Bridging the Gap between Ranking-based Semantics and Extension-ranking Semantics

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Abstract

In this paper, we discuss the relationship between ranking-based semantics and extension-ranking semantics in the area of abstract argumentation frameworks. In particular, we investigate approaches to transform these two semantics into each other, i.e. going from a ranking over arguments to a ranking over sets of arguments (*lifting*) and from a ranking over sets of arguments to a ranking over arguments (*social ranking*). Additionally, we analyse the principles and properties the resulting semantics do satisfy.

Keywords

Abstract Argumentation, Ranking-based semantics, Extension-ranking semantics, Social Ranking Problem, Lifting

1. Introduction

Formal argumentation [1] has gained attention as a ration decision-making model with a focus on the representation of arguments and their relationships. The well-know approach of *abstract argumentation frameworks* (AF) [2] uses directed graphs to reason, where arguments are the nodes and the edges are representing attacks between two arguments s.t. the source of the edge is *attacking* the target. One way to reason with these frameworks are the so called *extension-based semantics*, which are functions allowing us to state that a set of arguments is jointly *accepted*. So, these semantics can be used as a binary classifier for sets of arguments based on their "acceptability", a set is either acceptable based on some account of acceptability or not.

In recent years, this binary classification has been criticised as being too limiting in real world scenarios like online debates [3]. For any argument, we can only say if it is part of an acceptable set or not. The extension-based semantics do not give us any insight into the inherent strength of each argument. For that purpose the so called *ranking-based semantics* [4, 5] were proposed. These functions allow us to rank arguments based on their strength alone.

While we can use ranking-based semantics to rank arguments base on their strength, these functions do not allow us to state that a set is jointly acceptable. Further, two arguments with high strength degrees may not be allowed to be jointly acceptable, since they are in conflict with each other. To refine the reasoning based on extensions, *extension-ranking semantics* were defined [6]. Using these functions we can state whether a set of arguments is "better" than another set.

Both ranking-based semantics and extension-ranking semantics in a sense are focusing on ranking arguments. Both these approaches return preorders, ranking-based semantics ranks single arguments, while extension-ranking semantics return rankings over sets of arguments. Like already shown by Skiba et al. [6] these approaches are related and can be combined. We can project extension-ranking semantics into ranking-based semantics and the other way around. Similar projection tasks between rankings of objects and rankings of sets of objects were already discussed as part of the area of computational social choice and in particular voting theory (for an overview see [7]), where the best committee should be constructed based on the preferences of each voter. So, we take a set of preferences or preorders over candidates and *lift* them to a preorder over potential committee setups (sets of candidates). These lifting questions were already discussed in the context of argumentation by Maly and Wallner [8], however they focus on a structured approach, where the preferences are part of the input. In the AF setting, we do not have such preference data, hence we need to generate our preferences in another way. One possibility are the ranking-based semantics, which we can lift to extension-ranking semantics. The work by Maly and Wallner [8] represents a starting point for a discussion about lifting in formal argumentation and can be extended. Besides structured approaches the problem of lifting was also discussed for *Preference-based AFs* (PAFs) [9, 10, 11, 12], which are extensions of argumentation framework with a preferences order over the set of arguments as part of the input. Again, preference data is part of the input and needs to be generated separately.

The other direction (going from a ranking over sets of objects to a ranking over objects) is equally interesting.

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The *social ranking problem* has received considerable interest in the area of computation social choice in recent years. Here individual persons, like researchers, are evaluated based on their performances and their impact in different teams. A number of social ranking solutions can be found in the literature [13, 14, 15, 16], however a discussion in the context of argumentation is missing. Using the idea of a social ranking solution we want to project a ranking over sets of arguments down to a ranking over arguments, allowing us to reason on the argument level.

In this work, we look at the relationship between ranking-based semantics and extension-ranking semantics in detail and want to establish connections between these two reasoning formalisms in abstract argumentation. These connections allow us to bring the ranking-based semantics closer to an extension-based approach. We look at the social ranking and the lifting problems between these two approaches, hence we transform ranking-based semantics into extension-ranking semantics and the other way around. For the social ranking problem it turns out, that the resulting rankings are generalisations of the credulous acceptance problems. Additionally, we investigate the properties of the resulting semantics based on principles from the literature.

In Section 2, we recall preliminaries about argumentation frameworks, ranking-based semantics and extension-ranking semantics. The social ranking problem is discussed in Section 3. Section 4 introduces the lifting problem. Section 5 concludes this paper.

2. Preliminaries

Abstract Argumentation Frameworks

An *abstract argumentation framework* is a directed graph $F = (A, R)$ where A is a finite set of *arguments* and $R \subseteq A \times A$ is an *attack relation* [2]. An argument a is said to *attack* an argument b if $(a, b) \in R$. We say that an argument a is *defended by a set* $E \subseteq A$ if every argument $b \in A$ that attacks a is attacked by some $c \in E$. For $a \in A$ we define $a^-_F = \{b \mid (b, a) \in R\}$ and $a^+_F = \{b \mid (a, b) \in R\}$, so the sets of attackers of a and the set of arguments attacked by a in F. For a set of arguments $E \subseteq A$ we extend these definitions to E^-_F and E^+_F via $E^-_F = \bigcup_{a \in E} a^-_F$ and $E^+_F = \bigcup_{a \in E} a^+_F$, respectively. If the AF is clear in the context, we will omit the index.

Most semantics [17] for abstract argumentation are relying on two basic concepts: *conflict-freeness* and *admissibility.*

Definition 1. *Given $F = (A, R)$, a set $E \subseteq A$ is*

- conflict-free *iff $\forall a, b \in E$, $(a, b) \notin R$;*
- admissible *iff it is conflict-free, and every element of E is defended by E.*

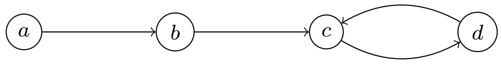

Figure 1: Abstract argumentation framework F from Example 1.

We use $cf(F)$ and $ad(F)$ to denote the sets of conflict-free and admissible sets of an AF F. The intuition behind these concepts is that a set of arguments may be accepted only if it is internally consistent (conflict-freeness) and able to defend itself against potential threats (admissibility). In order to define the remaining semantics proposed by [2] we use the *characteristic function.*

Definition 2. *For an AF $F = (A, R)$ the* characteristic function *for a set of arguments $E \subseteq A$, $\mathcal{F}_F(E) : 2^A \to 2^A$ is defined via:*

$$\mathcal{F}_F(E) = \{a \in A | E \textit{ defends } a\}$$

In words, the characteristic functions returns for every set of arguments all arguments defended by that set. Using this function we introduce all the remaining semantics defined by [2] and in addition the semi-stable semantics [18].

Definition 3. *Given $F = (A, R)$, an admissible set $E \subseteq A$ is*

- *a* complete *extension (co) iff $E = \mathcal{F}_F(E)$;*
- *a* preferred *extension (pr) iff it is a $\subseteq$-maximal admissible extension;*
- *the unique* grounded *extension (gr) iff E is the least fixed point of $\mathcal{F}_F$;*
- *a* stable *extension (stb) iff $E^+_F = A \setminus E$;*
- *a* semi-stable *extension (sst) iff it is a complete extension, where $E \cup E^+_F$ is $\subseteq$-maximal.*

The sets of extensions of an AF F for these five semantics are denoted as (respectively) $co(F)$, $pr(F)$, $gr(F)$, $stb(F)$ and $sst(F)$. Based on these semantics, we can define the status of any (set of) argument(s), namely *skeptically accepted* (belonging to each σ-extension), *credulously accepted* (belonging to some σ-extension) and *rejected* (belonging to no σ-extension). Given an AF F and an extension-based semantics σ, we use (respectively) $sk_\sigma(F)$, $cred_\sigma(F)$ and $rej_\sigma(F)$ to denote these sets of arguments.

Example 1. *Consider the AF $F = (A, R)$ depicted as a directed graph in Figure 1, with the nodes corresponding to arguments $A = \{a, b, c, d\}$, and the edges corresponding to attacks $R = \{(a, b), (b, c), (c, d), (d, c)\}$. We see that F has three complete extensions $\{a\}, \{a, c\}$ and $\{a, d\}$ only the last two are preferred in addition. Also, we see* *that, a is skeptically accepted w.r.t. complete semantics, c and d are credulously accepted w.r.t. complete semantics and b is rejected w.r.t complete semantics.*

An *isomorphism* γ between two AFs $F = (A, R)$ and $F' = (A', R')$ is a bijective function $\gamma : A \to A'$ such that $(a, b) \in R$ iff $(\gamma(a), \gamma(b)) \in R'$ for all $a, b \in A$.

Ranking-based Semantics

Instead of only reasoning based on the acceptance of sets of arguments, *ranking-based semantics* [4] were introduced to focus on the strength of a single argument with respect to the other arguments. Note that the order returned by a ranking-based semantics is not necessarily total, i. e. not every pair of arguments is comparable.

Definition 4. *A* ranking-based semantics ρ *is a function, which maps an AF $F = (A, R)$ to a preorder*[1] $\succeq^\rho_F$ *on A.*

Intuitively, $a \succeq^\rho_F b$ means, that a is at least as strong as b in F. We define the usual abbreviations as follows;

- $a \succ^\rho_F b$ denotes *strictly stronger*, i. e. $a \succeq^\rho_F b$ and $b \not\succeq^\rho_F a$;
- $a \simeq^\rho_F b$ denotes *equally strong*, i. e. $a \succeq^\rho_F b$ and $b \succeq^\rho_F a$;
- $a \bowtie^\rho_F b$ denotes *incomparability* so neither $a \succeq^\rho_F b$ nor $b \succeq^\rho_F a$.

One example for ranking-based semantics is the *h-categoriser ranking-based semantics* [19]. This ranking considers the direct attackers of an argument to calculate its strength value.

Definition 5 ([19])**.** *Let $F = (A, R)$. The* h-categoriser function $Cat : A \to (0, 1]$ *is defined as:*

$$Cat(a) = \begin{cases} 1 & \text{if } a^-_F = \emptyset \\ \frac{1}{1+\sum_{b \in a^-_F} Cat(b)} & \text{otherwise} \end{cases}$$

The h-categoriser ranking-based semantics *defines a ranking $\succeq^{Cat}_F$ on A s.t. for $a, b \in A$, $a \succeq^{Cat}_F b$ iff $Cat(a) \geq Cat(b)$.*

Pu et al. [20] have shown, that the *h-categoriser ranking-based semantics* is well defined, i. e., an h-categoriser function exists and is unique for every AF.

Example 2. *Given the AF F from Example 1. We can calculate for each argument a strength value using the h-categoriser function. Argument a is unattacked, hence, $Cat(a) = 1$. Based on the value of a, we can calculate the remaining values:*

$$Cat(b) = 0.5;$$
$$Cat(c) = 0.46;$$
$$Cat(d) = 0.69.$$

These values will result in the following ranking:

$$a \succ^{Cat}_F d \succ^{Cat}_F b \succ^{Cat}_F c.$$

So, argument a is ranked highest, then d, and the least ranked argument is c.

Traditionally, the development of ranking-based semantics is guided by a principle-based approach. Each principle embodies a different property for argument rankings. We recall some of the most fundamental principles [5] as well as newer ones, which are closer to the extension-based reasoning process [21]. Before we start, we need additional notations. Let $F = (A, R)$ be an AF with arguments $a, b \in A$. A *path* of length $l_P = n$ between two arguments a, b is a sequence of arguments $P(a, b) = (a, a_1, ..., a_{n-1}, b)$ with $(a_i, a_{i+1}) \in R$ for all i with $a_0 = a$ and $a_n = b$. The *strongly connected components* $cc(F)$ of an AF F are the maximal subgraphs $F' = (A', R')$, where for every pair of arguments $a, b \in A'$ there exists an undirected path $P_u(a, b) = (a = a_0, a_1, ..., a_{n+1}, a_n = b)$ s.t. for every i there is either $(a_i, a_{i+1}) \in R$ or $(a_{i+1}, a_i) \in R$. For an AF $F = (A, R)$ and an extension-based semantics σ, an argument a *weakly σ-supports* b if $b \in cred_\sigma(F)$ and for all $E \in \sigma(F)$, with $b \in E$ then $a \in E$ and a *strongly σ-supports* b if $b \in cred_\sigma(F)$ and for all $E \in \sigma(F)$, with $b \in E$ then there is $E' \in \sigma(F)$ with $E' \subseteq E$, $a \in E'$ and $b \notin E'$.

Definition 6. *A ranking-based semantics ρ satisfies the respective principle iff for all AFs $F = (A, R)$ and any $a, b \in A$:*

Abstraction (Abs). *Names of arguments should not be relevant.*

For a pair of AFs $F = (A, R)$ and $F' = (A', R')$ and every isomorphism $\gamma : A \to A'$, we have $a \succeq^\rho_F b$ iff $\gamma(a) \succeq^\rho_{F'} \gamma(b)$.

Independence (In). *Unconnected arguments should not influence a ranking.*

For every $F' = (A', R') \in cc(F)$ and for all $a, b \in A'$: $a \succeq^\rho_F b$ iff $a \succeq^\rho_{F'} b$.

Void Precedence (VP). *Unattacked arguments should be ranked better then attacked ones.*

If $a^-_F = \emptyset$ and $b^-_F \neq \emptyset$ then $a \succ^\rho_F b$.

Self-Contradicition (SC). *Self-attacking arguments should be ranked worse than any other argument.*

If $(a, a) \notin R$ and $(b, b) \in R$ then $a \succ^\rho_F b$.

[1] A preorder is a (binary) relation that is *reflexive* and *transitive*.

Cardinality Precedence (CP). *Two arguments are compared based on the number of attackers.*

If $|a_F^-| < |b_F^-|$ *then* $a \succ_F^\rho b$.

Quality Precedence (QP). *Two arguments are compared based on the strength of their attackers.*

If there is $c \in b_F^-$ *s.t. for all* $d \in a_F^-$ *it holds that* $c \succ_F^\rho d$ *then* $a \succ_F^\rho b$.

Counter-Transitivity (CT). *Two arguments are compared based on the number and quality of their attackers.*

If some injecitve $f : a_F^- \to b_F^-$ *exists s.t.* $f(x) \succeq_F^\rho x$ *for all* $x \in a_F^-$ *then* $a \succeq_F^\rho b$.

Strict Counter-Transitivity (SCT). *Strict version of* CT.

If some injecitve $f : a_F^- \to b_F^-$ *exists s.t.* $f(x) \succeq_F^\rho x$ *for all* $x \in a_F^-$ *and either* $|a_F^-| < |b_F^-|$ *or there exists some* $x \in a_F^-$ *with* $f(x) \succ_F^\rho x$*, then* $a \succ_F^\rho b$.

Defense Precedence (DP). *For two arguments with the same number of attackers, a defended argument is ranked better than a non-defended argument.*

If $|a_F^-| = |b_F^-|$, $(a_F^-)_F^- \neq \emptyset$ *and* $(b_F^-)_F^- = \emptyset$*, then* $a \succ_F^\rho b$.

Distributed Defense precedence (DDP). *Every defender should attack exactly one attacker.*

If $|a_F^-| = |b_F^-|$ *and* $|(a_F^-)_F^-| = |(b_F^-)_F^-|$*, and if defense of* a *is simple - every direct defender of* a *directly attacks exactly one direct attacker of* a *- and distributed - every direct attacker of* a *is attacked by at most one argument - and defense of* b *is simple but not distributed, then* $a \succ_F^\rho b$.

Non-attacked Equivalence (NaE) *Two unattacked arguments should be equally ranked.*

If $a_F^- = b_F^- = \emptyset$ *then* $a \simeq_F^\rho b$.

Attack vs. Full Defense (AvsFD). *Arguments without any unattacked indirect attackers should be ranked better than arguments only attacked by one unattacked argument.*

If F *acyclic and every path* $P(u, a)$ *in* F *from unattacked* u *to* a *has* $l_p = 0 \bmod 2$ *and there exists unattacked* $v \in b_F^-$*, then* $a \succ_F^\rho b$.

σ-Compatibility (σ-C). *Credulously accepted arguments should be ranked better than rejected arguments.*

For an extension-based semantics σ *it holds that if* $a \in cred_\sigma(F)$ *and* $b \in rej_\sigma(F)$*, then* $a \succ_F^\rho b$.

weak σ-Support (wσ-S). *If an argument* a *is an unavoidable side-effect of accepting another argument* b*, then* a *should be at least as acceptable as* b.

If a *weakly* σ*-supports* b*, then* $a \succeq_F^\rho b$.

strong σ-Support (sσ-S). *If an argument* a *is a prerequisite for accepting another argument* b *and* b *is irrelevant for accepting* a*, then* a *should be ranked better then* b.

If a *strongly* σ*-supports* b*, then* $a \succ_F^\rho b$.

Note that these principles are not always compatible with each other, especially *SC* and *CP* are not compatible [4]. *h-categoriser* satisfies *Abs, In, VP, CT, SCT, DP*, and *NaE*, every other principle from Definition 6 is violated [5, 21].

Extension-ranking Semantics

Extension-ranking semantics defined in [6] are a generalisation of extension-based semantics. These semantics are used to state that a set of arguments is not only jointly acceptable or not, but also whether a set E is more plausible than another set E'.

Definition 7. *Let* $F = (A, R)$ *be an AF. An* extension ranking *on* F *is a preorder over the powerset of arguments* 2^A*. An* extension-ranking semantics τ *is a function that maps each* F *to an extension ranking* $\sqsupseteq_F^\tau$ *on* F.

For an AF $F = (A, R)$, an extension-ranking semantics τ and two sets $E, E' \subseteq A$ we say E is *at least as plausible* as E' with respect to τ in F if $E \sqsupseteq_F^\tau E'$. We define the usual abbreviations as follows:

- E is *strictly more plausible* than E' (denoted as $E \sqsupset_F^\tau E'$) if $E \sqsupseteq_F^\tau E'$ and not $E' \sqsupseteq_F^\tau E$;
- E and E' are *equally as plausible* (denoted as $E \equiv_F^\tau E'$) if $E \sqsupseteq_F^\tau E'$ and $E' \sqsupseteq_F^\tau E$;
- E and E' are *incomparable* (denoted $E \asymp_F^\tau E'$) if neither $E \sqsupseteq_F^\tau E'$ nor $E' \sqsupseteq_F^\tau E$.

Skiba et al. [6] defined a family of approaches to define such extension-ranking semantics. Their semantics are generalisations of the classical extension-based semantics. Using these semantics we can state that a set is "closer" to be admissible, than another set. Skiba et al. [6] argued that the characteristic function does not behave intuitive if applied to conflicting sets. Therefore a variation of the characteristic function $\mathcal{F}^*$ was introduced.

Definition 8. *Let* $F = (A, R)$ *be an AF and* $E \subseteq A$*. The function* $\mathcal{F}_F^* : 2^A \to 2^A$ *is defined as* $\mathcal{F}_F^*(E) = \bigcup_{i=1}^\infty \mathcal{F}_{i,F}^*(E)$ *with*

$$\mathcal{F}_{1,F}^*(E) = E;$$
$$\mathcal{F}_{i,F}^*(E) = \mathcal{F}_{i-1,F}^*(E) \cup \mathcal{F}_F(\mathcal{F}_{i-1,F}^*(E)) \setminus E_F^-.$$

Before we define the semantics, we recall the *base functions*, each of them generalises one aspect of extension-based reasoning.

Definition 9 (Base Functions [6]). *Let* $F = (A, R)$ *be an AF and* $E \subseteq A$. *Define the* base functions $\alpha \in \{CF, UD, DN, UA\}$ *via*

$$CF(E, F) = \{(a, b) \in R | a, b \in E\};$$
$$UD(E, F) = E \setminus \mathcal{F}_F(E);$$
$$DN(E, F) = \mathcal{F}^*_F(E) \setminus E;$$
$$UA(E, F) = \{a \in A \setminus E | \neg\exists b \in E : (b, a) \in R\};$$

and the corresponding α base extension ranking $\sqsupseteq^\alpha_F$ *for* $E, E' \in A$ *via:*

$$E \sqsupseteq^\alpha_F E' \textit{ iff } \alpha(E, F) \subseteq \alpha(E', F)$$

By lexicographically combining these base functions, we denote the extension-ranking semantics.

Definition 10. *Let* $F = (A, R)$ *be an AF and* $E, E' \subseteq A$. *We define:*

- Admissible extension-ranking semantics $r\text{-}ad$ *via* $E \sqsupseteq^{r\text{-}ad}_F E'$ *iff* $E \sqsupset^{CF}_F E'$ *or (*$E \equiv^{CF}_F E'$ *and* $E \sqsupseteq^{UD}_F E'$*).*
- Complete extension-ranking semantics $r\text{-}co$ *via* $E \sqsupseteq^{r\text{-}co}_F E'$ *iff* $E \sqsupset^{r\text{-}ad}_F E'$ *or (*$E \equiv^{r\text{-}ad}_F E'$ *and* $E \sqsupseteq^{DN}_F E'$*).*
- Preferred extension-ranking semantics $r\text{-}pr$ *via* $E \sqsupseteq^{r\text{-}pr}_F E'$ *iff* $E \sqsupset^{r\text{-}ad}_F E'$ *or (*$E \equiv^{r\text{-}ad}_F E'$ *and* $E' \subseteq E$*).*
- Grounded extension-ranking semantics $r\text{-}gr$ *via* $E \sqsupseteq^{r\text{-}gr}_F E'$ *iff* $E \sqsupset^{r\text{-}co}_F E'$ *or (*$E \equiv^{r\text{-}co}_F E'$ *and* $E \subseteq E'$*).*
- Semi-stable extension-ranking semantics $r\text{-}sst$ *via* $E \sqsupseteq^{r\text{-}sst}_F E'$ *iff* $E \sqsupset^{r\text{-}co}_F E'$ *or (*$E \equiv^{r\text{-}co}_F E'$ *and* $E \sqsupseteq^{UA}_F E'$*).*

In words, one set E is at least as plausible as E' with respect to the admissible ranking, if E has fewer conflicts than E' or if E and E' have the same conflicts, then we compare the undefended arguments. For complete we first look at the admissible ranking and in case of equality we look at the DN ranking. Note that since AFs without a stable extension exists it is impossible to define a generalisation of the stable semantics, hence we define a generalisation of the semi-stable semantics instead.

Example 3. *Continuing Example 1. Comparing sets* $E_1 = \{c, d\}$ *and* $E_2 = \{a, c, d\}$ *with the admissible ranking, we see* E *and* E' *have the same conflicts* (c, d) *and* (d, c)*, however* E_2 *defends argument c from b, therefore* $E_2 \sqsupset^{r\text{-}ad}_F E_1$*, so* E_2 *is closer to be admissible, then* E_1*.*

Extension-ranking semantics also follows a principle-based approach. Before we recall the principles defined in [6], we need to introduce the notion of most plausible sets i.e. sets for which we can not find any other sets ranked strictly better.

Definition 11 (Most plausible sets). *Let* $F = (A, R)$ *be an AF,* $E, E' \subseteq A$ *two sets of arguments and* τ *an extension-ranking semantics. We denote by* $min_\tau(F)$ *the minimal (or* most plausible*) elements of the extension ranking* $\sqsupseteq^\tau_F$*, i.e.,*

$$min_\tau(F) = \{E \subseteq A \mid \nexists E' \subseteq A \textit{ with } E' \sqsupset^\tau_F E\}.$$

The principle σ*-generalisation* states, that most plausible sets should coincide with the σ-extensions.

Definition 12 (σ*-Gen*). *Let* σ *be an extension-based semantics and* τ *an extension-ranking semantics.* τ *satisfies*

- σ-soundness *iff for all* $F = (A, R)$*:* $min_\tau(F) \subseteq \sigma(F)$*.*
- σ-completeness *iff for all* $F = (A, R)$*:* $min_\tau(F) \supseteq \sigma(F)$*.*
- σ-generalisation *iff* τ *satisfies both* σ*-soundness and* σ*-completeness.*

The next two properties (*composition* and *decomposition*) state that unconnected arguments should not influence the ranking.

Definition 13 (*(De)Comp*). *Let* τ *be an extension-ranking semantics.* τ *satisfies* composition *if for every* F *s.t.* $F = F_1 \cup F_2 = (A_1, R_1) \cup (A_2, R_2)$ *with* $A_1 \cap A_2 = \emptyset$ *and* $E, E' \subseteq A_1 \cup A_2$ *it holds that if* $\left\{ \begin{array}{l} E \cap A_1 \sqsupseteq^\tau_{F_1} E' \cap A_1 \\ E \cap A_2 \sqsupseteq^\tau_{F_2} E' \cap A_2 \end{array} \right\}$ *then* $E \sqsupseteq^\tau_F E'$*.*

τ *satisfies* decomposition *if for every* F *s.t.* $F = F_1 \cup F_2 = (A_1, R_1) \cup (A_2, R_2)$ *with* $A_1 \cap A_2 = \emptyset$ *and* $E, E' \subseteq A_1 \cup A_2$ *it holds that if* $E \sqsupseteq^\tau_F E'$ *then* $\left\{ \begin{array}{l} E \cap A_1 \sqsupseteq^\tau_{F_1} E' \cap A_1 \\ E \cap A_2 \sqsupseteq^\tau_{F_2} E' \cap A_2 \end{array} \right\}$.

The *reinstatement* principles are used to establish that the addition of defended arguments, which do not create new conflicts, is preferred.

Definition 14 (*RI*). *Let* τ *be an extension-ranking semantics.* τ *satisfies* weak reinstatement *iff for all* $F = (A, R)$ *with* $E \subseteq A$ *holds* $a \in \mathcal{F}_F(E)$*,* $a \notin E$ *and* $a \notin (E^-_F \cup E^+_F)$ *implies* $E \cup \{a\} \sqsupset^\tau_F E$*.*

τ *satisfies* strong reinstatement *iff for all* $F = (A, R)$ *with* $E \subseteq A$ *holds* $a \in \mathcal{F}_F(E)$*,* $a \notin E$ *and* $a \notin (E^-_F \cup E^+_F)$ *implies* $E \cup \{a\} \sqsupset^\tau_F E$*.*

Names of arguments should not influence the ranking as stated by *syntax independence.*

Definition 15 (*SI*). *An extension-ranking semantics τ satisfies* syntax independence *if for every pair of AFs $F = (A, R)$, $F' = (A', R')$ and for every isomorphism $\gamma : A \to A'$, for all $E, E' \subseteq A$, we have $E \sqsupseteq^{\tau}_{F} E'$ iff $\gamma(E) \sqsupseteq^{\tau}_{F'} \gamma(E')$.*

The extension-ranking semantics based on semantics $\sigma \in \{ad, co, pr, gr, sst\}$ do satisfy their corresponding *σ-Gen, Comp, DeComp, SI* and *wRI.* $\{r\text{-}co, r\text{-}pr, r\text{-}gr, r\text{-}sst\}$ also satisfies *sRI* [6].

3. Social Ranking Problem

Like shown by Skiba et al. [6] extension-ranking semantics and ranking-based semantics are related to each other. We use extension-ranking semantics and their resulting preorders to construct a ranking over arguments, where arguments are ranked based on the strength of the corresponding sets.

Definition 16 ([6]). *Let $F = (A, R)$ be an AF, $a, b \in A$, and τ be an extension-ranking semantics. We define an ranking-based semantics $\succeq^{\tau}_{F}$ via $a \succeq^{\tau}_{F} b$ iff there is a set E with $a \in E$ s.t. for all sets E' with $b \in E'$ we have $E \sqsupseteq^{\tau}_{F} E'$.*

In words, an argument a is at least as plausible as b if a is contained in a set E, which is ranking better than any set E' containing b.

Example 4. *Continuing with Example 1. Using $r\text{-}ad$ as the underlying extension-ranking semantics, we see that $\{a, c\}$ and $\{a, d\}$ are admissible sets. Since $r\text{-}ad$ satisfies ad-Gen, we know that there can not be any set ranked better than these two sets and especially no set containing b is ranked better. This observation result in the ranking:*

$$a \simeq^{r\text{-}ad}_{F} c \simeq^{r\text{-}ad}_{F} d \succ^{r\text{-}ad}_{F} b.$$

Since $\{a, c\}$ and $\{a, d\}$ are also complete, preferred and semi-stable sets, we see that $r\text{-}co$, $r\text{-}pr$, and $r\text{-}sst$ do induce the same ranking. Only for $r\text{-}gr$ the induced ranking differs:

$$a \succ^{r\text{-}gr}_{F} c \simeq^{r\text{-}gr}_{F} d \succ^{r\text{-}gr}_{F} b.$$

In the example above we see that the credulously accepted arguments w.r.t. admissible semantics are the best ranked arguments. This holds for every AF, if the most plausible sets coincide with the admissible sets, hence there is at least one set that contains a credulously accepted argument and is ranked most plausibly. Similar observations can be done for the complete, grounded, preferred, and semi-stable semantics and their corresponding extension-ranking semantics. By comparing the resulting rankings from Example 2 and Example 4, we see that these two rankings are quite different. They have in common that a is one of the best ranked arguments, however these two semantics disagree on the strength of argument b quite strongly. In the ranking induced by $r\text{-}ad$, b is the weakest argument, while in the ranking based on h-categoriser, c is the weakest argument. Hence, the discussion about counter-intuitive behaviour of ranking-based semantics from Blümel and Thimm [21] can be recalled. Blümel and Thimm argued that credulously accepted arguments with respect to an extension-based semantics should be ranked better, than rejected arguments. Hence, c should be ranked better than b.

Next, we investigate the principles the ranking induced by an extension-ranking semantics does satisfy. Skiba et al. [6] have discussed *Abs*, *In* and *SC* and have stated that $\succeq^{\tau}$ for $\tau \in \{r\text{-}ad, r\text{-}co, r\text{-}gr, r\text{-}pr, r\text{-}sst\}$ satisfies these principles.

Starting with *Abs*, we see that the induced ranking is not influenced by the names of the arguments if the underlying extension-ranking semantics is also not influenced by the names.

Proposition 1. *If extension-ranking semantics τ satisfies* SI*, then $\succeq^{\tau}$ satisfies* Abs.

Proof. Let $F = (A, R)$ be an AF, $E, E' \subseteq A$ and τ an extension-ranking semantics. Assume τ satisfies syntax independence, then we know that for every isomorphism γ we have $E \sqsupseteq^{\tau}_{F} E'$ it also hold that $\gamma(E) \sqsupseteq^{\tau}_{\gamma(F)} \gamma(E')$. Hence, if a set E exists s.t. for all E' we have $E \sqsupseteq^{\tau}_{F} E'$ the same behaviour also happens in the isomorphic AF. So, for all pairs of arguments $a, b \in A$ with $a \succeq^{\tau}_{F} b$ it holds that $\gamma(a) \succeq^{\tau}_{\gamma(F)} \gamma(b)$. □

The connection between *In*, *Comp* and *DeComp* is apparent. *In* stats that unconnected arguments should not influence each others, while *DeComp* stats that by splitting a disjoint AF into two AFs, the resulting ranking should stay the same.

Proposition 2. *If extension-ranking semantics τ satisfies* Comp *and* DeComp*, then $\succeq^{\tau}$ satisfies* In.

Proof. Let $F = (A, R)$ be an AF, $E, E' \subseteq A$ and τ an extension-ranking semantics. Assume τ satisfies composition and decomposition, then for any AF, which can be partitioned into two disjoint AFs, we know that the relationship between E and E' is still the same in the subAFs. So, if we split an AF into its connected components, we see that for every pair of sets of arguments the relationship is still the same. Hence, the induced ranking is also the same. □

The main motivation of extension-ranking semantics was to generalise the extension-based reasoning process and the first step of this reasoning process is always the conflict-freeness check. A set can only be accepted, if it is conflict-free otherwise this set will be rejected by any extension-based semantics. Similar for the extension-ranking semantics the first step is to compare two sets of arguments based on their conflicts. A conflict-free set is always ranked better than a conflicting set. Since the addition of a self-attacking argument into a set always coincides with a conflicting set, we see that any self-attacking argument will be ranked worse, than any non self-attacking argument.

Proposition 3. *If extension-ranking semantics τ satisfies* cf-soundness, *then $\succeq^\tau$ satisfies* SC.

Proof. Let $F = (A, R)$ be an AF, $E, E' \subseteq A$ and τ an extension-ranking semantics. Assume τ satisfies cf-soundness, then for any non self-attacking argument a and for any self-attacking argument b, we have $\{a\} \sqsupset^\tau_F E'$ for any E' with $b \in E'$, since $CF(\{a\}, F) = \emptyset$ and $CF(E', F) \neq \emptyset$. So, the induced ranking will be $a \succeq^\tau_F b$. □

Based on the satisfaction of *SC*, we know that $\succeq^\tau$ violates *CP*, *CT* and *SCT* [4].

Proposition 4. *If extension-ranking semantics τ satisfies* cf-soundness, *then $\succeq^\tau$ violates* CP, CT *and* SCT.

Since two credulously accepted arguments are always ranked equally in the induced ranking, we can not identify whether one argument is attacked or not. Hence, *VP* is violated.

Example 5. *Let*

$$F = (\{a, b, c\}, \{(a, b), (b, c)\}$$

be an AF and τ any extension-ranking semantics satisfying ad-soundness. We know that $\{a, c\}$ is an admissible extension and therefore we have

$$a \simeq^\tau_F c \succ^\tau_F b.$$

However, $a^-_F = \emptyset$ and $c^-_F = \{b\}$ and this violates VP.

Thimm and Kern-Isberner [22], proposed a weak version of VP, *where equivalence is enough. We see that $\succeq^\tau$ satisfies the weak version.*

Similar to the example for showing the violation of *VP*, we can show that two arguments are ranked equally despite their attackers not being equally strong, which violates *QP*.

Example 6. *Let*

$$F = (\{a, b, c, d, e\}, \{(e, d), (d, d), (d, a), (c, b), (b, c)\})$$

depicted in Figure 2 and τ any extension-ranking semantics satisfying cf-soundness, then since d is self-attacking, we know that $c \succ^\tau_F d$. So, the attacker of b is stronger than the attacker of a, therefore QP *would imply $a \succ^\tau_F b$. However, $a, b \in cred_{ad}(F)$ and therefore $a \simeq^\tau_F b$. This is a contradiction to* QP.

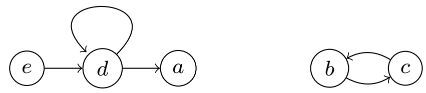

Figure 2: Counterexample for *QP* from Example 6.

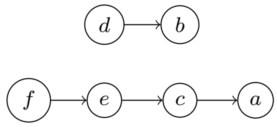

Figure 3: Counterexample for *DP* from Example 7.

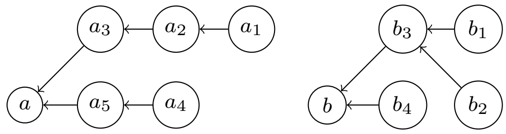

Figure 4: Counterexample for *DDP* from Example 8.

Two arguments, which are rejected and not self-attacking, can be ranked equally. This fact entails the violation of *DP*. Even-though one argument has a defender while the other one has none, both these arguments can be rejected.

Example 7. *Let*

$$F = (\{a, b, c, d, e, f\}, \{(c, a), (e, c), (f, e), (d, b)\}$$

depicted in Figure 3 and

$$\tau \in \{r\text{-}cf, r\text{-}ad, r\text{-}co, r\text{-}pr, r\text{-}gr, r\text{-}sst\}.$$

Then $|a^-_F| = |b^-_F| = 1$, while a has one defender e and b has no defender. DP would entail $a \succ^\tau_F b$, however the induced ranking entails $a \simeq^{r\text{-}cf}_F b$ respectively $a \bowtie^\tau_F b$ for the remaining semantics.

The idea of *DDP* is that every attacker should be defeated by exactly one defender. This behaviour can not be depicted, while using extension-ranking semantics, since the number of defenders is not relevant for the acceptance of an argument. An argument is credulous accepted w.r.t. admissible semantics if we find at least one set of defenders for this argument, it is not important that these defenders are simple or distributed. Hence, *DDP* is violated.

Example 8. *Let F be the AF depicted in Figure 4 and*

$$\tau \in \{r\text{-}cf, r\text{-}ad, r\text{-}co, r\text{-}pr, r\text{-}gr, r\text{-}sst\}.$$

For a, b it holds that the number of attackers and defenders are the same. However, the defense of a is distributed and simple, while the defense of b is only simple. So, if DDP *is satisfied it has to hold that $a \succeq^{\tau}_{F} b$. For $\tau = r\text{-}cf$, we have that both $\{a\}$ and $\{b\}$ are conflict-free, hence both these sets are most plausible sets and therefore we have $a \simeq^{\tau}_{F} b$, which violates* DDP.

Let $\tau = r\text{-}ad$, we know that

$$S = \{a_1, a_3, a_4, b_1, b_2, b_4\}$$

and its subsets are the admissible extensions and therefore also the most plausible sets. In particular, $a, b \in rej_{ad}(F)$. For set $\{a\}$ we have $CF(\{a\}, F) = \emptyset$ and $UD(\{a\}, F) = \{a\}$ and for $\{b\}$ we have $CF(\{b\}, F) = \emptyset$ and $UD(\{b\}, F) = \{b\}$, so there can not be a set $E \not\subseteq S$ with $E \succ^{r\text{-}ad}_{F} \{a\}$, otherwise this entails $CF(E) = \emptyset$ and $UD(E) \subset UD(\{a\})$, which means $UD(E) = \emptyset$. However, Skiba et al. [6] have shown that this entails $E \in ad(F)$ meaning $E \subseteq S$. Similar can be reason for $\{b\}$. Hence, $\{a\}$ and $\{b\}$ are on the second level, but these two sets are incomparable, which entails $a \bowtie^{\tau}_{F} b$ and DDP *is violated.*

For the remaining extension-ranking semantics $\tau \in \{r\text{-}co, r\text{-}pr, r\text{-}gr, r\text{-}sst\}$, a and b are incomparable, since all these semantics are based on the admissible one. Hence, DDP *is violated.*

AvsFD can be interpreted as an admissible credulously accepted argument should be ranked better than any rejected argument. So, the induced argument ranking satisfies this principle.

Proposition 5. *If extension-ranking semantics τ satisfies ad-soundness, then $\succeq^{\tau}$ satisfies* AvsFD.

Proof. Let $F = (A, R)$ be an AF and τ an extension-ranking semantics. Assume τ satisfies ad-soundness, then for any argument a without an unattacked indirect attacker, we know that a has to be admissible credulous accepted and an argument b with an unattacked attacker is admissible rejected. So, there exists $E \in min_{ad}(F)$ with $a \in E$ such that for every $E' \subseteq A$ with $b \in E'$ we have $E \sqsupset^{\tau}_{F} E'$, this implies $a \succ^{\tau}_{F} b$. □

For an ranking-based semantics to satisfy *NaE*, two unattacked argument should be handled equally. To recap, unattacked arguments are always admissible credulously accepted, and therefore this principle is satisfied by ranking-based semantics defined based on Definition 16.

Proposition 6. *If extension-ranking semantics τ satisfies ad-soundness, then $\succeq^{\tau}$ satisfies* NaE.

Proof. Let $F = (A, R)$ be an AF and τ an extension-ranking semantics. Assume τ satisfies ad-soundness and a, b are unattacked arguments. Since a and b are unattacked, we know that they are admissible credulous accepted and there are sets $E \in min_{ad}(F)$ and $E' \in min_{ad}(F)$ with $a \in E$ and $b \in E'$, this implies $a \simeq^{\tau}_{F} b$. □

The main motivation of extension-ranking semantics was to generalise the extension-based reasoning. The goal was to keep σ-extensions intact and compare the remaining sets. For every credulously accepted argument, we can find one σ-extension containing that argument. Hence, $\succeq^{\tau}$ satisfies σ-C.

Proposition 7. *If extension-ranking semantics τ satisfies σ-Gen, then $\succeq^{\tau}$ satisfies σ-*C.

Proof. Let $F = (A, R)$ be an AF and τ an extension-ranking semantics. Assume τ satisfies σ-generalisation, then for any argument $a \in A$ that is σ credulously accepted, there is a set $E \in min_{\sigma}(F)$ with $a \in E$ and for every σ rejected argument b there is no set E' s.t. $E' \in min_{\sigma}(F)$ and $b \in E'$. This implies that $E \sqsupset^{\tau}_{F} E'$ and therefore $a \succ^{\tau}_{F} b$. □

If the acceptance of an argument is the unavoidable side effect of accepting another argument, then both these arguments are credulously accepted and ranked equally. Hence, wσ-S is satisfied.

Proposition 8. *If extension-ranking semantics τ satisfies σ-Gen, then $\succeq^{\tau}$ satisfies* wσ-S.

Proof. Let $F = (A, R)$ be an AF and τ an extension-ranking semantics. Assume τ satisfies σ-generalisation, if b is credulously accepted wrt. σ and for all $E \in \sigma(F)$ with $b \in E$ we have $a \in E$, then a has to be credulously accepted as well. This implies that there exists a set $E \in min_{\sigma}(F)$ with $a, b \in E$ and therefore we can entail that $a \simeq^{\tau}_{F} b$. □

As shown in the proof for Proposition 8 we know that two credulously accepted arguments are ranked equally strong with respect to $\succeq^{\tau}$ and therefore *sσ-S* is violated.

Since a number of principles are violated a discussion about Definition 16 is evident. One obvious change of this definition is to change the quantifiers used. So, for an AF $F = (A, R)$, a extension-ranking semantics τ and $a, b \subseteq A$ instead of searching for one set E with $a \in E$ it has to hold for all sets E with $a \in E$ that there is a set E' with $b \in E'$ such that $E \sqsupset^{\tau}_{F} E'$. The problem with this definition is that the resulting ranking is flat, i.e. it has only one level, since the set $E' = A$ is always ranked at least as bad as any other set. Hence, this change results in an even worse ranking. Besides Definition 16 a number of different social ranking approaches can be found in the literature like in [13, 14, 15, 16]. A detailed analysis of the resulting ranking-based semantics will be done in future work.

Another interesting observation is that the induced ranking based on an extension-ranking semantics are in a sense a generalisation of credulous acceptance. Every credulously accepted argument is ranked best, however we can say even more. If we look at the ranking induced by the grounded extension-ranking semantics in Example 4, we see that argument $a \in cred_{gr}(F)$, while $b, c, d \notin cred_{gr}(F)$. However, we can still compare c and d with b and can state that c and d are stronger arguments than b. Such a statement is not possible with classical credulously acceptance.

4. Lifting to Extension-ranking Semantics

A number of approaches to *lift* a ranking over objects into a ranking over sets of objects were already discussed in the area of computation social choice [23] and applied to structured argumentation [8] or preference-based argumentation frameworks [9, 10, 11, 12]. For both these frameworks, the preference data is part of the input and not based on the structure of the argumentation framework. The goal of this section is to discuss an approach to define lifting operators without the need of additional information like preference data.

Definition 17 (Lifting Operator)**.** *For an AF $F = (A, R)$ an* lifting operator *ι takes as input a set of arguments $E \subseteq A$ and a ranking-based semantics ρ on E and outputs an extension-ranking semantics $\sqsupseteq^{\rho}_F$ on the powerset of E.*

Lifting operators were already discussed in the context of structured argumentation and in particular for the ASPIC$^+$ framework [24]. Maly and Wallner [8] have shown that the *elitist lifting operator* satisfies a number of interesting principles in the structured case. The elitist lifting operator states that for a set E to be preferred over another set E', there has to be an argument a in E which is ranked better than any argument in E' wrt. a ranking-based semantics ρ.

Definition 18. *Let $F = (A, R)$ be an AF, ρ and ranking-based semantics and $E, E' \subseteq A$. We define an extension-ranking semantics $\sqsupseteq^{\rho\text{-}eli}$ by: $E \sqsupseteq^{\rho\text{-}eli}_F E'$ iff there is an argument $a \in E$ s.t. for all arguments $b \in E'$ we have $a \succeq^{\rho}_F b$.*

In a sense, the elitist operator focuses on the *best* argument of each set.

Example 9. *Let us continue with Example 1. The h-categoriser function returns*

$$a \succ^{Cat}_F d \succ^{Cat}_F b \succ^{Cat}_F c,$$

hence applying $\sqsupseteq^{Cat\text{-}eli}_F$ will return an extension-ranking for F. Looking at the sets $\{d\}$ and $\{a, b\}$, we see that $\{a, b\} \sqsupset^{Cat\text{-}eli}_F \{d\}$, since $a \succ^{Cat}_F d$.

The example shows a few shortcomings. Since $\{a, b\} \sqsupset^{Cat\text{-}eli}_F \{d\}$, we know that cf-soundness is violated. If we focus on extensions alone, then conflict-freeness is the most basic propriety for the acceptance of a set. In addition to the violation of cf-soundness, we see that the set containing all arguments A is ranked highly, despite being the most conflicting set. If we follow the justification of Skiba et al. [6], we argue that this is a disadvantage of this approach. However, the goal of the work of Skiba et al. was to generalise the extension-based reasoning process. Blümel and Thimm [21] have discussed the counter-intuitive behaviour of ranking-based semantics from the viewpoint of extension-based reasoning. The majority of the ranking-based semantics used in the literature do not satisfy *σ-C*, hence it is not surprising, that $\sqsupseteq^{Cat\text{-}eli}$ is not a generalisation of extension-based reasoning.

Next, we discuss the principles $\sqsupseteq^{Cat\text{-}eli}_F$ does satisfy. For *SI* we see easily, that $\sqsupseteq^{Cat\text{-}eli}_F$ satisfies this principle, since the names of the arguments are not important. We show a more general statement.

Proposition 9. *If the underlying ranking-based semantics ρ satisfies* Abs*, then $\sqsupseteq^{\rho\text{-}eli}_F$ satisfies* SI.

Proof. Let $F = (A, R)$ be an AF, ρ a ranking-based semantics, which satisfies abstraction, and two arguments $a, b \in A$, then for any isomorphism $\gamma(F)$ it holds that if $a \succeq^{\rho}_F b$, then $\gamma(a) \succeq^{\rho}_{\gamma(F)} \gamma(b)$. Hence, for two sets $E, E' \subseteq A$ we have if $E \sqsupseteq^{\rho\text{-}eli}_F E'$, then there is an argument $a \in E$ s.t. $a \succeq^{\rho}_F b$ and this argument a also exists in $\gamma(F)$ therefore $\gamma(E) \sqsupseteq^{\rho\text{-}eli}_{\gamma(F)} \gamma(E')$. □

In Example 9, we have seen that $\sqsupseteq^{Cat\text{-}eli}_F$ does not satisfies cf-soundness and therefore this semantics can not satisfy *σ-Gen* for $\sigma \in \{ad, co, gr, pr, sst\}$. In order to show that one set of arguments is ranked better than another one, we only have to find one argument in the first set which is better than any argument in the second set. This observation yield to conflicting sets being ranked highly, if these sets contain one highly ranked argument, especially the set containing all arguments A will be ranked as a most plausible set. This shows that for no ranking-based semantics ρ the induced extension-ranking semantics $\sqsupseteq^{\rho\text{-}eli}_F$ satisfies cf-soundness.

For every set E, $\sqsupseteq^{Cat\text{-}eli}_F$ focuses only on the best ranked element, hence this best ranked argument dominates the whole set. This domination of one argument also holds in disjoint AFs. Hence, we can show that $\sqsupseteq^{Cat\text{-}eli}_F$ satisfies *Comp*. We can show an even stronger proposition.

Proposition 10. *If the underlying ranking-based semantics ρ satisfies* In*, then $\sqsupseteq^{\rho\text{-}eli}_F$ satisfies* Comp.

Proof. Let F be an AF s.t. $F = F_1 \cup F_2 = (A_1, R_1) \cup (A_2, R_2)$ with $A_1 \cap A_2 = \emptyset$, $E, E' \subseteq A_1 \cup A_2$ and ρ an ranking-based semantics satisfying independence. Assume $E \sqsupseteq^{\rho\text{-}eli}_{F_1} E'$ and $E \sqsupseteq^{\rho\text{-}eli}_{F_2} E'$, so there is an argument $a \in E \cap A_1$ s.t. $a \succeq^{\rho}_{F_1} b$ for all $b \in E' \cap A_1$ and there is an argument $c \in E \cap A_2$ s.t. $c \succeq^{\rho}_{F_2} d$ for all $d \in E' \cap A_2$. So $\{a, c\} \subseteq E$, assume $E' \sqsupset^{\rho\text{-}eli}_{F} E$, then there has to be an argument $e \in E'$ s.t. $e \succ^{\rho}_{F} a$ and $e \succ^{\rho}_{F} c$, since ρ satisfies independence, we know that if $e \succ^{\rho}_{F} a$ then $e \succ^{\rho}_{F_1} a$, same holds for c. That means there is an argument e, which is ranked better than a in F_1, hence a can not be ranked better than every argument in E' and therefore $E \not\sqsupseteq^{\rho\text{-}eli}_{F_1} E'$, which is a contradiction to our assumption and proofing that *composition* is satisfied. □

While one dominating argument is helpful for satisfying *Comp*, this behaviour violates *DeComp*.

Example 10. *Let* $F = (\{a, b, c, d\}, \{(a, b), (c, d)\})$ *and the resulting ranking for every ranking-based semantics* ρ*, which satisfies* VP *like h-categoriser is:*

$$a \simeq^{\rho}_{F} c \succ^{\rho}_{F} b \simeq^{\rho}_{F} d.$$

Let us look at the two sets $E = \{a, d\}$ *and* $E' = \{b, c\}$*, then these two sets are equally ranked in* F *wrt.* $\sqsupseteq^{\rho\text{-}eli}_{F}$*. However, if we split this AF into its connected components* $F_1 = (\{a, b\}, \{(a, b)\})$ *and* $F_2 = (\{c, d\}, \{(c, d)\})$*, then we see that* $E \sqsupset^{\rho\text{-}eli}_{F_1} E'$ *and* $E' \sqsupset^{\rho\text{-}eli}_{F_2} E$ *and therefore* DeComp *is violated.*

Before we discuss the reinstatement principles, we have to discuss the empty set. The elitist lifting operator is not applicable for the empty set. Hence, the empty set has to be handled differently. Modgil and Prakken [24] argued that the empty set should not be ranked better than any non-empty set and any non-empty set should be ranked better than the empty set. In the context of extension-based reasoning we can argue that this should not be the case, since the empty set is always an admissible extension and if *ad-Gen* should be satisfied, then the empty set should be among the best ranked sets. However, in Example 9 we have seen that $\sqsupseteq^{Cat\text{-}eli}_{F}$ does not satisfies cf-soundness and therefore will not satisfy any version of σ-*Gen*. Investigating $\sqsupseteq^{\rho\text{-}eli}_{F}$ further, we see that for every non-empty set E every superset of E is at least as plausible as E. Following this observation, then we should agree with Modgil and Prakken and rank the empty set as the least preferred set. In the remainder of this section, we will discuss both approaches to handle the empty set.

Using the fact that for every non-empty set E their supersets are ranked at least as good as E, we see that adding any argument a into E the ranking of E will not get worse and this implies that *wRI* is satisfied.

Proposition 11. *If* ρ *satisfies* VP *and* NaE*, then* $\sqsupseteq^{\rho\text{-}eli}_{F}$ *satisfies* wRI.

Proof. Let $F = (A, R)$ be an AF, ρ any ranking-based semantics and $E \subseteq A$. We know that for every superset E' of E it holds that $E' \sqsupseteq^{\rho\text{-}eli}_{F} E$, since every argument in E is also in E', therefore if an argument a from E is ranked better any argument inside another set E'' and we can follow that $E \sqsupseteq^{\rho\text{-}eli}_{F} E''$, we know that $a \in E'$ and therefore also $E' \sqsupseteq^{\rho\text{-}eli}_{F} E''$. Since $E \cup \{a\}$ for $a \in \mathcal{F}_F(E)$, $a \notin E$ and $a \notin (E^-_F \cup E^+_F)$ is a superset of E, we know that weak reinstatement is satisfied.

Next, we discuss the two variations of handling $\emptyset$, the empty set is either among the best ranked sets or the worst ranked set. If $E = \emptyset$ and $E' \sqsupset^{\rho\text{-}eli}_{F} E$ for every non-empty set $E' \subseteq A$, then weak reinstatement is satisfied, since every set is ranked better then $\emptyset$. Especially the set $\{a\}$, with $a \in \mathcal{F}_F(E)$, $a \notin E$ and $a \notin (E^-_F \cup E^+_F)$. If $E = \emptyset$ and $E \sqsupseteq^{\rho\text{-}eli}_{F} E'$ for every E', then for $a \in \mathcal{F}_F(E)$, $a \notin E$ and $a \notin (E^-_F \cup E^+_F)$, we know that $a^-_F = \emptyset$ otherwise $a \notin \mathcal{F}_F(E)$. If ρ satisfies *void-precedence* and *non-attack equivalence*, then there is no $b \in A$ s.t. $b \succ^{\rho}_{F} a$ this implies that for every set E' containing a, we know that $E' \sqsupseteq^{\rho\text{-}eli}_{F} E''$ for any set $E'' \subseteq A$ and especially $E'' = \emptyset$. This implies that weak reinstatement is satisfied. □

While $\sqsupseteq^{Cat\text{-}eli}_{F}$ satisfies *wRI*, we can easily see that *sRI* is violated.

Example 11. *Let* $F = (\{a, b, c\}, \{(a, b), (b, c)\}$ *be an AF and the resulting ranking based on h-categoriser is* $a \succ^{Cat}_{F} c \succ^{Cat}_{F} b$ *and this results in* $\{a\} \equiv^{Cat\text{-}eli}_{F} \{a, c\}$*, despite the fact that* c *is defended by* $\{a\}$ *and does not attack* a*. Hence,* sRI *is violated.*

Looking back at the proofs, we see that there is no difference between ranking the empty set as the best or the worst set w.r.t. the satisfied principles. Hence, the position of the empty set should be based on the application. If admissible sets should be ranked highly, then the empty set should be ranked highly.

5. Discussion

In this work, we discussed the connection between ranking-based semantics and extension-ranking semantics. We proposed approaches to transform extension-ranking semantics into ranking-based semantics (*social ranking*) and the other way around, from ranking-based semantics to extension-ranking semantics (*liftings*). Additionally, we analysed the properties of the resulting semantics based on principles out of literature for ranking-based and extension-ranking semantics.

Combining ranking-based semantics and extension-based reasoning was already discussed in [25]. One of

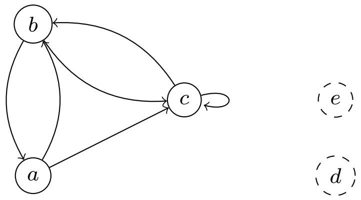

Figure 5: AF for Example 12, where arguments d and e are added later.

the goals of the authors was to refine the extension-based reasoning process with the help of ranking-based semantics. The strength of a set is the aggregated strength of each argument contained inside that set. One way to establish the strength of each argument are ranking-based semantics and the resulting rankings over arguments. Based on the number of arguments ranked better than an argument, the strength of that argument is established.

Example 12. *Let*

$$F = (\{a, b, c\}, \{(a, b), (b, a), (a, c), (b, c), (c, b), (c, c)\})$$

as depicted in Figure 5 be an AF. The corresponding ranking based on h-categoriser is $a \succ_F^{Cat} b \succ_F^{Cat} c$*. Argument* a *receives a score of* 0*,* b *a score of* 1 *and* c *a score of* 2*. If we sum up the strength values of each argument inside set we can establish the strength of each set. For example, set* $E = \{a, b\}$ *has the value of* 1 *while set* $E' = \{c\}$ *has the value of* 2*, meaning* E *should be ranked better than* E'*. If we add two arguments* d, e*, which are independent from any other argument i.e. they do not attack* a, b *or* c *nor are attacked by these arguments, the resulting ranking is:*

$$d \simeq_F^{Cat} e \succ_F^{Cat} a \succ_F^{Cat} b \succ_F^{Cat} c.$$

However, for the two sets E *and* E' *we see that* E *has the aggregate strength value of* $2 + 3 = 5$ *and for* E' *the score is* 4*, meaning that* E' *should be ranked better than* E*. Hence, this ranking is influenced by unconnected and independent elements, which can be seen as a big disadvantage. This example shows that aggregation of the strength values of each argument inside a set is possible, however the resulting ranking has undesired properties. Hence, a discussion about the used ranking-based semantics and aggregation functions is needed.*

Finding the "best" extensions is one of the focal points for discussing *preference-based argumentation frameworks* (short *PAF*) [26], which are extensions of AFs, where in addition to sets of arguments and attacks, a preorder $\geq$ over the set of arguments is given. Using these preorders the "best" extensions can be found with the help of operators similar to Definition 18 [9, 10, 11, 12]. The difference between *PAFs* and our work is that *PAF* receive the rankings as a fixed input. This ranking is not based on the behaviour of the arguments or the structure of the underlying AFs. Additionally, *PAFs* are only focused on σ-extensions, while our work focuses on the powerset of arguments.

The semantics proposed in this work have some undesirable behaviour. For the social ranking case, we receive a generalisation of credulous acceptance, however a number of desirable principles are violated. For the lifting case, the most conflicting set is ranked among the best sets. Additionally, applying these two semantics one after another result in a flat ranking, i.e. every argument resp. set of arguments are ranked equally. These behaviours can be used as a motivation to define new social ranking solutions and lifting operators specifically tailored to abstract argumentation.

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