|Aut(\Gamma)| = (numAllEdges - |\mathscr{E}(\Gamma)|)$, we are able to construct a complement hypergraph Γ^C to hypergraph Γ. The hypergraph Γ^C is a hypergraphical regular representation via k-regular $(maxDegree - d)$-uniform hypergraph for group G, where $G \cong Aut(\Gamma)$. Third, if $|Aut(\Gamma)| = |\mathscr{E}(H^C)|$, we can obtain a complement hypergraph H^C to hypergraph Γ. The hypergraph H^C is a hypergraphical regular representation via k-regular d'-uniform hypergraph, where $G \cong Aut(\Gamma)$ and d' is the regularity of H^C.

Algorithm 1 Pseudocode (one group): identifying HRR of groups of greater order

```
function(order, i, k)
group = SmallGroup(order, i)
permutations = Action(group,AsList(group),OnRight)
orbits = OrbitsDomain(permutations, Combinations([1..order], k), OnSets)
numAllEdges = Binomial(order, k)
maxDegree = numAllEdges * k / order
bound = Int(Size(orbits)/2)
for 1 ≤ numOfOrbits ≤ bound do
  for all combinations of orbits of size numOfOrbits do
    edges = Concatenation(combination of orbits)
    gamma = BlockDesign(order, edges)
    if gamma is regular then
      autGroup = AutomorphismGroup(gamma)
      if |autGroup| > 32 then
        if |autGroup| = |edges| then
          print found k-regular d-uniform HRR
        end if
        if |autGroup| = (numAllEdges − |edges|) then
          print found k-regular (maxDegree − d)-uniform HRR (complement 1)
        end if
        if |autGroup| = NrBlockDesignBlocks(ComplementBlockDesign(gamma)) then
          print found (order − k)-regular d'-uniform HRR (complement 2)
        end if
      end if
    end if
  end for
end for
end function
```

6. Results of experiments

We ran our algorithm for groups of orders smaller than 15. Groups of order greater than 10 were not fully explored for greater values of k since the computational complexity and time increased with increasing k. Also, the density of printed results decreased with increasing k. We attribute the smaller amount of printouts to the fact that with increasing k, there is a higher probability of finding hypergraphical regular representation via k-uniform hypergraph for the starting group G. However, we are more interested in k-uniform hypergraphs Γ that are not regular representations of the starting group G as $Aut(\Gamma) \ncong G$ and $|Aut(\Gamma)|$ is greater (or smaller) than $|G|$. Thus, these k-uniform hypergraphs can be regular representations for groups of greater (or smaller) order than G.

We first launched our algorithm without the restriction $|Aut(\Gamma)| > 32$ to verify our algorithm by obtaining hypergraphical regular representations for groups of orders smaller than 33. We obtained numerous hy-

Table 1
Hypergraphical regular representations via k-uniform hypergraph for groups of orders greater than 32

G **with HRR**	$\lvert G\rvert$	**k**
C3 x (C3 : C4)	36	12; 24
C3 x A4	36	12; 24
C6 x S3	36	12; 24
S3 x S3	36	12; 16; 20; 24
C13 : C3	39	9; 30
C2 x (C5 : C4)	40	12; 16; 24; 28
C2 x (C7 : C3)	42	9; 33
C7 : C6	42	9; 33
A4 : C4	48	12; 16; 32; 36
C2 x C2 x A4	48	12; 16; 32; 36
C2 x S4	48	8; 12; 16; 32; 36
C4 x A4	48	12; 16; 32; 36
D8 x S3	48	12; 16; 32; 36
GL(2,3)	48	18
C13 : C4	52	8; 12; 40
C9 : C6	54	24; 30
C11 : C5	55	20; 35
(C6 x S3) : C2	72	18; 54
(S3 x S3) : C2	72	32
C3 x S4	72	18; 54
C13 : C6	78	18; 60
C2 x (C7 : C6)	84	18; 66
(A4 : C4) : C2	96	24; 72
(C2 x C2 x A4) : C2	96	24; 72
(C2 x S4) : C2	96	24; 72
(C4 x A4) : C2	96	24; 72
C2 x C2 x S4	96	24; 72
C4 x S4	96	24; 72
C11 : C10	110	30
(C3 x C3) : ((C4 x C2) : C2)	144	36
C2 x ((S3 x S3) : C2)	144	36
S3 x S4	144	36
S4 x S3	144	36
C2 x ((C2 x C2 x C2 x C2) : C5)	160	64

pergraphical regular representations for those groups that confirmed the correctness of our algorithm. Then, we looked for hypergraphical regular representations of groups of orders greater than 32. Groups and values of k, for which we found hypergraphical regular representations via k-uniform hypergraphs, are presented in Table 1. The names of the groups are in *GAP* notation. Most of the hypergraphical regular representations are from complementary hypergraphs H^C.

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